Branes and the Swampland

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This conference is part of the String Geometry and String Phenomenology Institute (17-28 June).

The annual String Phenomenology conference discusses recent progress in compactifications of string theory and their relation to particle physics and cosmology.

Topics include:

- Swampland and Quantum Gravity Conjectures
- Formal and Mathematical Aspects of string compactifications (such as F-theory, G2 compactifications, heterotic compactifications, and other corners of the string landscape, e.g. non-geometric compactifications)
- String Model Building in particle physics and cosmology
- Machine Learning Techniques to explore the String Landscape
In this talk, I’ll discuss our recent work in using strings/branes to probe the consistency of EFTs coupled to gravity:

- **Branes and the Swampland** [Kim, GS, Vafa, 1905.08261]
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In the interest of time, I won’t be able to cover other interesting works on:

- **Weak Gravity Conjecture (WGC):**
  - WGC, Black Hole Entropy, & Modular Invariance [Aalsma, Cole, GS, ’19]
  - WGC from Unitarity and Causality [Hamada, Noumi, GS, 18]
  - Tower WGC [Andriolo, Junghans, Noumi, GS, ’18] (see Andriolo’s talk)
  - Strong WGC & Modular Bootstrap [Montero, GS, ’19] (see Montero’s talk)

- **de Sitter in String theory**
  - Distance and de Sitter Conjectures [Ooguri, Palti, GS, Vafa, ’18]
  - Understanding KKLT from a 10d perspective [Hamada, Hebecker, GS, Soler, ’18, ’19] (see Hebecker & Soler’s talks)

- **Data Science and String Theory:**
  - Topological Data Analysis [Cole, GS, ’18] and Genetic Algorithm [Cole, Schachner, GS, ’19] (see Schachner’s talk) for the string landscape.
Landscape vs Swampland
Landscape vs Swampland

Landscape

$10^{500}$ IIB flux vacua

[Douglas, '03]
Landscape vs Swampland

Swampland
[Vafa, ’05]

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What properties delineate the landscape from the swampland?
What are the phenomenological implications? *Is there a swampland?*
Landscape vs Swampland

Swampland
[Vafa, ’05]

Landscape
$10^{272,000}$ F-theory vacua
[Taylor, Wang, ’15]

What properties delineate the landscape from the swampland?
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Swampland Criteria

There are varying degrees of understanding for different swampland criteria and their interconnections:

Weak Gravity Conjecture
[Arkani-Hamed, Motl, Nicolis, Vafa. ’06]

Distance Conjecture
[Ooguri, Vafa. ’06]

dS Conjecture
[Obied, Ooguri, Spodyneiko, Vafa,’18];
[Ooguri, Palti, GS, Vafa, ’18]

These criteria do not follow from purely low-energy EFT considerations. Why are they necessary for consistency of quantum gravitational theories?
Branes and the Swampland

[Kim, GS, Vafa, ’19]

“Putting strings back into string pheno”
Branes and the Swampland

- Completeness of spectrum of charged branes [Polchinski ’03], [Banks, Seiberg, ’10]: use them to probe consistency of EFTs coupled to gravity.

- First consider N=(1,0) SUGRA theories in 10d & 6d as gauge and gravitational anomaly cancellations severely limit the possibilities.

- We illustrate the power of this approach with just a few examples and with only string probes but we expect this program of using brane probes to understand swampland criteria has wider applicabilities.

- We showed the 10d anomaly-free theories with E₈ x U(1)²⁴₈ and U(1)⁴⁹₆ gauge groups that have no string realizations are in the swampland.

- Infinite families of anomaly-free 6d theories [Kumar, Morrison, Taylor, ’10] with unbounded gauge group rank, or unbounded number of tensors or matter in exotic representations. We showed that unitarity of current algebra on string probes can rule out some of these infinite families.
Strings in 10d N=(1,0) SUGRA
Anomaly Cancellation in 10d

- The gauge and gravitational anomalies of 10d N=(1,0) SUGRA theories can be cancelled by the Green-Schwarz mechanism [Green, Schwarz, ’84], allowing only 4 choices for gauge groups:

  \[ SO(32), \quad E_8 \times E_8, \quad E_8 \times U(1)^{248}, \quad U(1)^{496} \]

- The latter two were conjectured to be in the swampland [Vafa ‘05].

- It was argued that anomaly cancellation cannot be made compatible with SUSY & Abelian gauge invariance [Adams, DeWolfe, Taylor, ’10].

- In [Kim, GS, Vafa, ’19], we presented an independent argument ruling out the latter 2 theories by showing that the central charges on BPS strings in these theories are too small to realize the current algebra.
Strings in 10d

- Strings are sources for the 2-form tensor field $B_2$, which by the completeness assumption should exist; they are also stable (::* BPS).
- A string with tensor charge $Q$ adds to the 10d action:
  \[ S^{\text{str}} = Q \int_{\mathcal{M}_{10}} B_2 \wedge \prod_{a=1}^{8} \delta(x^a)dx^a = Q \int_{\mathcal{M}_2} B . \]
- The 2-form $B$ transforms under local gauge & Lorentz symmetries:
  \[ B_2 \rightarrow B_2 - \frac{1}{4} \sum_i \text{Tr}(\Lambda_i F_i) + \text{tr}(\Theta R) . \]
- The string action is not invariant:
  \[ \delta_{\Lambda,\Theta} S^{\text{str}} = Q \int_{\mathcal{M}_2} \left[ -\frac{1}{4} \sum_i \text{Tr}(\Lambda_i F_i) + \text{tr}(\Theta R) \right] . \]
**Anomaly Inflow**

- Anomaly inflow along the worldsheet:
  
  \[ I_4^{\text{inflow}} = Q \left( -\frac{1}{4} \sum_i \text{Tr} F_i^2 + \text{tr} R^2 \right) \]

- A **half-BPS string** in 10d gives rise to N=(0,8) SCFT at low energy: supercurrent on the worldsheet has a definite chirality (right-moving) and is opposite to that of the current for the group (left-moving).

- To cancel the anomaly inflow, the anomalies of the worldsheet SCFT:
  
  \[ I_4 = -I_4^{\text{inflow}} = Q \left[ \frac{1}{2} p_1(T_2) - c_2(SO(8)) + \frac{1}{4} \sum_i \text{Tr} F_i^2 \right] \]

  where we used the decomposition:

  \[ \text{tr} R^2 = -\frac{1}{2} p_1(T_2) + c_2(SO(8)), \]

- The above result includes contributions from **center of mass dofs**.
Anomaly Inflow

- The com modes form a free (0,8) multiplet \((X_\mu , \lambda^I_+)\) with \(\mu, I = 1, \ldots, 8\)
  \[X_\mu : \text{motion in 8 transverse directions}\]
  \[\lambda^I_+ : \text{right – moving SO}(8) \text{ spinor}\]
- The anomaly contribution from the center of mass modes:
  \[I^\text{com}_4 = -\frac{1}{6} p_1(T_2) - c_2(SO(8))\ .\]
- The anomaly of the interacting sector in the worldsheet SCFT (Q=1):
  \[I'_4 = I_4 - I^\text{com}_4 = \frac{2}{3} p_1(T_2) + \frac{1}{4} \sum_i \text{Tr}F^2_i\ .\]
- The central charges & level of gauge algebras can be read off from:
  \[I'_4 = -\frac{c_R - c_L}{24} p_1(T_2) - \frac{c_R}{6} c_2(SO(8)) + \sum_i \frac{k_i}{4} F^2_i\]
Anomaly Cancellation on String Probes

- This gives: \( c_L = 16, \quad c_R = 0, \quad k_i = 1 \)
- The central charges are constrained by unitarity conditions on the 2d CFTs which describe the IR dofs on the string.
- The central charge realizing level-k Kac-Moody algebra:
  \[
  c_G = \frac{k \cdot \text{dim}G}{k + h^\vee}
  \]
- For unitary CFT on a string:
  \[
  \sum_i c_i = \sum_i \frac{k_i \cdot \text{dim}G_i}{k_i + h_i^\vee} \leq c_L .
  \]
- The unitary bound is violated for \( E_8 \times U(1)^{248} \) and \( U(1)^{496} \) (they are in the swampland) while it is saturated for \( SO(32) \) and \( E_8 \times E_8 \).
Strings in 6d N=(1,0) SUGRA
Strings in 6d

- N=(1,0) SUGRA in 6d has 4 kinds of massless supermultiplets:

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- The **chiral fields** (two-forms, gravitino, and other chiral fermions) contribute to the gauge and gravitational anomalies:
Green-Schwarz-Sagnotti Mechanism

- The anomaly polynomial factorizes:
  \[ I_{8-loop}^1 = \frac{1}{2} \Omega_{\alpha \beta} X_4^\alpha X_4^\beta , \]
  \[ X_4^\alpha = \frac{1}{2} a^\alpha \text{tr} R^2 + \frac{1}{4} \sum_i b_i^\alpha \frac{2}{\lambda_i} \text{tr} F_i^2 , \]
  \( \Omega_{\alpha \beta} : \) symmetric with (1,T) signature
  \( a^\alpha , b_i^\alpha : \) vectors \( \in \mathbb{R}^{1,T} \)

- Factorization of the anomaly polynomial requires:
  \[ H - V = 273 - 29T , \quad a \cdot a = 9 - T , \]
  \[ 0 = B^i_{\text{adj}} - \sum_R n_R^i B_R^i , \]
  \[ a \cdot b_i = \frac{\lambda_i}{6} \left( A^i_{\text{adj}} - \sum_R n_R^i A_R^i \right) , \]
  \[ b_i \cdot b_i = \frac{\lambda_i^2}{3} \left( \sum_R n_R^i C_R^i - C_{\text{adj}}^i \right) , \]
  \[ b_i \cdot b_j = 2\lambda_i \lambda_j \sum_{R,S} n_R^{ij} n_S^{ij} A_R^i A_S^j \quad (i \neq j) , \]
  \( n_R^i = \# \) hypermultiplets in rep. \( R \) of \( G_i \)
  \( A_R, B_R, C_R \) are group theory facotrs :
  \[ \text{tr}_R F^2 = A_R \text{tr} F^2 , \]
  \[ \text{tr}_R F^4 = B_R \text{tr} F^4 + C_R (\text{tr} F^2)^2 . \]

- Anomaly cancelled by the Green-Schwarz term:
  \[ S_{GS} = \int \Omega_{\alpha \beta} B_2^\alpha \wedge X_4^\beta \]
String Probes of 6d SUGRA

- There are infinitely many anomaly-free N=(1,0) SUGRA in 6d. Are they all UV-completeable in quantum gravity?
- Strings are sources for the (1+T) B2 which should exist by the completeness assumption. BPS strings preserve 1/2 SUSY.
- The worldsheet theory is a (0,4) SCFT at low energy, with anomaly:

\[ I_4 = \Omega_{\alpha\beta} Q^\alpha \left( \frac{1}{2} a^\alpha \text{tr} R^2 + \frac{1}{4} \sum_i b_i^\alpha \text{Tr} F_i^2 + \frac{1}{2} Q^\beta \chi_4(N_4) \right) \]

\[ = -\frac{Q \cdot a}{4} p_1(T_2) + \frac{1}{4} \sum_i Q \cdot b_i \text{Tr} F_i^2 - \frac{Q \cdot Q - Q \cdot a}{2} c_2(R) + \frac{Q \cdot Q + Q \cdot a}{2} c_2(l) \]

- This anomaly includes the com contributions: 4 bosons common to left- and right-movers, and 4 right-moving fermions:

\[ I_4^{\text{com}} = -\frac{1}{12} p_1(T_2) - c_2(l) . \]
Central Charges

- The anomaly polynomial after removing contributions from the commensurate dofs that decouple in the IR SCFT:

\[
I'_4 = I_4 - I_4^{com} = -\frac{3Q \cdot a-1}{12} p_1(T_2) + \frac{1}{4} \sum_i Q \cdot b_i \text{Tr} F_i^2 - \frac{Q \cdot Q - Q \cdot a}{2} c_2(R) + \frac{Q \cdot Q + Q \cdot a + 2}{2} c_2(l).
\]

- The central charges can be read off from the coefficients of the gravitational anomaly and the R-symmetry anomaly.

- It is possible that an accidental symmetry emerges in the IR and becomes the R-symmetry. It is also possible that the worldsheet theory degenerates to a product of SCFTs with different IR R-symmetries.

- This happens for strings in local SCFTs or little string theories embedded in SUGRA.

- Completeness allows us to focus on strings that give rise to a single interacting SCFT at low energy without the accidental symmetry.
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\[ SO(4) = SU(2)_L \times SU(2)_R : \text{transverse } \mathbb{R}^4 \]

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Central Charges and F-theory

- For strings with a non-degenerate IR SCFT, R-symmetry = SU(2)_R:
  \[ c_L = 3Q \cdot Q - 9Q \cdot a + 2, \quad c_R = 3Q \cdot Q - 3Q \cdot a \]
- The 't Hooft anomalies for the bulk gauge symmetries G_i and SU(2)_l:
  \[ k_i = Q \cdot b_i, \quad k_l = \frac{1}{2}(Q \cdot Q + Q \cdot a + 2) \]
- F-theory on elliptic Calabi-Yau 3-folds → 6d N=(1,0) SUGRA.

Strings with charge Q: D3-branes wrapping genus g curve C=Q in the base
Comparison to F-theory

- The 2d SCFT for a D3-brane wrapping a genus g curve C inside B:

\[c_L' = 3C \cdot C - 9K \cdot C + 6, \quad c_R' = 3C \cdot C - 3K \cdot C + 6,\]

\[k_i' = g - 1.\]  

[Haghighat, Murthy, Vafa, Vandoren, ’16]

where K=canonical class of B, and the genus g of the curve can be computed by the Riemann-Roch theorem:

\[C \cdot C + K \cdot C = 2g - 2\]

- Again, this includes the com contributions:

\[c_L^{com} = 4, \quad c_R^{com} = 6, \quad k_i^{com} = -1\]

- Removing the com contributions gives perfect agreement with the anomaly inflow results by identifying:

\[\Omega \leftrightarrow \text{intersection form in } H_2(B, \mathbb{Z}), \quad a \leftrightarrow K\]
Consistency Conditions

- Consider the moduli space of a 6d SUGRA theory parametrized by scalars in the tensor multiplets + a scalar controls the overall volume.

- For this moduli space to be well-defined, \( \exists \) a linear combination of these scalars \( J \) such that:

\[
J \cdot J > 0, \quad J \cdot b_i > 0, \quad -J \cdot a > 0.
\]

- In F-theory, \( J \) is the Kahler form \( J \in H^{1,1}(B) \). The above conditions define a positive-definite Kahler cone on \( B \).

- The tension of a BPS string with charge \( Q \) is non-negative if

\[
Q \cdot J \geq 0
\]

- Unitarity on the IR SCFT of such a string give constraints on:

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c_L, \quad c_R, \quad k_l, \quad k_i
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$$c_L, \quad c_R, \quad k_l, \quad k_i$$
Unitary CFT

- For a unitary CFT, the central charges must be non-negative
  \[ c_L, c_R \geq 0 \]
- The SU(2)_i current algebra is realized on the left-movers and has:
  \[ k_l \geq 0 \]

  In F-theory, this amounts to requiring \( g \geq 0 \).
- The level of the \( G_i \) current algebra:
  \[ k_i = Q \cdot b_i \geq 0 \]

  In F-theory, this is the condition that the curve class \( Q \) is effective and irreducible within the Mori cone of the Kahler base \( B \).
- Unitarity of the SCFT requires:
  \[ \sum_i \frac{k_i \cdot \dim G_i}{k_i + h_i^\vee} \leq c_L \]
Example 1

- 6d SUGRA coupled to T=9 tensors with SU(N) x SU(N) gauge group and 2 bifund. [Kumar, Morrison, Taylor ‘10] ([Schwarz ‘96] for T=1 models).
- Anomaly polynomial factorizes for any N but these models do not admit F-theory realizations at large enough N.
- The bilinear form $\Omega$ and the vectors $a, b_1, b_2$ are given by:
  \[
  \Omega = \text{diag}(+1, (-1)^9), \quad a = (-3, (+1)^9), \\
  b_1 = (1, -1, -1, -1, 0^6), \quad b_2 = (2, 0, 0, 0, (-1)^6).
  \]
- In this basis, J=$(1,0^9)$ satisfies $J^2 > 0, J \cdot b > 0, J \cdot a < 0$.
- Consider a string with charge $Q=(q_0, q_1, \cdots, q_9)$ with $q_0 > 0$ for positive tension w.r.t. J. The unitarity conditions $c_R \geq 0, k_l \geq 0$: 
  \[
  q_0^2 - \sum_{i=1}^{9} q_i^2 \geq -1, \quad q_0^2 - \sum_{i=1}^{9} q_i^2 - 3q_0 - q_{1:3} - q_{4:9} \geq -2, 
  \]
Example 1

- The positivity of the level of the current algebra $G_i$:

$$k_1 = q_0 + q_{1:3} \geq 0, \quad k_2 = 2q_0 + q_{4:9} \geq 0,$$

where

$$q_{1:3} \equiv \sum_{i=1}^{3} q_i \text{ and } q_{4:9} \equiv \sum_{i=4}^{9} q_i.$$  

- The **unitarity bound** on the level:

$$\frac{k_1(N^2 - 1)}{k_1 + N} + \frac{k_2(N^2 - 1)}{k_2 + N} \leq c_L = 3(q_0^2 - \sum_{i=1}^{9} q_i^2) + 9(q_0 + q_{1:3} + q_{4:9}) + 2$$

- The strongest bound on $N$ is given by a string with

$$q_0^2 - \sum_{i=1}^{9} q_i^2 = -1 \quad \text{and} \quad k_1 = 0, \ k_2 = 1$$

which occurs for $Q=(1,-1,0,0,-1,0^5)$. 
Example 1

- The central charge bound for the string worldsheet to be unitary:

\[
\frac{k_2(N^2 - 1)}{k_2 + N} \leq c_L \rightarrow \frac{N^2 - 1}{1 + N} \leq 8 \rightarrow N \leq 9.
\]

- 6d anomaly-free theories with \(N > 9\) belong to the swampland.

- **Stronger than the Kodaira condition in F-theory: \(N \leq 12\):** anomaly argument can teach us something about elliptic Calabi-Yau 3-folds!

- Reassuring that the swampland bound does not rule out the string realization for \(N=8\) in terms of a K3 orientifold [Dabholkar, Park, ’96].

- Remarkably, we obtain a bound on the rank of the gauge groups that is consistent with F-theory argument and known string realization.

- Interesting to see if the \(N=9\) case can be constructed.
Example 2

- 6d SUGRA with T=1 & SU(N) gauge group coupled to one symmetric + (N-8) fundamental hypermultiplets is anomaly free for N \leq 30.

\[ \Omega = \text{diag}(1, -1) , \quad a = (-3, 1) , \quad b = (0, -1) \]  

[Kumar, Morrison, Taylor ‘10]

- The Kahler form can be chosen as \( J = (n, 1) \) with \( n^2 > 1 \) and \( n > 0 \).

- No F-theory realization: when the base B is identified with Hirzebruch surface \( F^1 \), \( b \) cannot be mapped to an effective curve class.

- Consider a string with charge \( Q=(q_1, q_2) \) satisfying \( c_R \geq 0, k_l \geq 0 \):

\[ q_1^2 - q_2^2 \geq -1 , \quad q_1^2 - q_2^2 - 3q_1 - q_2 \geq -2 , \]
\[ k = Q \cdot b = q_2 \geq 0 . \]

- This string has positive tension if:

\[ nq_1 > q_2 \text{ from } J \cdot Q > 0. \]
Example 2

- These constraints can be simplified as:

\[ q_1 \geq 3 \quad q_1 - 2 \geq q_2 > 0 \]

- The unitarity bound on the central charge:

\[ \frac{q_2(N^2 - 1)}{q_2 + N} \leq 3(q_1^2 - q_2^2) + 9(3q_1 + q_2) + 2 \]

- The most stringent bound is \( N \leq 117 \) which occurs when \( Q=(3,1) \).

- This bound is weaker than the bound from anomaly cancellation.

- Interesting to see if there are inconsistencies revealed by other means or else construct \( N \leq 30 \) models from other string theories.
Example 3

- We found a bound on a family of 6d models with $T=8k+9$ and gauge group $G=(E_8)^k$ for arbitrary $k$ introduced in [Kumar, Morrison, Taylor ‘10]
- The anomaly coefficients of 6d SUGRA:
  \[ a \cdot b_i = 10, \quad b_i \cdot b_j = -2\delta_{ij} \text{ with } i, j = 1, \ldots, k. \]
- For $k \geq 3$, one can choose a basis of tensors:
  \[ \Omega = \text{diag}(1, (-1)^{8k+9}), \quad a = (-3, 1^{8k+9}), \]
  \[ b_i = (-1, -1, 0^{4(i-1)}, (-1)^3, -3, 0^{8k+8-4i}) , \]
- The Kahler form can be chosen:
  \[ J = (-j_0, 0^{4k+1}, 1^{4k+8}), \quad (4k+8)/3 > j_0 > \sqrt{4k+8}. \]
- Consider a string with charge $Q=(-q, 0^{8k+9})$ whose tension is positive if $q > 0$. Furthermore, $c_R \geq 0$, $k_l \geq 0$ can be satisfied for $q > 2$. 
Example 3

- However, the bound on the levels of the current algebras $k_i = Q \cdot b_i = q$: 
  \[
  \sum_{i=1}^{k} \frac{248k_i}{k_i + 30} \leq c_L \quad \to \quad k \frac{248q}{q + 30} \leq 3q(q - 9) + 2
  \]
  is not satisfied by strings with $3 \leq q \leq 14$ for any $k \geq 3$ (\textit{\rightarrow swampland})

- This anomaly inflow argument does not rule out models for $k \leq 2$.

- For $k=1, 2$, there exists another solutions of $\Omega$ and $a, b_i$:
  
  \[
  \Omega = \text{diag}(1, (-1)^{17}), \quad a = (-3, 1^{17}), \quad b_1 = (0, 1, (-1)^{11}, 0^5),
  \]
  
  \[
  \Omega = \text{diag}(1, (-1)^{25}), \quad a = (-3, 1^{25}), \quad b_1 = (0, 1, (-1)^{11}, 0^{13}), \quad b_2 = (0, 0^{13}, 1, (-1)^{11}),
  \]

- The above analysis do not apply. Indeed, the $k=2$ model can be realized by compactification of M theory on K3 x $(S^1/\mathbb{Z}_2)$ with 24 M5-branes on the interval [Seiberg, Witten ’96].
Example 4

- 6d SUGRA with $T=0$ and gauge group $SU(8)$ coupled to an exotic hypermultiplet in the “box” representation [Kumar, Park, Taylor, ’10].
- Admits no F-theory realization. 6d anomaly cancellation sets
  \[ a = -3, \quad b = 8 \]
- Strings with $Q > 0$ satisfy $c_R \geq 0$, $k_i \geq 0$, $k_i \geq 0$.
- Most stringent constraint on $c_L$ is given by strings with minimal $Q=1$:
  \[ \frac{k \times 63}{k + 8} \leq c_L \quad \Rightarrow \quad 31.5 \leq 32 \quad \text{for } k = Q \cdot b = 8 \]

which is marginally satisfied!
Summary
String pheno continues to be a vital field that aims at extracting fundamental predictions on low energy physics from string theory/quantum gravity.

The Swampland program aims to identify the boundary of possibilities for QFTs that can be consistently coupled to gravity. Results of such investigations have broad phenomenological implications.

We initiated a program of using brane probes for a deeper understanding of the swampland conditions.

The method presented in [Kim, GS, Vafa] was recently used to bound the number of abelian gauge group factors in 6d gravitational theories with minimal SUSY and in their F-theoretic realizations [Lee, Weigand].

Consistency between various types of branes and their interactions with one another may provide further constraints on the swampland.

This program may allow us to substantially cut down the # of string vacua, and more broadly deepens our understanding of the swampland criteria.