

# Swampland Conjectures at the Limits of Field Space

Thomas W. Grimm

Utrecht University



**Based on:** 1811.02571 with Chongchuo Li, Eran Palti  
1812.07548 with Irene Valenzuela, Pierre Corvilain  
1905.00901 with Damian van de Heistee

Work in Progress with Irene & Chongchuo - asymptotic flux compactification

# Introduction and general comments

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# Quantum consistent effective theories?

→ Important Question:

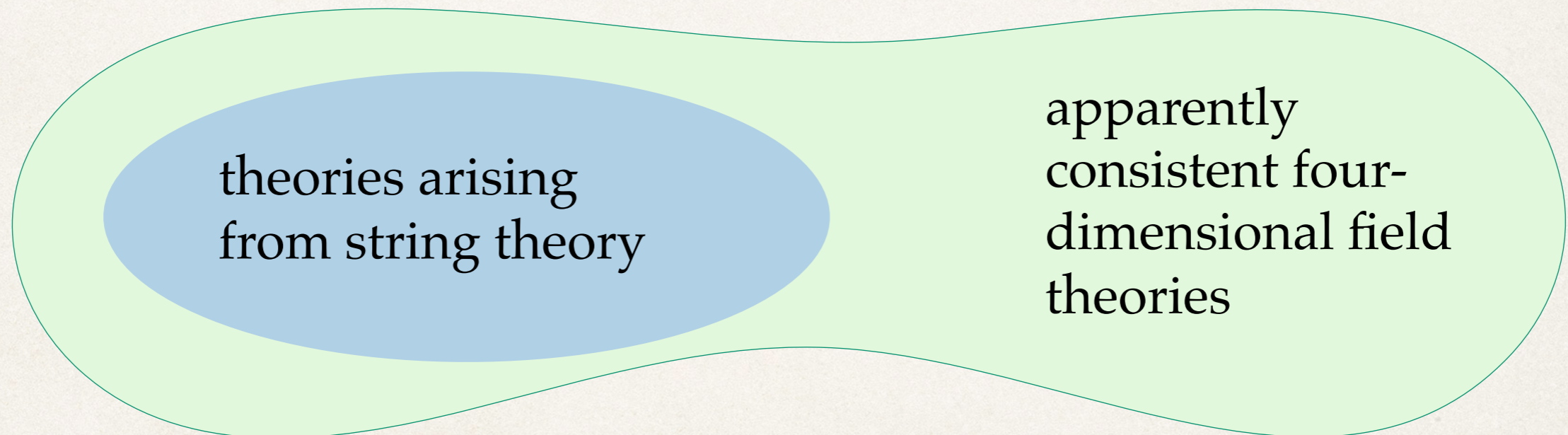
Which low energy effective theories are consistent with Quantum Gravity? What are the imprints of the underlying theory?

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- Map out the set of four-dimensional effective theories from String Theory

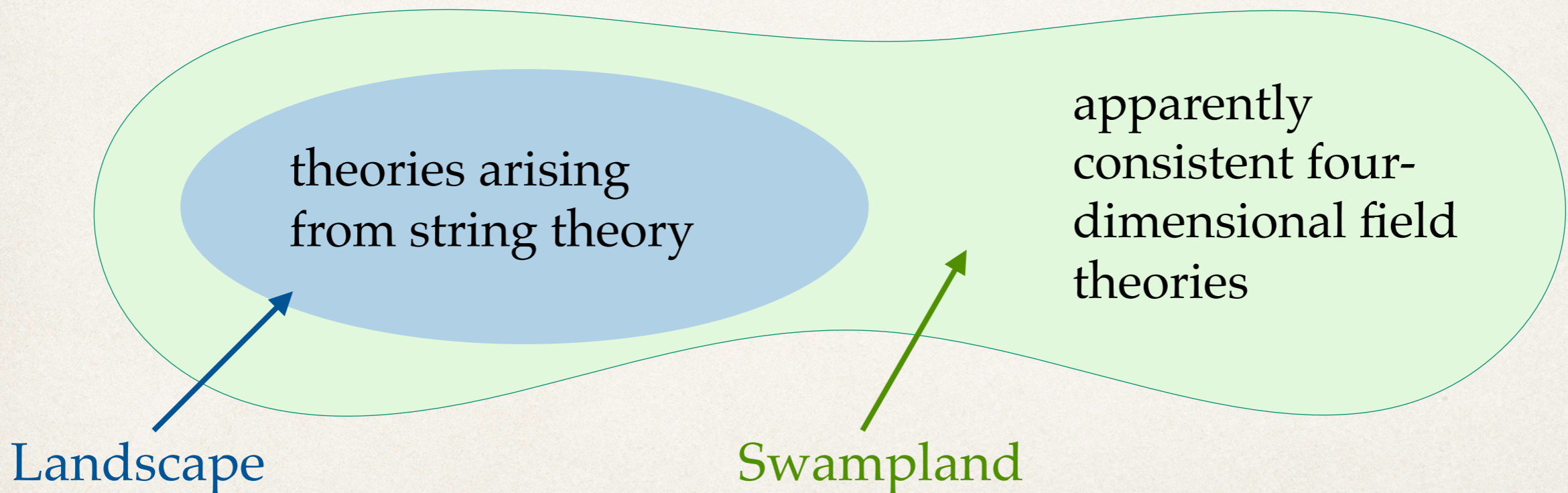


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# Many important works on the Swampland

Plenary talks at this conference about the landscape / swampland:

*Shiu, Montero, Marchesano, Savelli, Gray, Parameswaran, McAllister, Lukas, Anderson, Vafa, Valenzuela, Heidenreich, Grana, Blumenhagen, Wrase, Palti, Uranga, Taylor, Westphal, Van Riet, Hebecker, Ibanez, Tomasiello, Sethi, Halverson, Nelson, Krippendorf, Schafer-Nameki, Martucci, Garcia-Etxebarria, Quevedo, Cvetič, Faraggi, Choi, Heckman, Nilles, Dudas, Lüst*

Many interesting parallel session talks!

And important works of our organizers:

*Andriot, [Lee, Lerche, Weigand], Ruehle*

Not possible to do justice!

# Related motivation: Search for structure

- Well-known setting: Large volume compactification
  - Couplings in the effective action are determined by intersection numbers, Chern classes of compact CY space

$$K = -\text{Log} \left( \frac{1}{6} \mathcal{K}_{IJK} v^I v^J v^K + \frac{\zeta(3)\chi}{32\pi^2} \right)$$

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- What is the structure dictating 'allowed' couplings?

Example from the Kreuzer-Skarke list:

$$\{\mathcal{K}_{IJK}\} = \{ \{ \{2, 2, 2\}, \{2, 0, 2\}, \{2, 2, 2\} \}, \{ \{2, 0, 2\}, \{0, 0, 0\}, \{2, 0, 0\} \}, \\ \{ \{2, 2, 2\}, \{2, 0, 0\}, \{2, 0, 0\} \} \}$$



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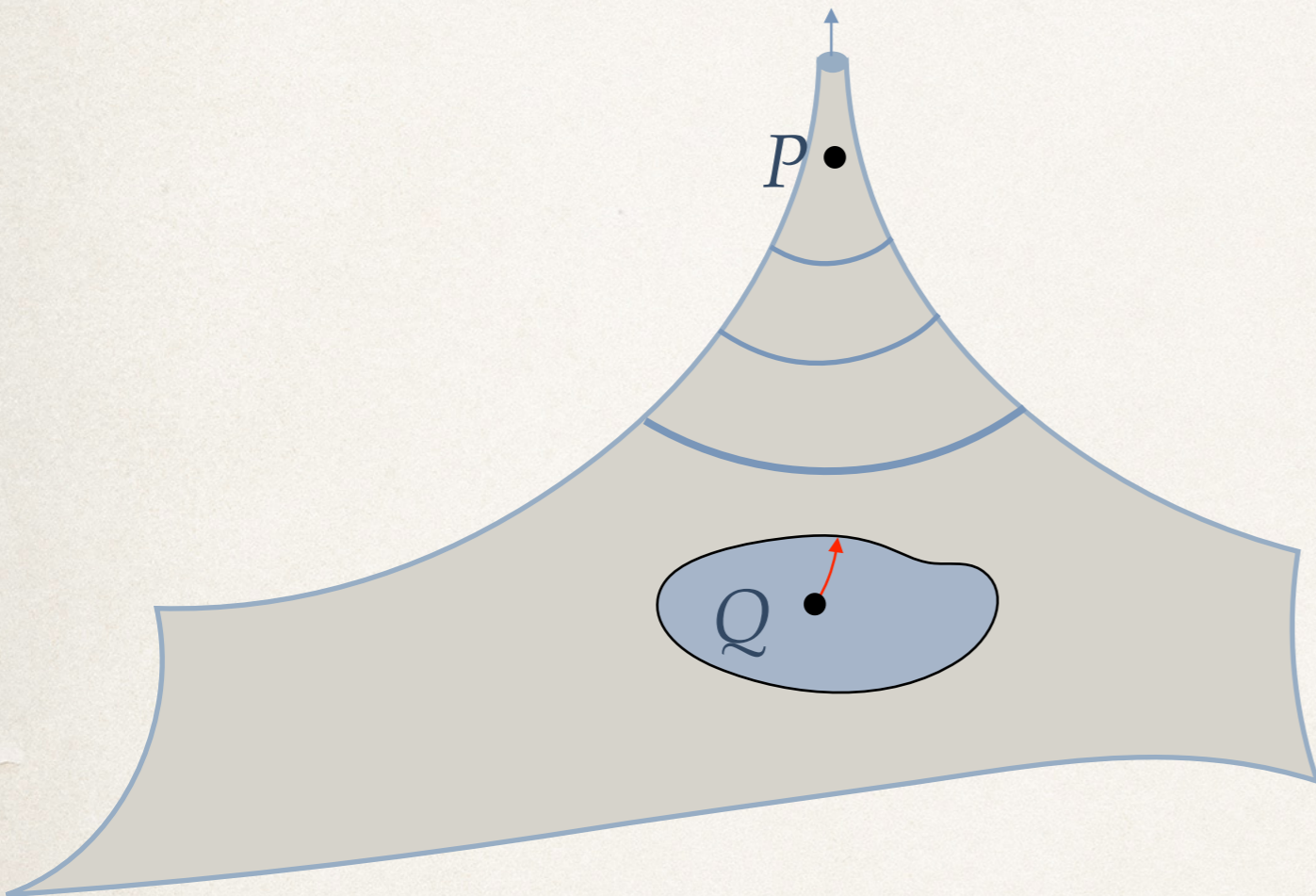
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Can one set the **red** numbers to zero?

# Swampland Distance Conjecture as a Guide

consider a moduli space and two points

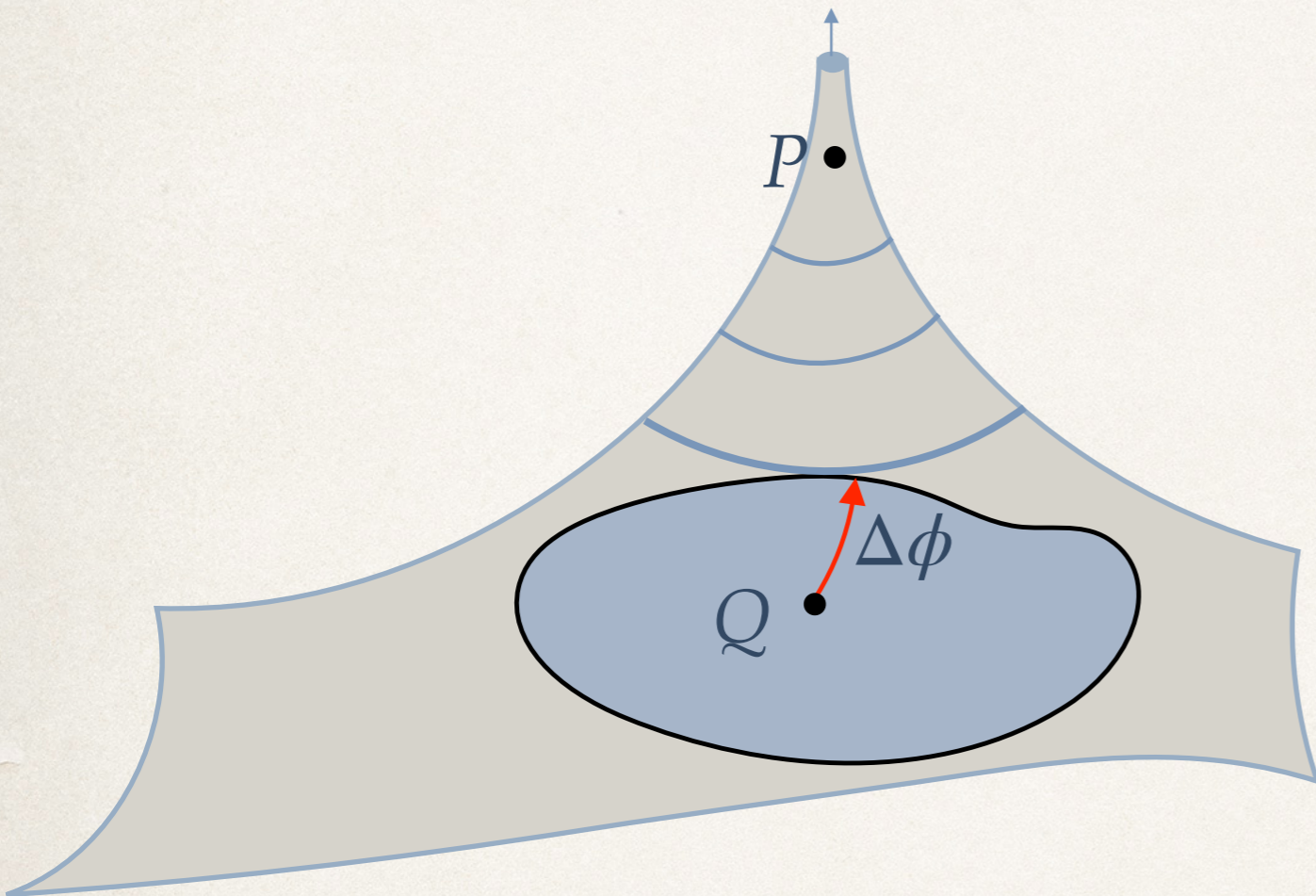
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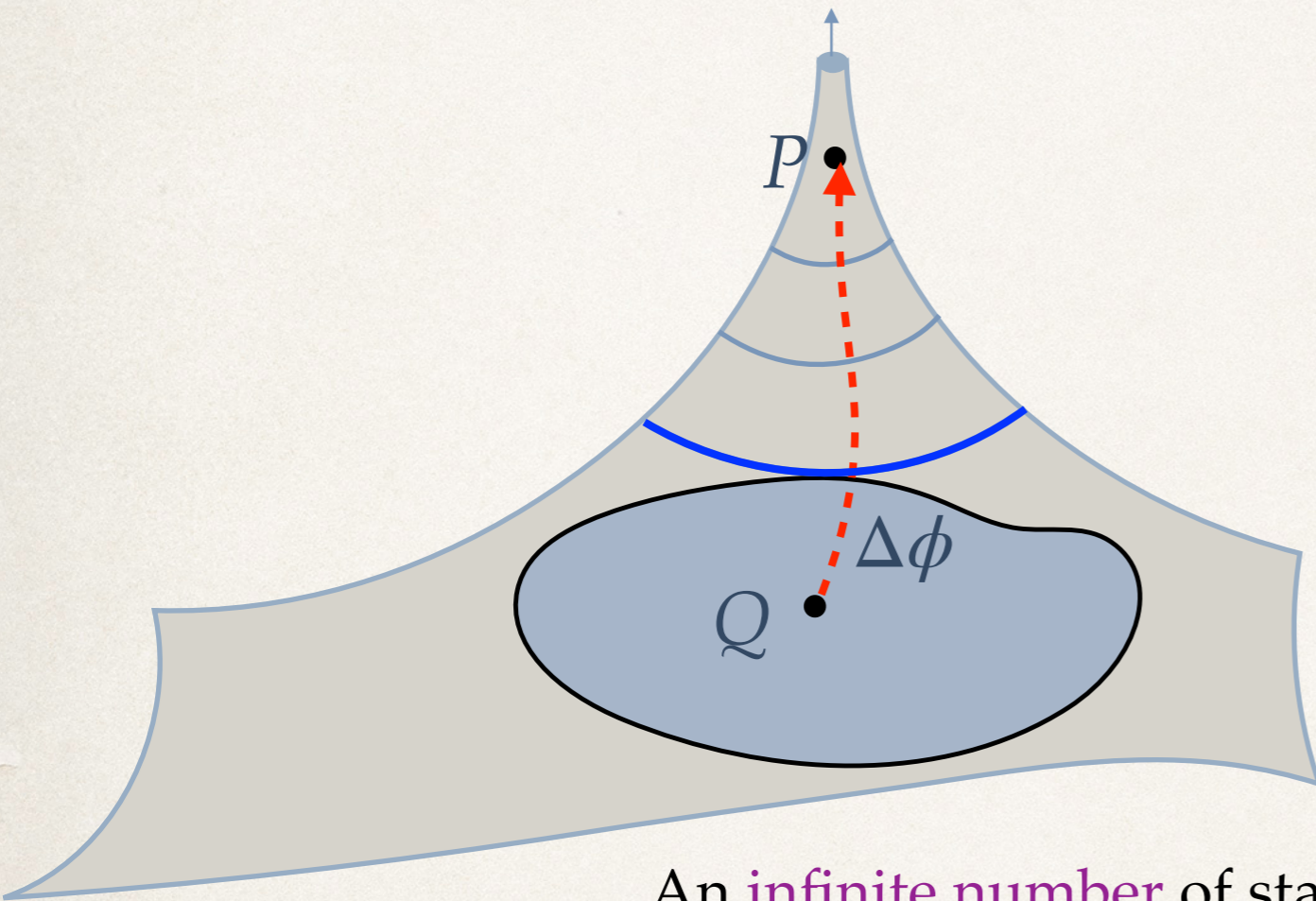
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# Swampland Distance Conjecture as a Guide

consider a moduli space and two points

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shortest geodesic between  $P, Q$   
(length  $d(P, Q)$ )

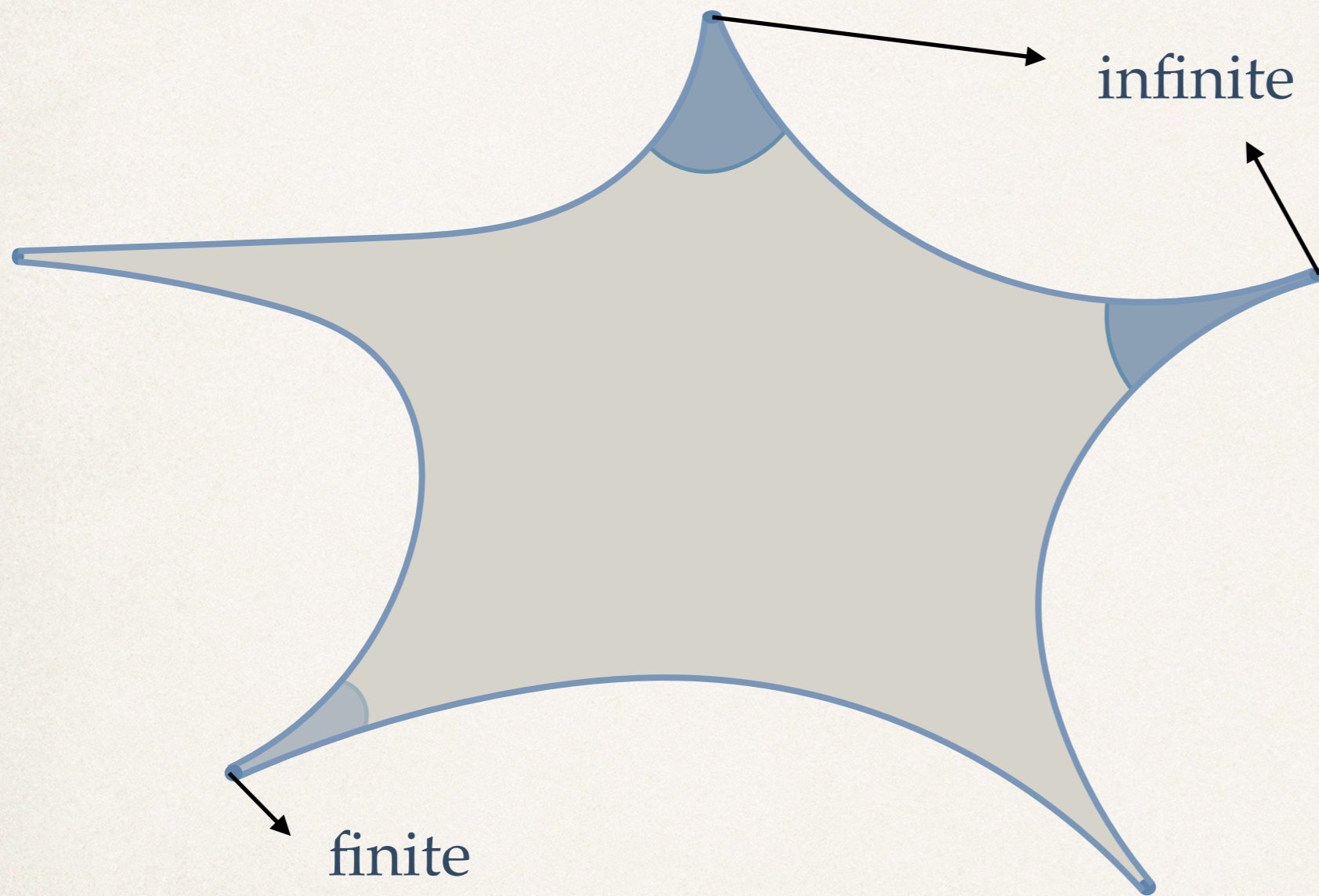
An **infinite number** of states become light on paths approaching an infinite distance point:

$$m(P) \propto M_p e^{-\gamma d(P, G)} \text{ as } d(P, Q) \gg 1$$

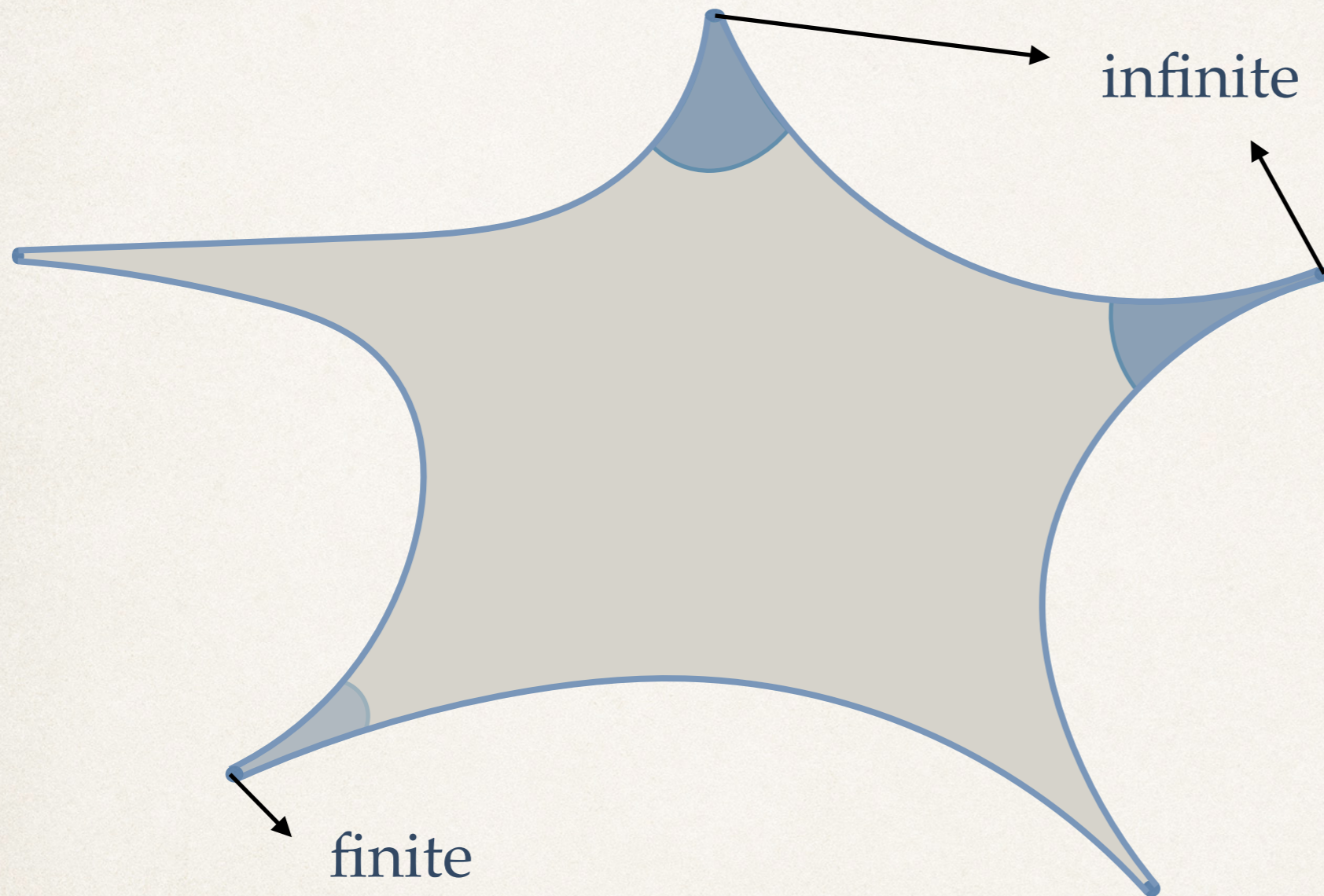
signaling the breakdown of an effective description

$\Rightarrow$  universal structure near infinite distance points

# Limits in Moduli Space



# Limits in Moduli Space



- In the following: restrict to geometric moduli spaces arising in Calabi-Yau compactifications:  $T^2$ ,  $K3$ ,  $CY_3$ ,  $CY_4$

⇒ Universal structure?!

# Universal Structure at the Limits in Moduli Space

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# Moduli space of Calabi-Yau compactifications

- Consider complex structure moduli space  $\mathcal{M}_{\text{CS}}$  (Kähler as a mirror)

$$\text{Kähler metric: } g_{I\bar{J}} = \partial_{z^I} \partial_{\bar{z}^J} K \qquad K = -\log \left[ i \int_{\text{CY}_n} \Omega \wedge \bar{\Omega} \right]$$



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- Periods of the  $(n,0)$ -form  $\Omega$

$$\Omega = \mathbf{\Pi}^T \boldsymbol{\gamma} \qquad \mathbf{\Pi}^{\mathcal{I}} = \int_{\Gamma_{\mathcal{I}}} \Omega$$

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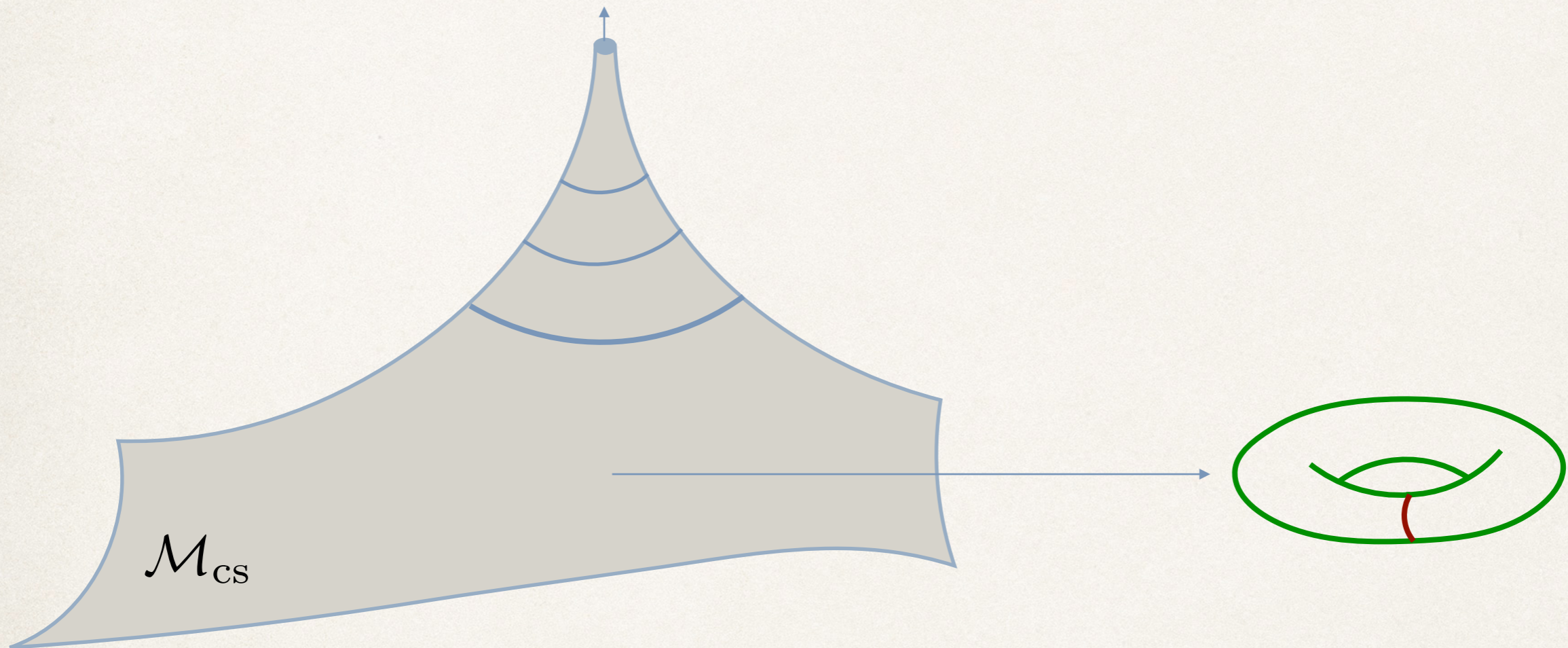
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Question: Is there a universal behavior of  $\mathbf{\Pi}$  at the limits of the moduli space?

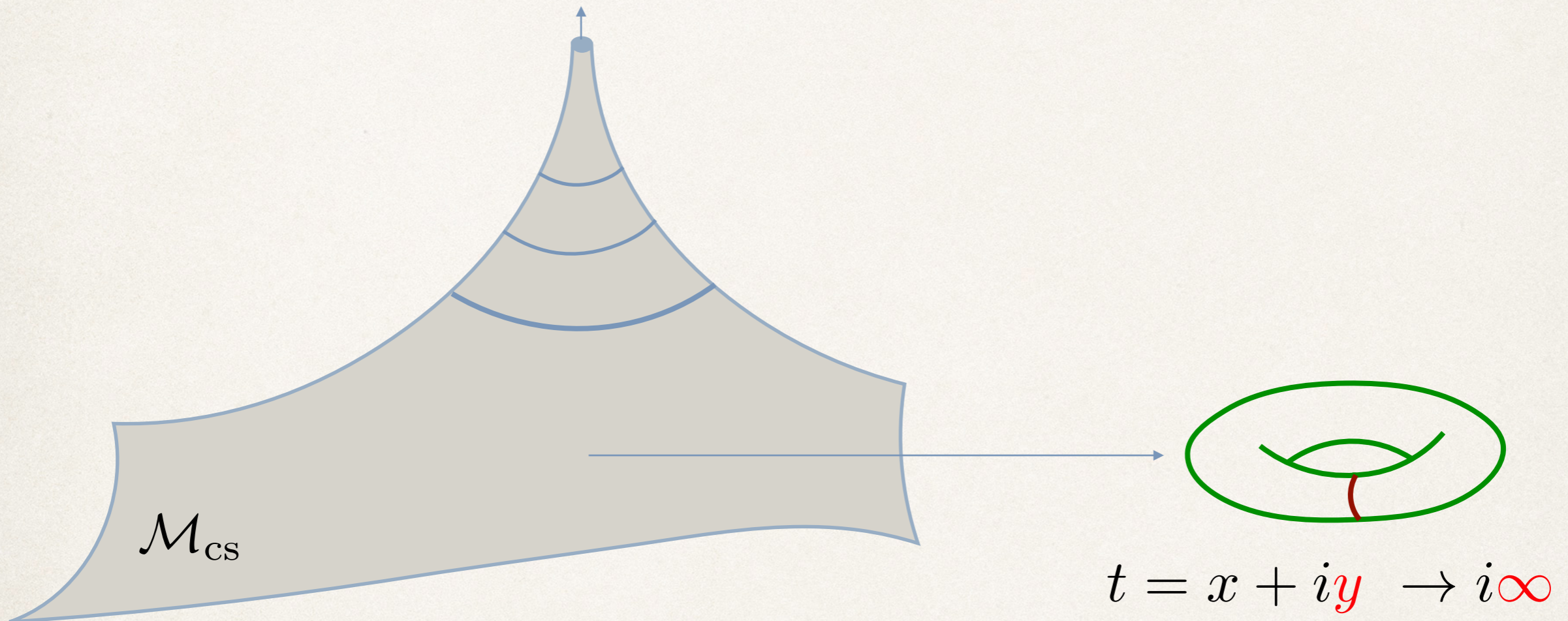
# Limits in complex structure moduli space

- Limits are the points where Calabi-Yau manifold degenerates!



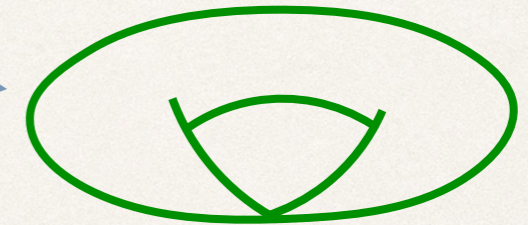
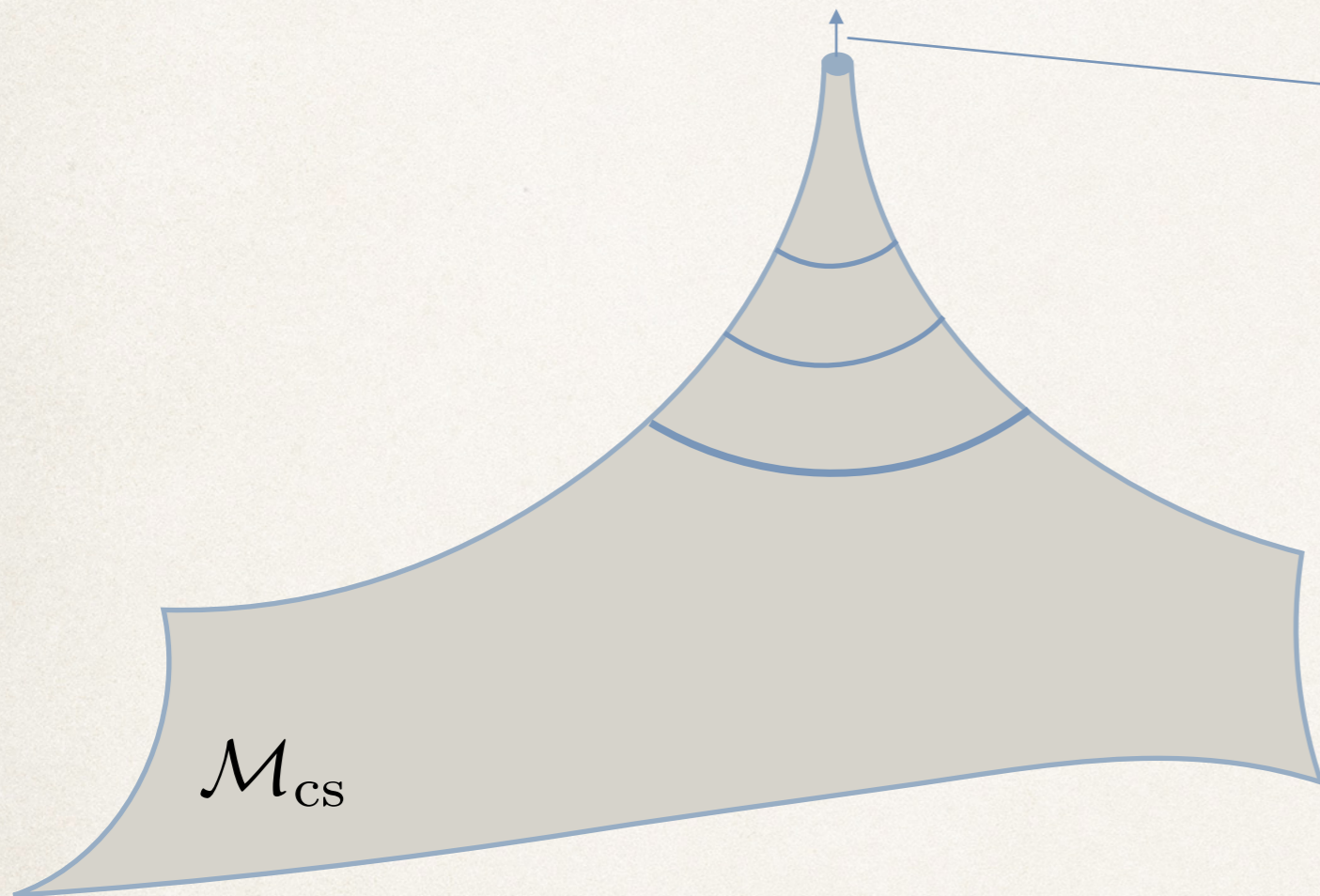
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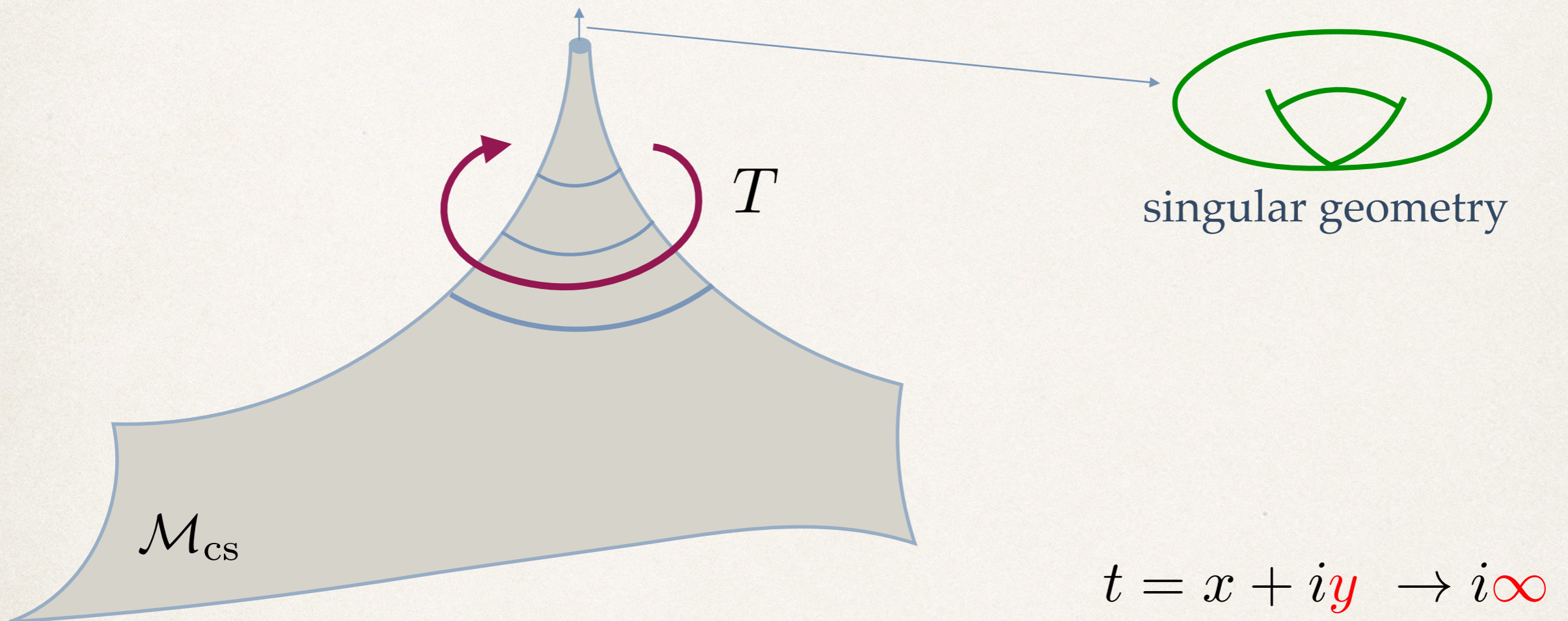


singular geometry

$$t = x + iy \rightarrow i\infty$$

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⇒ monodromy around singular loci:  $\Pi(t+1, \dots) = T \cdot \Pi(t, \dots)$

# Universal behavior of periods

- Limiting behavior of  $\Pi$  near degeneration points

$$t^1, \dots, t^n \rightarrow i\infty \quad \zeta^\kappa \text{ finite}$$

$$\mathbf{\Pi} = e^{t^i N_i} \mathbf{a}_0 + \mathcal{O}(e^{2\pi i t})$$

[Schmid]

(up to rescaling)

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- 'limiting' vector  $\mathbf{a}_0(\zeta)$  - can depend on the coords not send to limit

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[Schmid]

(up to rescaling)

Polynomial in  $t^i$   
nilpotent orbit  
("perturbative part")

Strongly suppressed in the limit  
 $\implies$  neglect  
("non-perturbative part")

# Emergence of an $\mathfrak{sl}(2)^n$ - algebra

- Remarkably: can associate an  $\mathfrak{sl}(2)^n$  - algebra to  $N_i, \mathbf{a}_0$  [Cattani, Kaplan, Schmid]

$n$  commuting  $\mathfrak{sl}(2)$ -triples:  $N_i^-, N_i^+, Y_i$

⇒ raising, lowering, level-operator

aside: need to fix sector in moduli space, or enhancement chain...later

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- Can split form into fine splitting associated to the asymptotic region

$$H^n(Y_n, \mathbb{Q}) = \sum_{l_1, \dots, l_n} V_{l_1, \dots, l_n}$$

eigenspaces of  
 $Y_{(i)} = Y_1 + \dots + Y_i$

# Asymptotic of the Hodge norm

- Hodge norm is omnipresent in string compactifications:

$$\|\alpha\|^2 = \int_{CY_n} \alpha \wedge \star \alpha \quad \alpha \in H^n(Y_n, \mathbb{Q})$$

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Type IIB /  $Y_3$  :  $-\|\alpha\|^2$  determines gauge coupling of  
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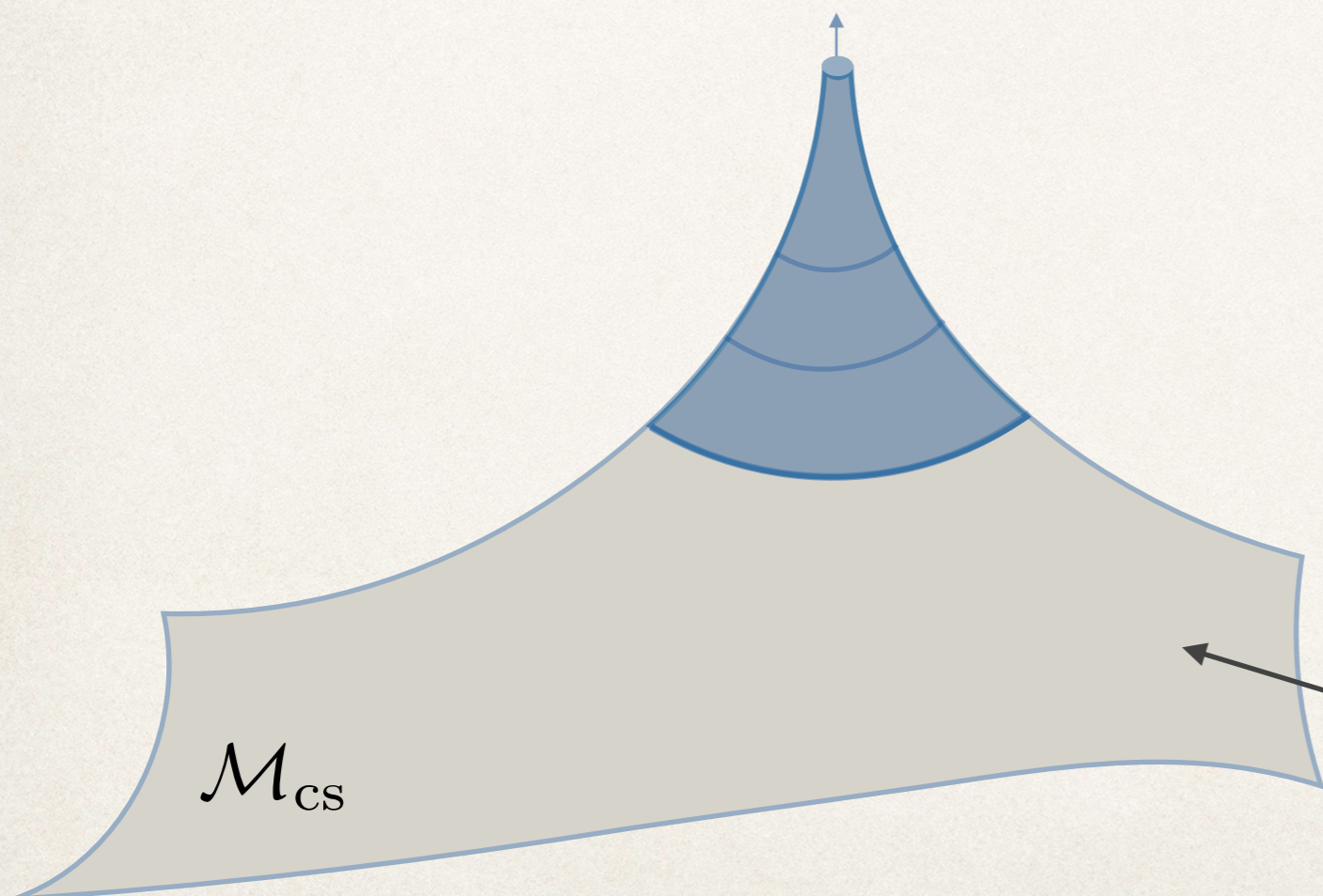
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WGC

dSC  
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# Asymptotic of the Hodge norm

→ Hodge norm in asymptotic region:  $t^i = x^i + iy^i \rightarrow i\infty$

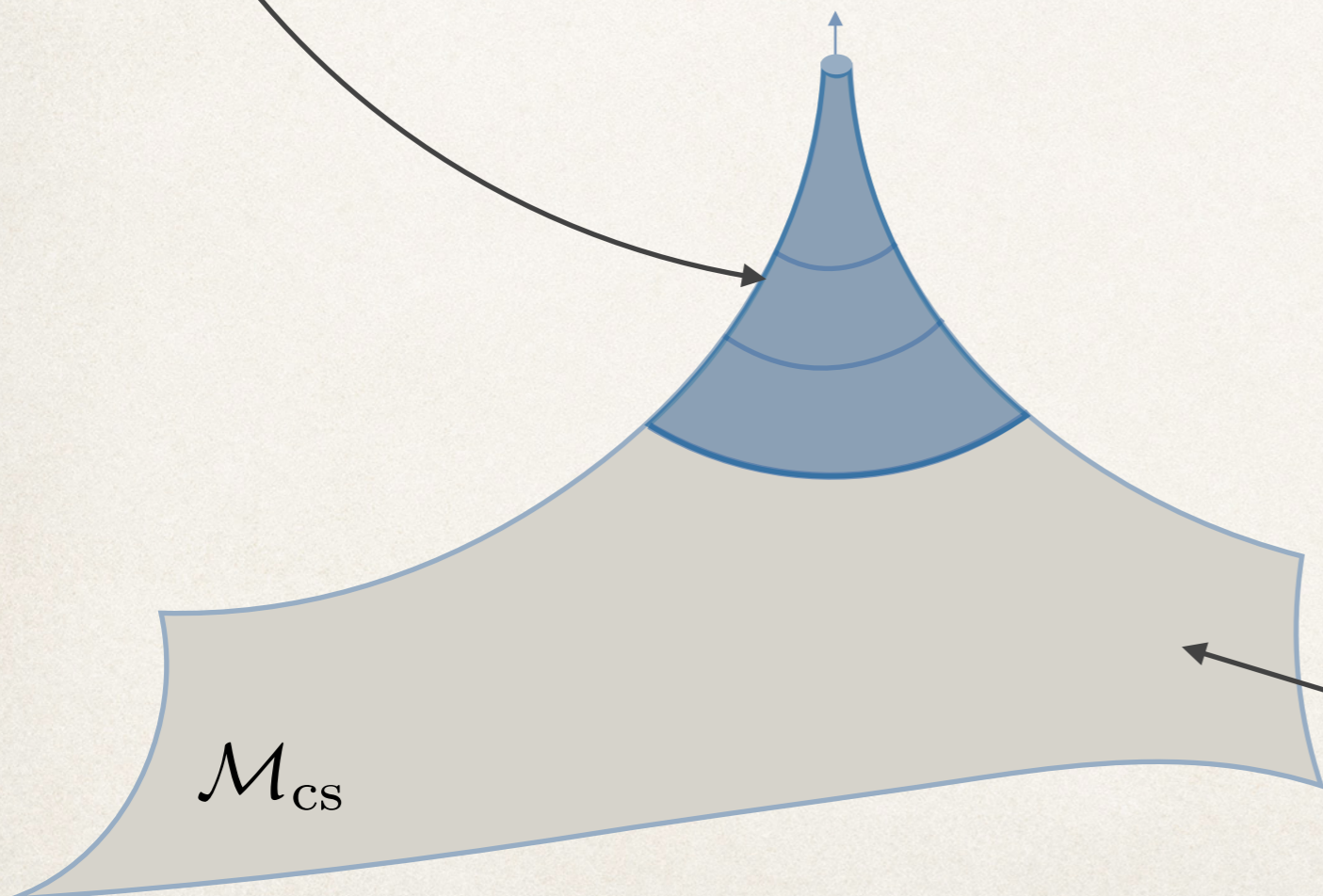


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$$\|\alpha\|^2 \sim \sum_{l_1, \dots, l_n} (y^1)^{l_1 - n} (y^2)^{l_2 - l_1} \dots (y^n)^{l_n - 1 - l_n} \|\rho_{l_1 \dots l_n}\|_\infty$$



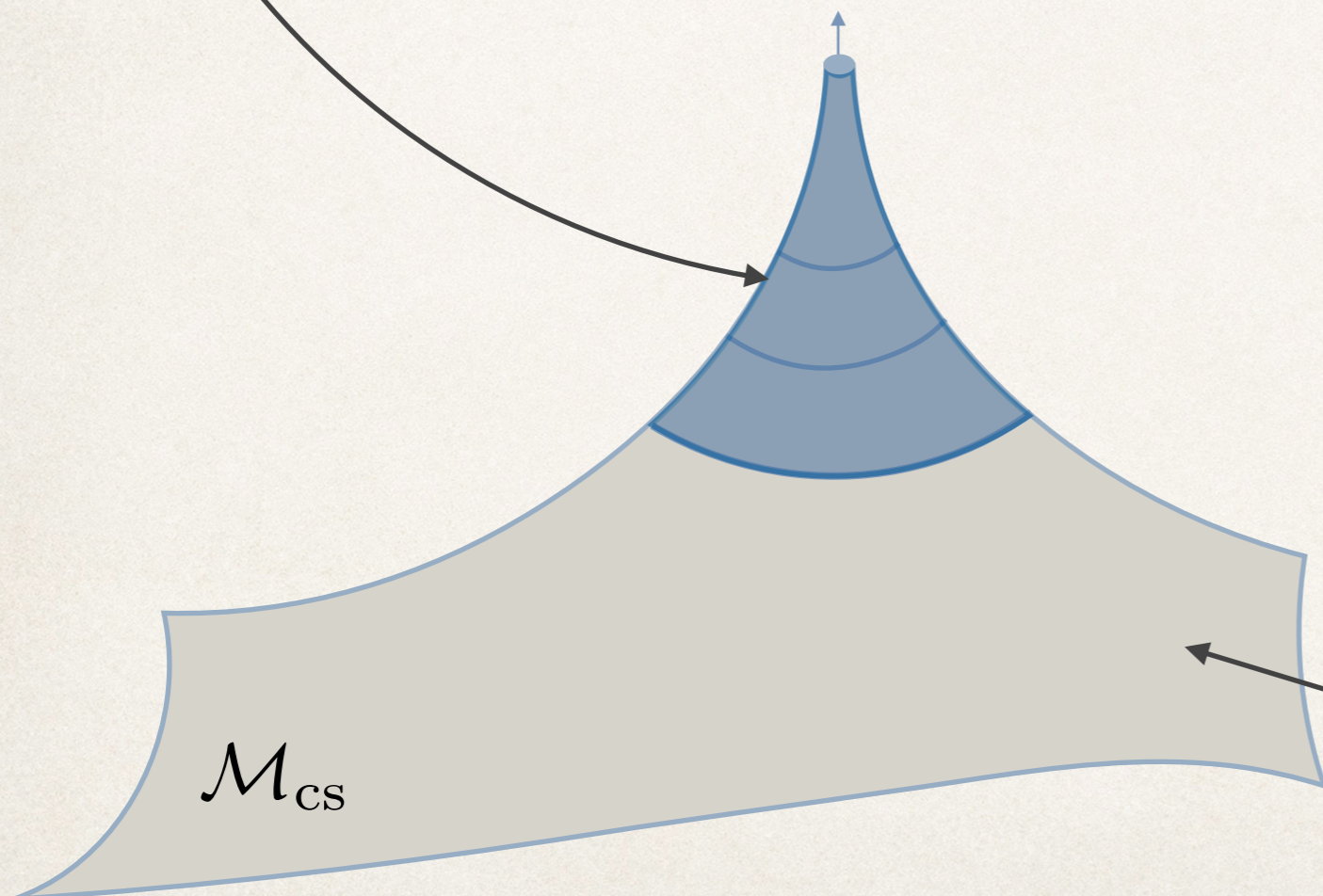
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restriction of  $e^{-x^i N_i} \alpha$   
to subspaces  $V_{l_1 \dots l_n}$



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Can split

$$H^n(Y_n, \mathbb{Q}) = V_{\text{light}} \oplus V_{\text{heavy}} \oplus V_{\text{rest}}$$

$$\|\alpha\|^2 \rightarrow 0$$

$$\|\alpha\|^2 \rightarrow \infty$$

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## Upshot:

In the asymptotic regime the dependents on  $y^i$  (saxions) and  $x^i$  (axions) is explicit  
⇒ classification requires to **classify all asymptotic limits**

restriction of  $e^{-x^i N_i} \alpha$   
to subspaces  $V_{l_1 \dots l_n}$

related expression:  
[Herraez, Ibanez,  
Marchesano, Zoccarato]

# Classification of asymptotic limits

- K3 surface:

[Kulikov]

Types: I, II, III

- Calabi-Yau threefolds:  $4 h^{2,1}$  types of limits

[Kerr,Pearlstein,Robles 2017]  
[Green,Griffiths,Robles]...

Types:  $I_a, II_b, III_c, IV_d$

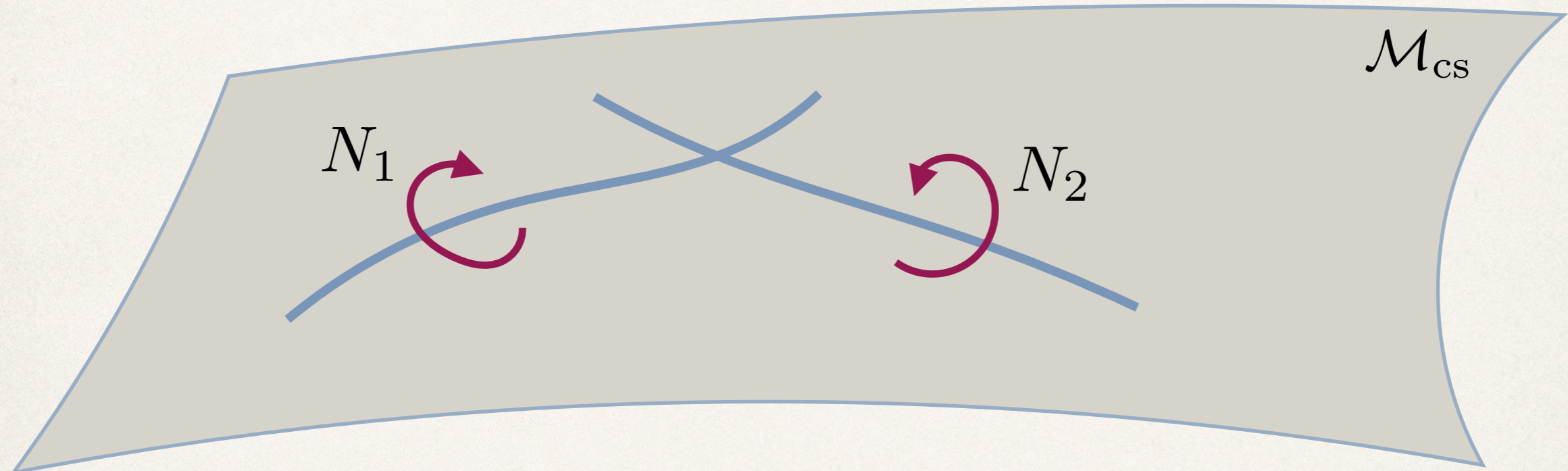
- Calabi-Yau fourfolds:  $8 h^{3,1}$  types of limits

[TG,Li,Zimmermann]  
[TG,Li,Valenzuela]

Types:  $I_{a,a'}, II_{b,b'}, III_{c,c'}, IV_{d,d'}, V_{e,e'}$

# Classification of singularity enhancements

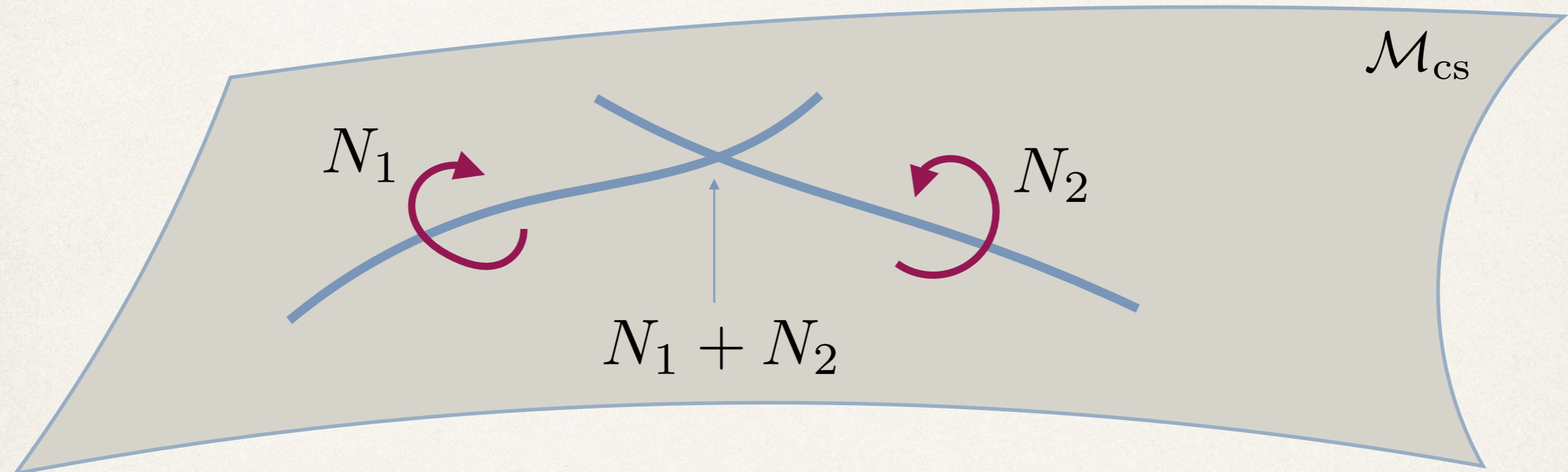
- multi-dimensional moduli spaces:





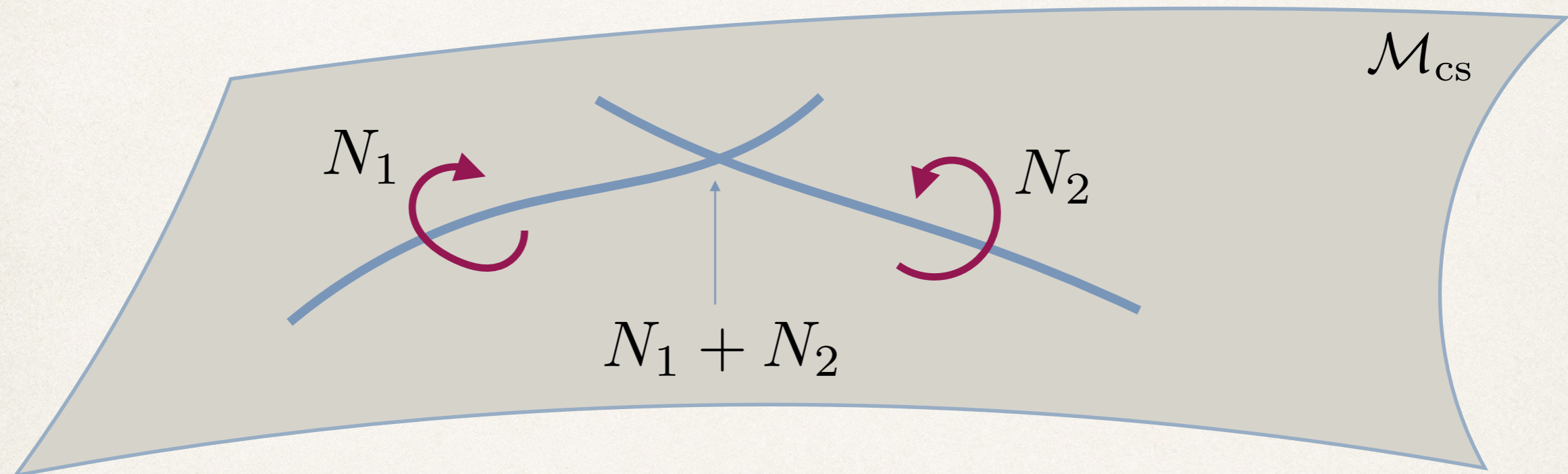
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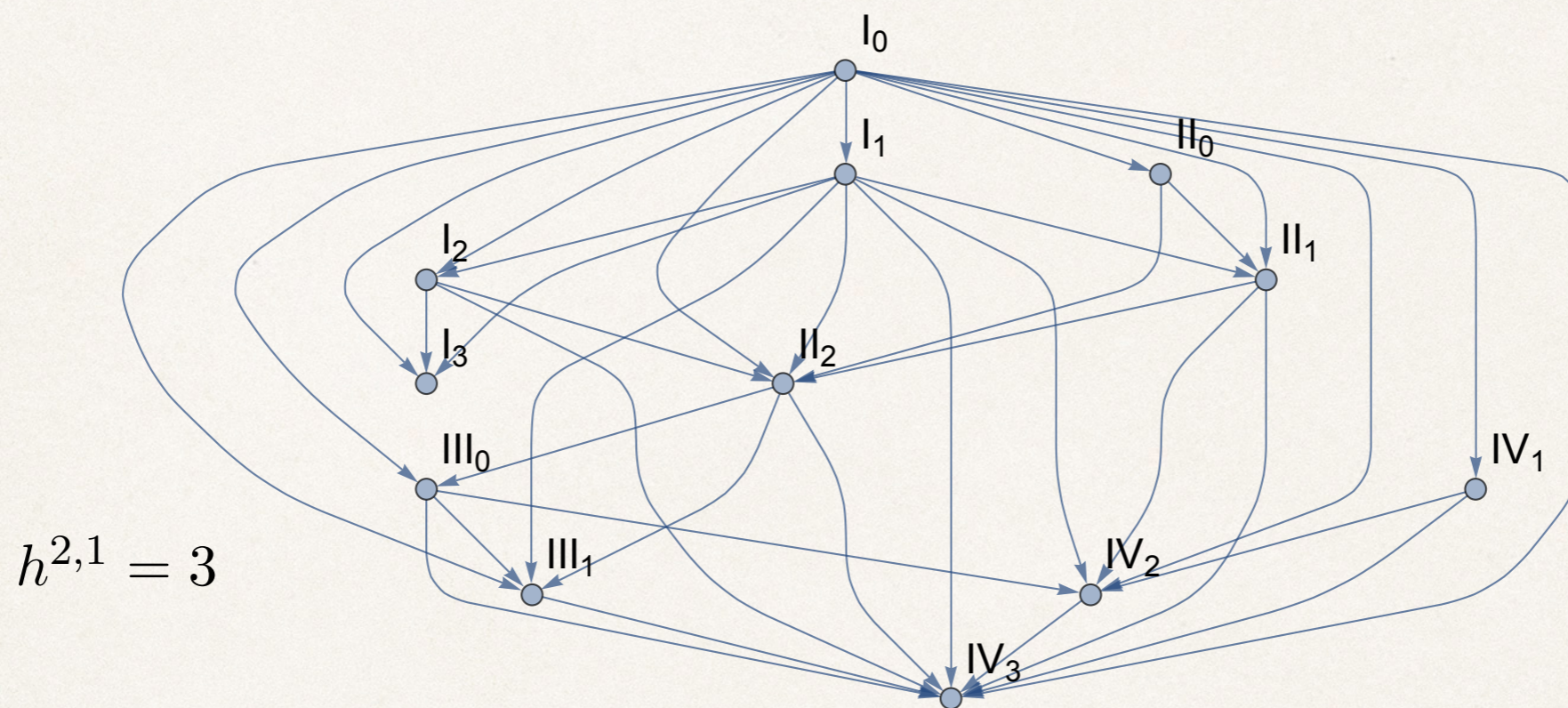
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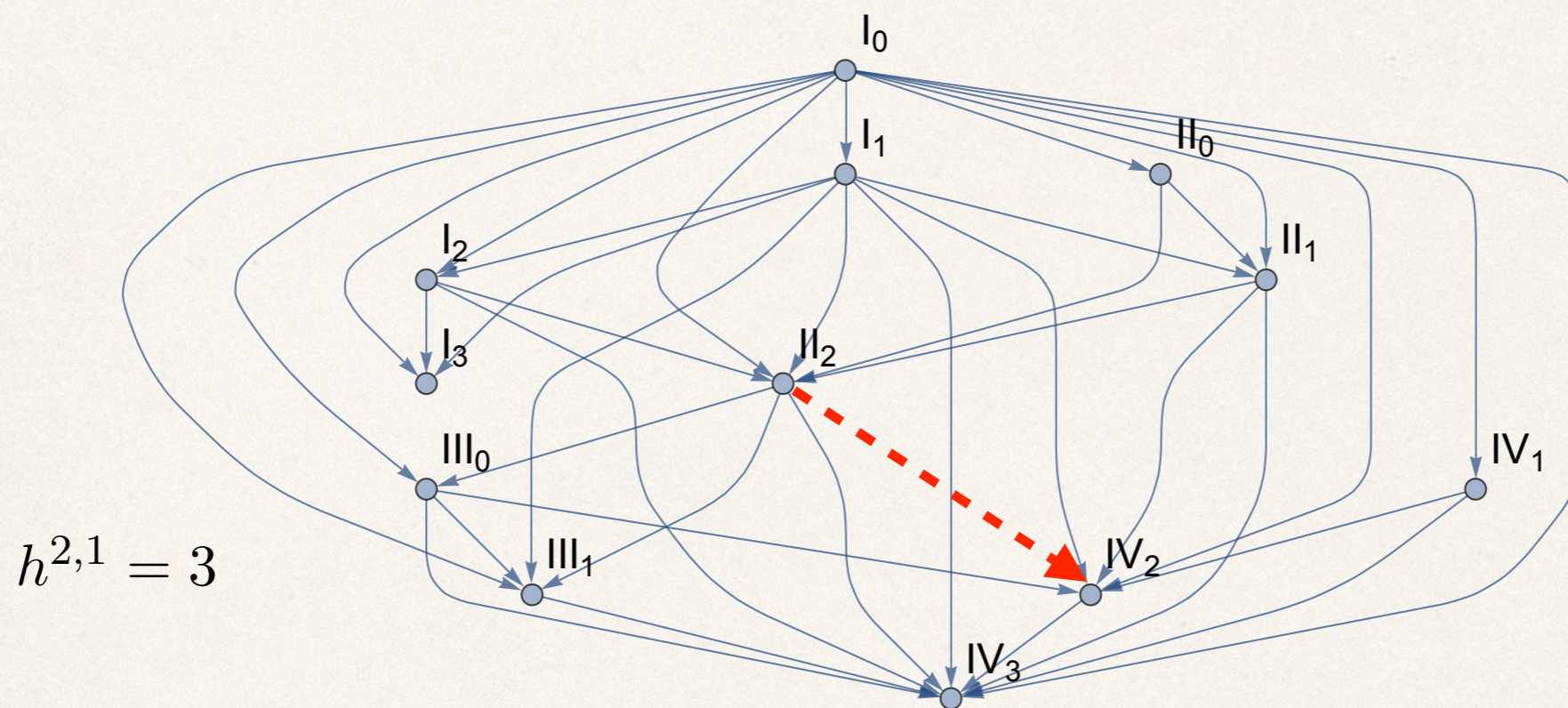
What enhancements are allowed?

- Enhancement rules can be systematically determined:
  - K3,  $CY_3$  [Kerr,Pearlstein,Robles]
  - $CY_4$  [TG,Li,Valenzuela], [TG,Li,Zimmermann]

# An example: general $CY_3$ , with 3 moduli

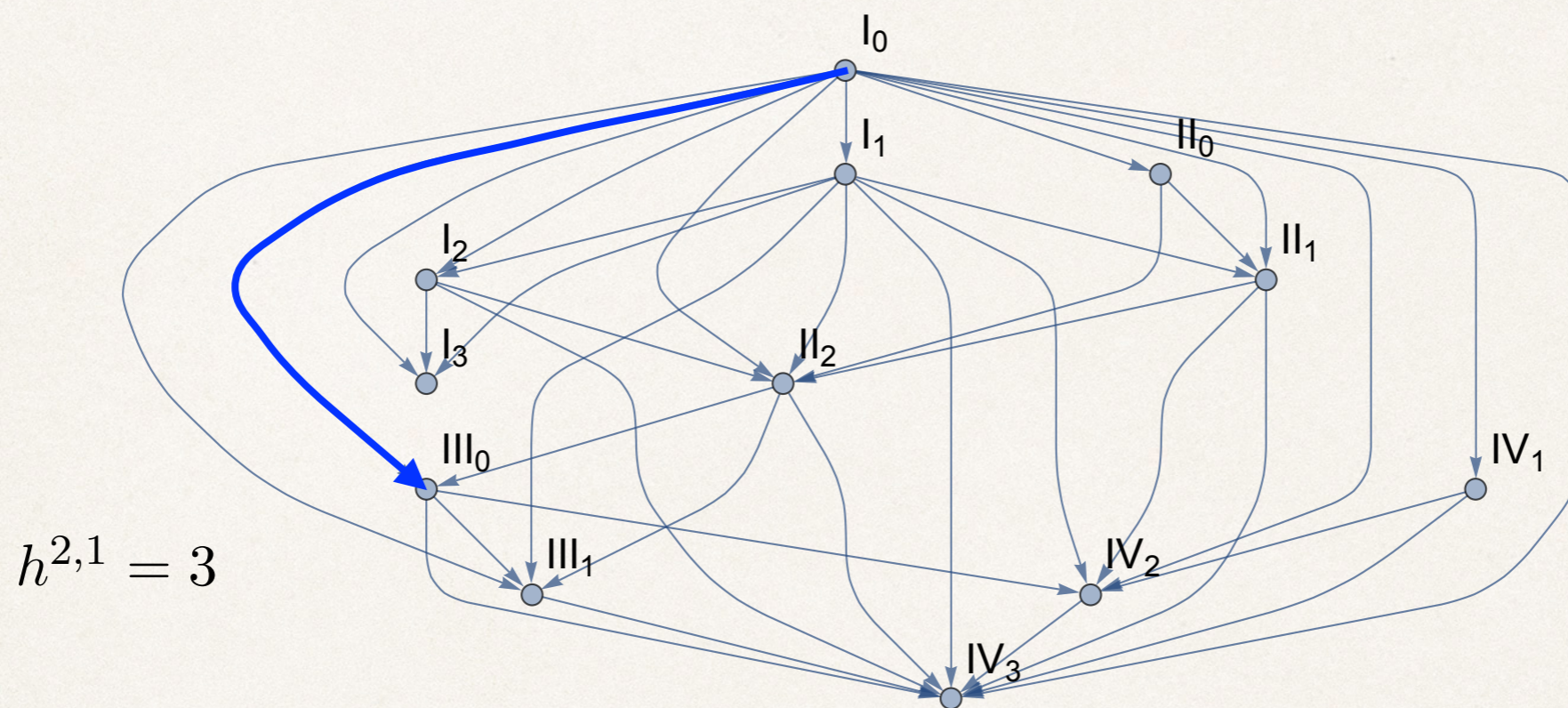


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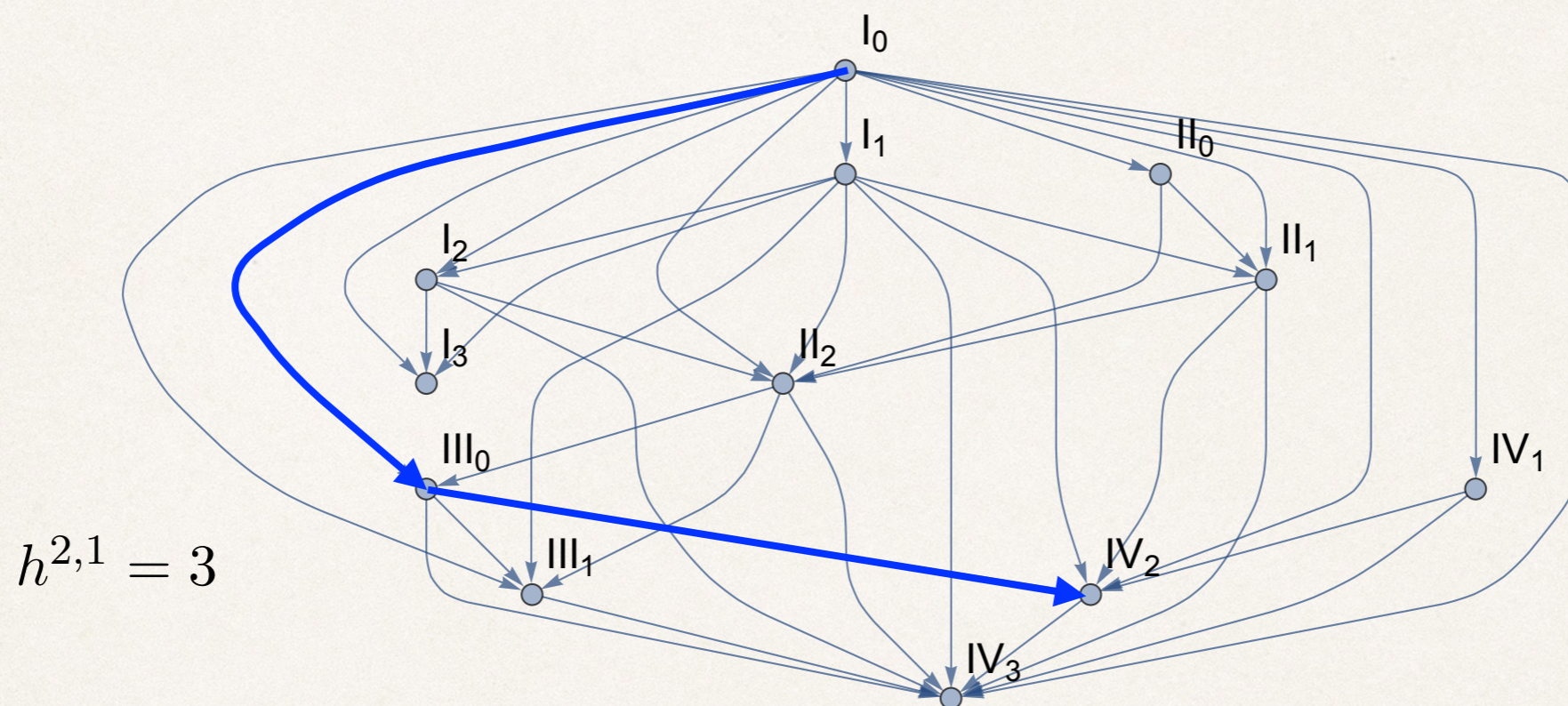
$II_2 \rightarrow IV_2$  not possible

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$$I_0 \rightarrow III_0$$

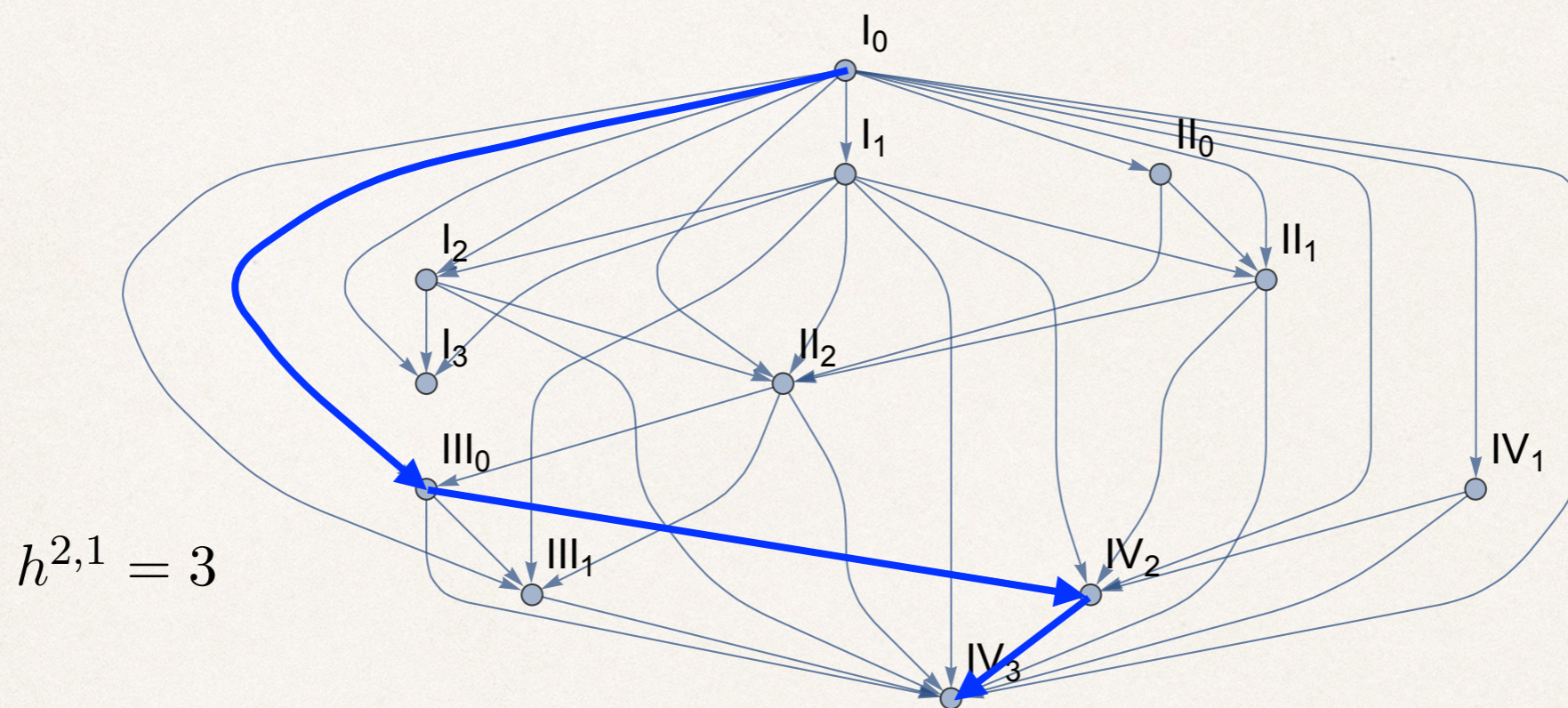
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$$h^{2,1} = 3$$

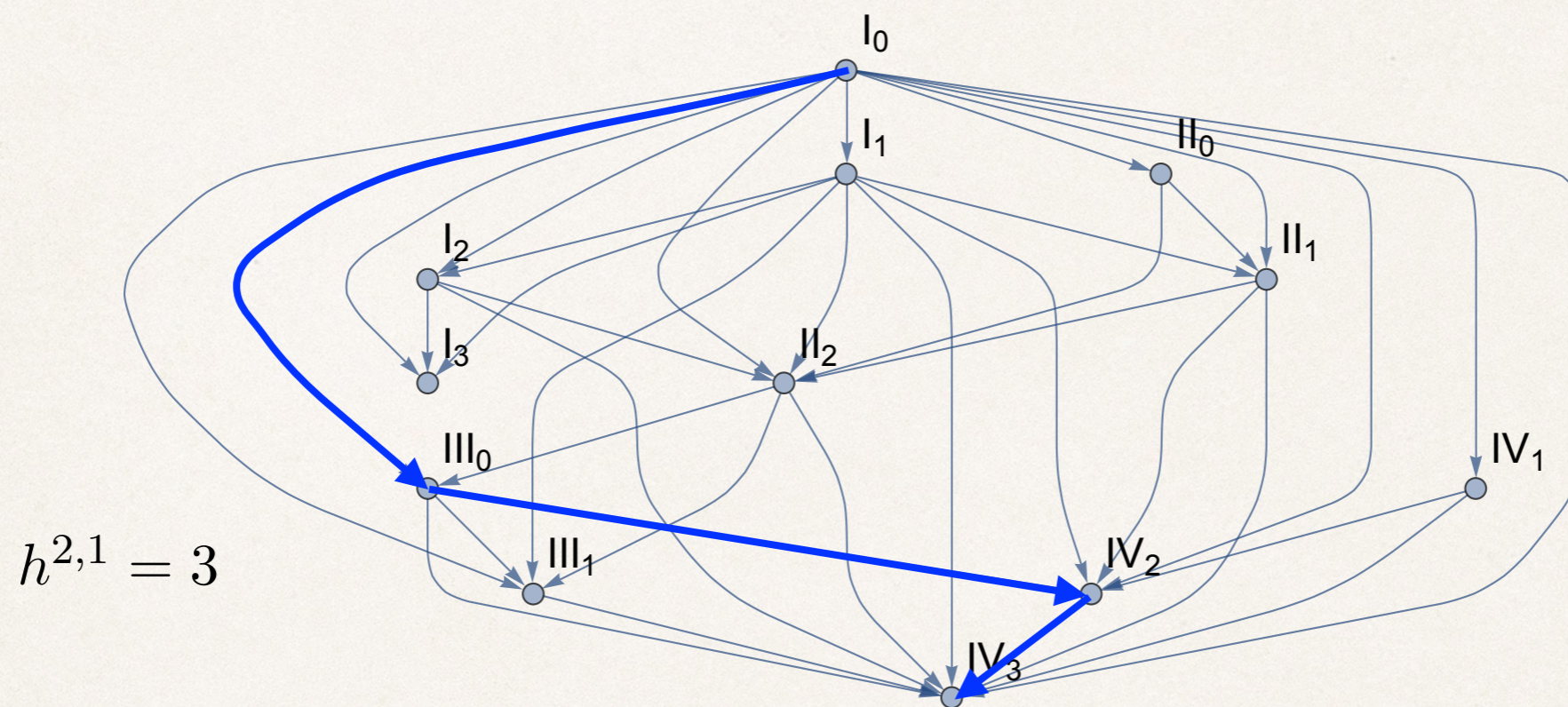
$$I_0 \rightarrow III_0 \rightarrow IV_2$$

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# An example: general $CY_3$ , with 3 moduli



$$I_0 \rightarrow III_0 \rightarrow IV_2 \rightarrow IV_3 \implies \mathfrak{sl}(2)^3\text{-algebra}$$

- each enhancement chain has its associated  $\mathfrak{sl}(2)$ -algebra and highest weight states relevant in the limit  $\rightarrow$  can be computed from  $\mathbf{a}_0, \mathbf{N}_i$

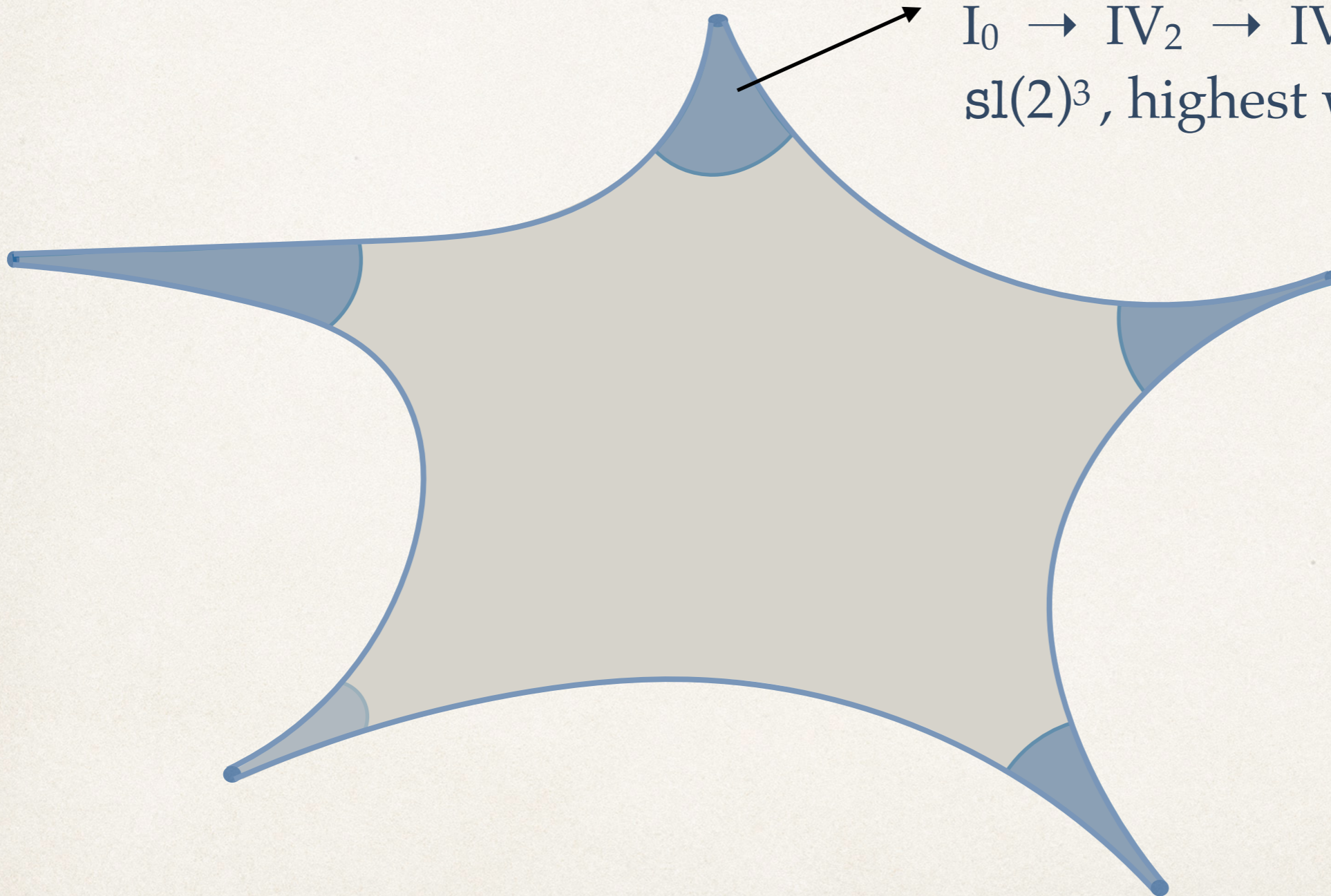


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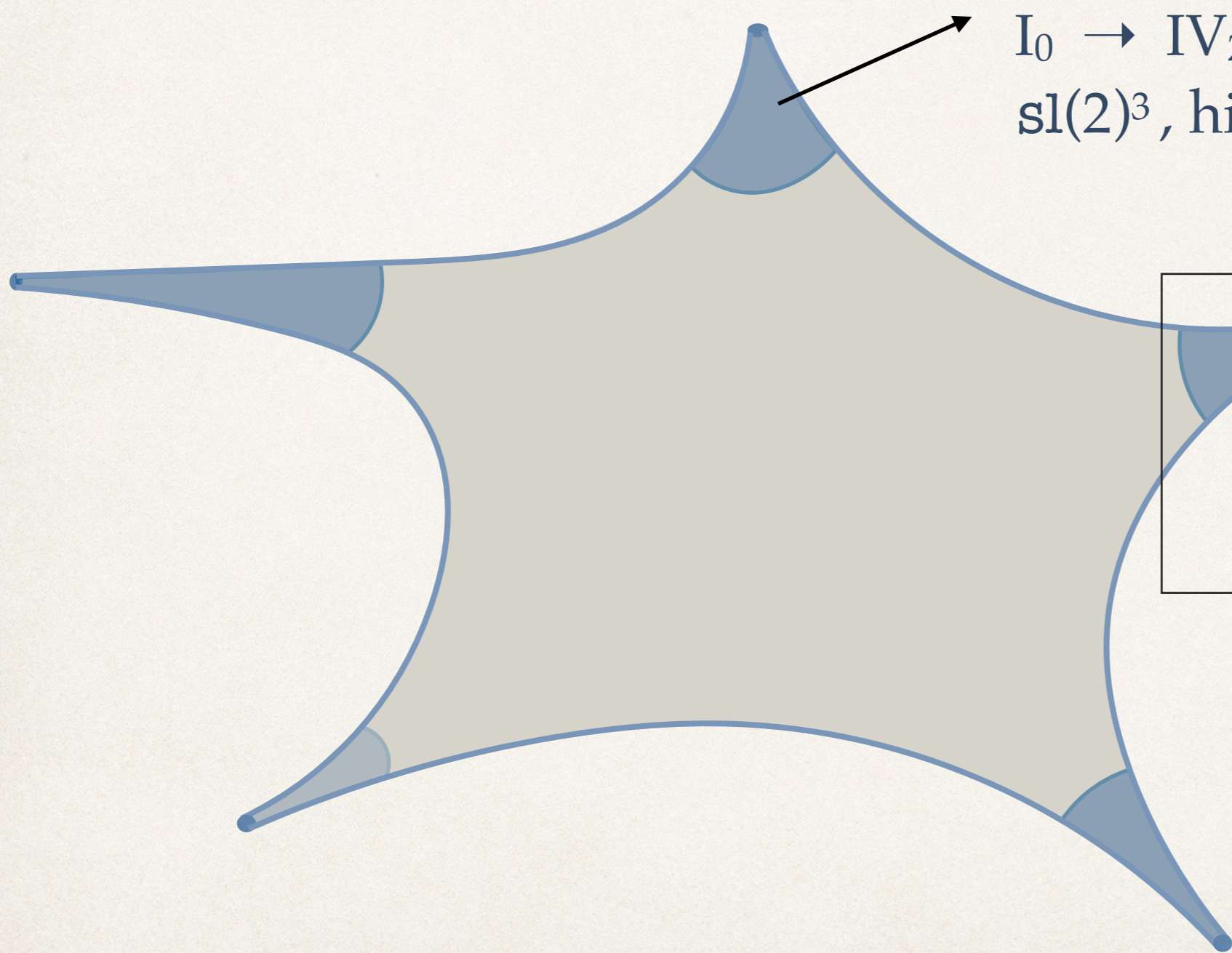


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$sl(2)^3$ , highest weight states



Large CS  
Large volume  
 $\rightarrow \dots \rightarrow IV_n$

highest singularity  
type

# New perspective on Kähler sector

→ Monodromies in Kähler moduli spaces (CY<sub>3</sub>):

[TG, Li, Palti]  
[Corvilain, TG, Valenzuela]

$$N_A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\delta_{AI} & 0 & 0 & 0 \\ -\frac{1}{2}\mathcal{K}_{AAI} & -\mathcal{K}_{AIJ} & 0 & 0 \\ \frac{1}{6}\mathcal{K}_{AAA} & \frac{1}{2}\mathcal{K}_{AJJ} & -\delta_{AJ} & 0 \end{pmatrix}$$

$$\mathbf{a}_0 = \begin{pmatrix} 1 \\ 0 \\ -c_2 I \\ \frac{i\zeta(3)\chi}{8\pi^3} \end{pmatrix}$$

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- ▶ Arising singularities: II<sub>b</sub>, III<sub>c</sub>, IV<sub>d</sub> and enhancements among them  
⇒ distinguished by: rank( $N$ ), rank( $N^2$ ), rank( $N^3$ )

see also [Bloch, Kerr, Vanhove]

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see also [Bloch, Kerr, Vanhove]

- ▶ Enhancement rules allow to rule out non-consistent intersection numbers  
(→ example from the intro is forbidden, would be II<sub>2</sub> → IV<sub>2</sub>) MSc thesis S.Bruning

# New perspective on Kähler sector

- Monodromies in Kähler moduli spaces (CY<sub>3</sub>):

[TG, Li, Palti]  
[Corvilain, TG, Valenzuela]

$$N_A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\delta_{AI} & 0 & 0 & 0 \\ -\frac{1}{2}\mathcal{K}_{AAI} & -\mathcal{K}_{AIJ} & 0 & 0 \\ \frac{1}{6}\mathcal{K}_{AAA} & \frac{1}{2}\mathcal{K}_{AJJ} & -\delta_{AJ} & 0 \end{pmatrix} \quad \mathbf{a}_0 = \begin{pmatrix} 1 \\ 0 \\ -c_2 I \\ \frac{i\zeta(3)\chi}{8\pi^3} \end{pmatrix}$$

- Arising singularities: II<sub>b</sub>, III<sub>c</sub>, IV<sub>d</sub> and enhancements among them  
⇒ distinguished by: rank( $N$ ), rank( $N^2$ ), rank( $N^3$ )

see also [Bloch, Kerr, Vanhove]

- Enhancement rules allow to rule out non-consistent intersection numbers  
(→ example from the intro is forbidden, would be II<sub>2</sub> → IV<sub>2</sub>) MSc thesis S. Bruning
- Can do much more! ‘massless states’, other limits in moduli space  
⇒ Talk of Pierre Corvilain

# Application to Swampland Conjectures

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# Swampland Distance conjecture

- Condition for limit to be at infinite distance:

[Wang]

exists  $N_i$ :  $N_i \mathbf{a}_0 \neq 0 \Rightarrow$  Type II, III, ... limit



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 $Q_3$  - charge                      mass:  $M^2 \leq \|Q_3\|^2$   
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- Idea: start with a single BPS state  $q_0 \in V_{\text{light}}$  (constructed using  $\mathfrak{sl}(2)$ ) [GLP]

$$Q_3(m_1, \dots, m_n) = e^{m^i N_i} q_0 \quad \text{infinite orbit of charges}$$

[TG, van de Heisteeg]

stable BPS states?  $\rightarrow$  talk of Markus Dierigl

# Reasons for being anti de Sitter

→ Irene's talk

- Consider F-theory with  $G_4$ -flux:

$$V_M = \frac{1}{\mathcal{V}_4^3} \left( \int_{Y_4} G_4 \wedge *G_4 - \int_{Y_4} G_4 \wedge G_4 \right)$$

Can be considered in all asymptotic regions of moduli space  
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- Example: 2 moduli  $\text{Im}t^1 = \tau$ ,  $\text{Im}t^2 = \rho$  :  $\text{II}_{0,\hat{m}-2} \rightarrow V_{2,\hat{m}}$  (out of 46)

$$\|G_4\|^2 \propto \frac{1}{\tau^3} \left[ \frac{c_1}{\rho^3 \tau} + \frac{c_2}{\rho \tau} + \frac{c_3 \rho}{\tau} + \frac{c_4 \rho^3}{\tau} + \frac{c_5 \tau}{\rho^3} + \frac{c_6 \tau}{\rho} + c_7 \rho \tau + c_8 \rho^3 \tau \right]$$

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⇒ exactly as in Type IIA flux compactifications

⇒ de Sitter no-go! [Hertzberg et al] What about the other 45 cases?

# Axion Weak Gravity Conjecture

- Connect (axion) Weak Gravity Conjecture with Distance conjecture  
[TG,Palti,Valenzuela] [Lee,Lerche,Weigand] [Font,Herraez,Ibanez] [Marchesano,Wiesner]
- Type IIA R-R axions from  $C_3 = \phi\alpha$ 
  - decay constants  $f \propto \|\alpha\|$  grow parametrically for  $\alpha \in V_{\text{heavy}}$   
[TG,van de Heisteeg]
- Can we find a D2-instanton with decreasing action  $S_{\text{inst}}$  s.t.  $S_{\text{inst}} f \leq qM_p$  ?
  - use distance conjecture to argue for instantons on  $\alpha \in V_{\text{light}}$
- $V_{\text{heavy}}$  and  $V_{\text{light}}$  are dual spaces

→ talk of Damian van de Heisteeg

# Conclusions

- Motivated by the Swampland Conjectures we uncovered a **universal structure emerging in the asymptotic regimes** of geometric moduli spaces
  - ⇒ limits characterized by  $sl(2)^n$  and its representations
  - ⇒ asymptotic of Hodge norm
  - ⇒ attainable for a classification
- New **general evidence** for the Conjectures at the limits of moduli space
- New way to study the Kähler moduli sector: structure behind **intersection numbers, Chern classes** determining type of Calabi-Yau
- **Numerous further questions**
  - control over numerical behavior: important in conjectures
  - going beyond geometry: what else is there in the landscape?

limiting mixed  
Hodge structures

Thank you for  
your attention!

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