A STRONG WEAK GRAVITY CONJECTURE FROM THE MODULAR BOOTSTRAP

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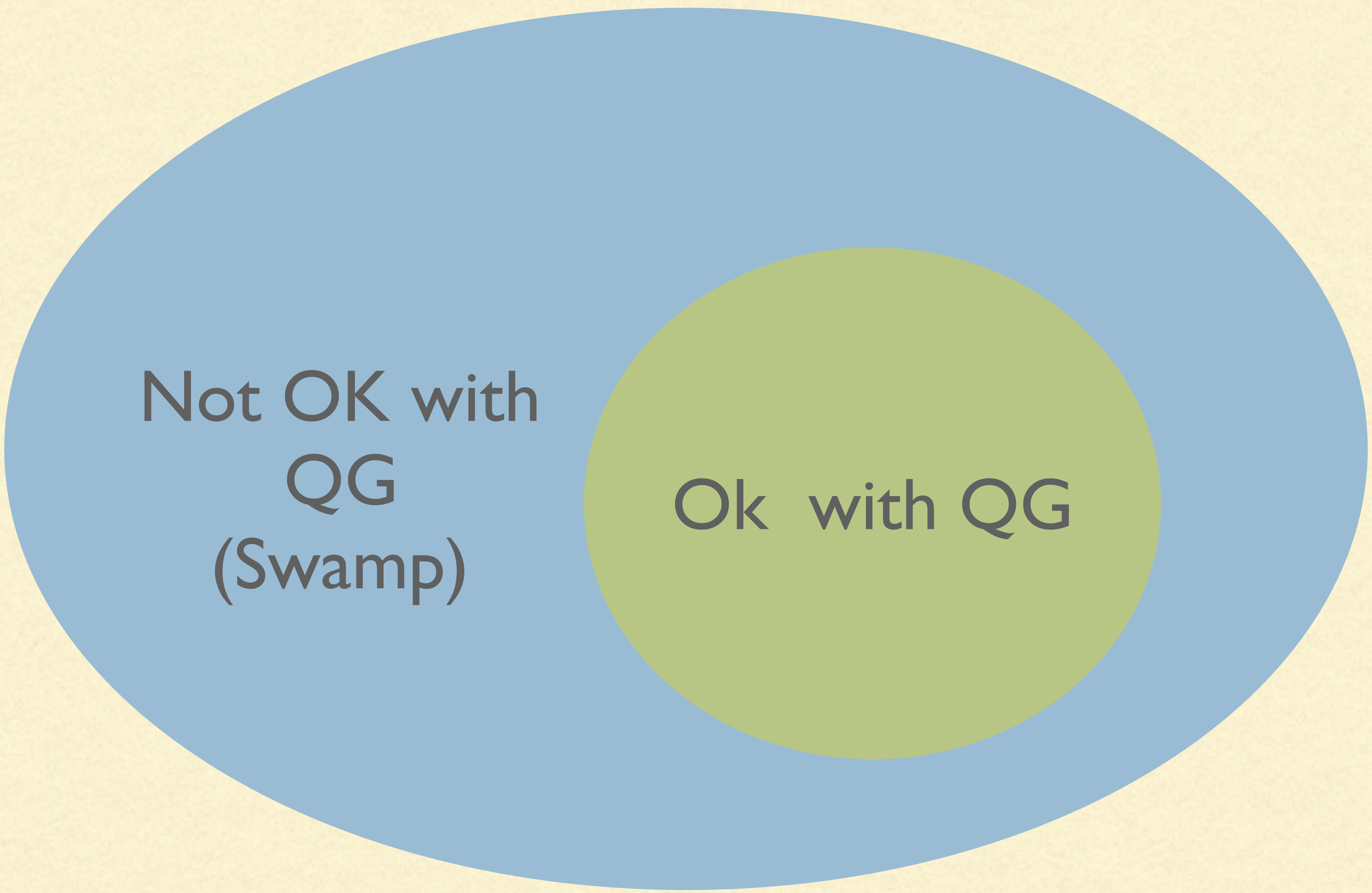
- **Swampland**: Constraints on **EFT**'s that can be consistently coupled with QG



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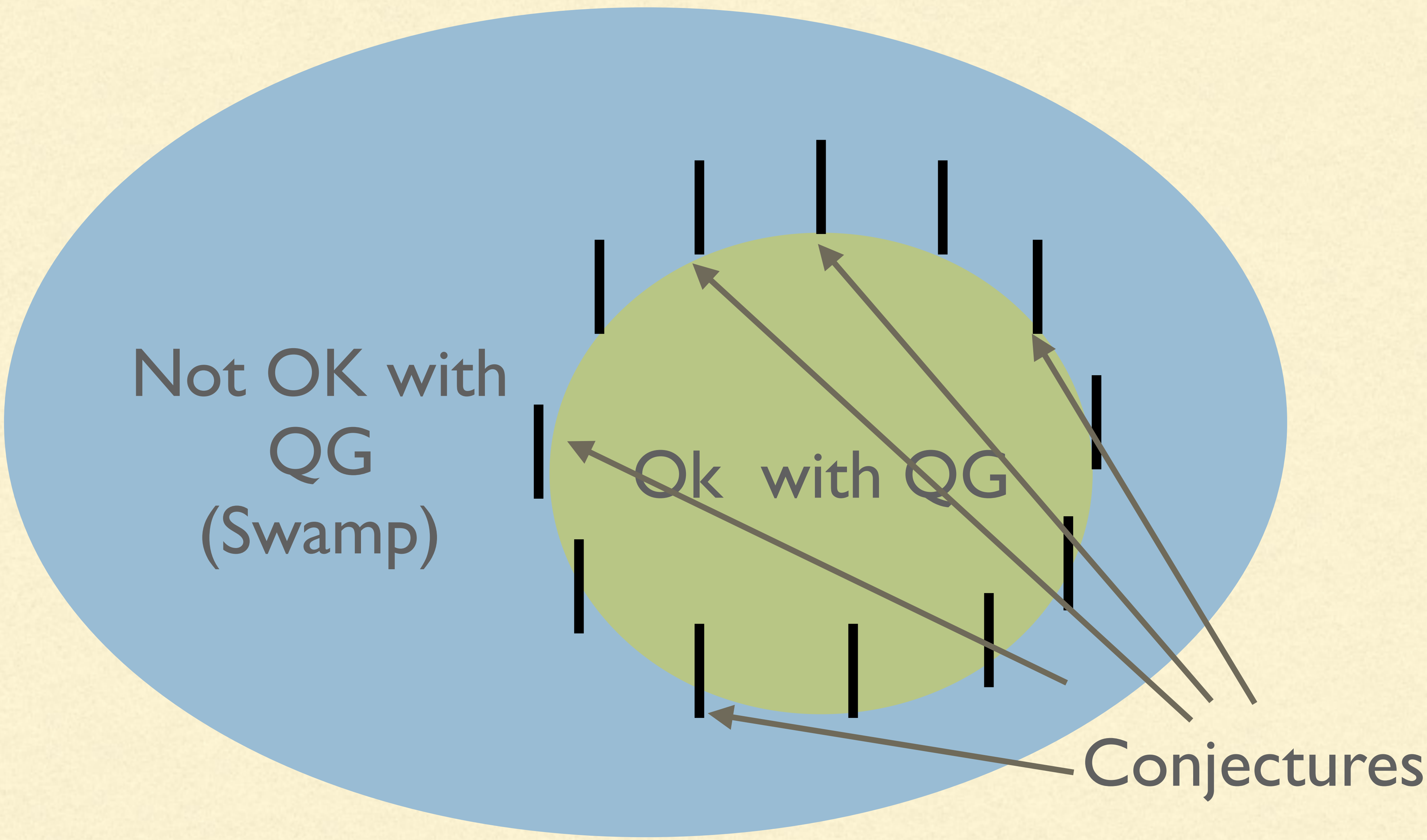

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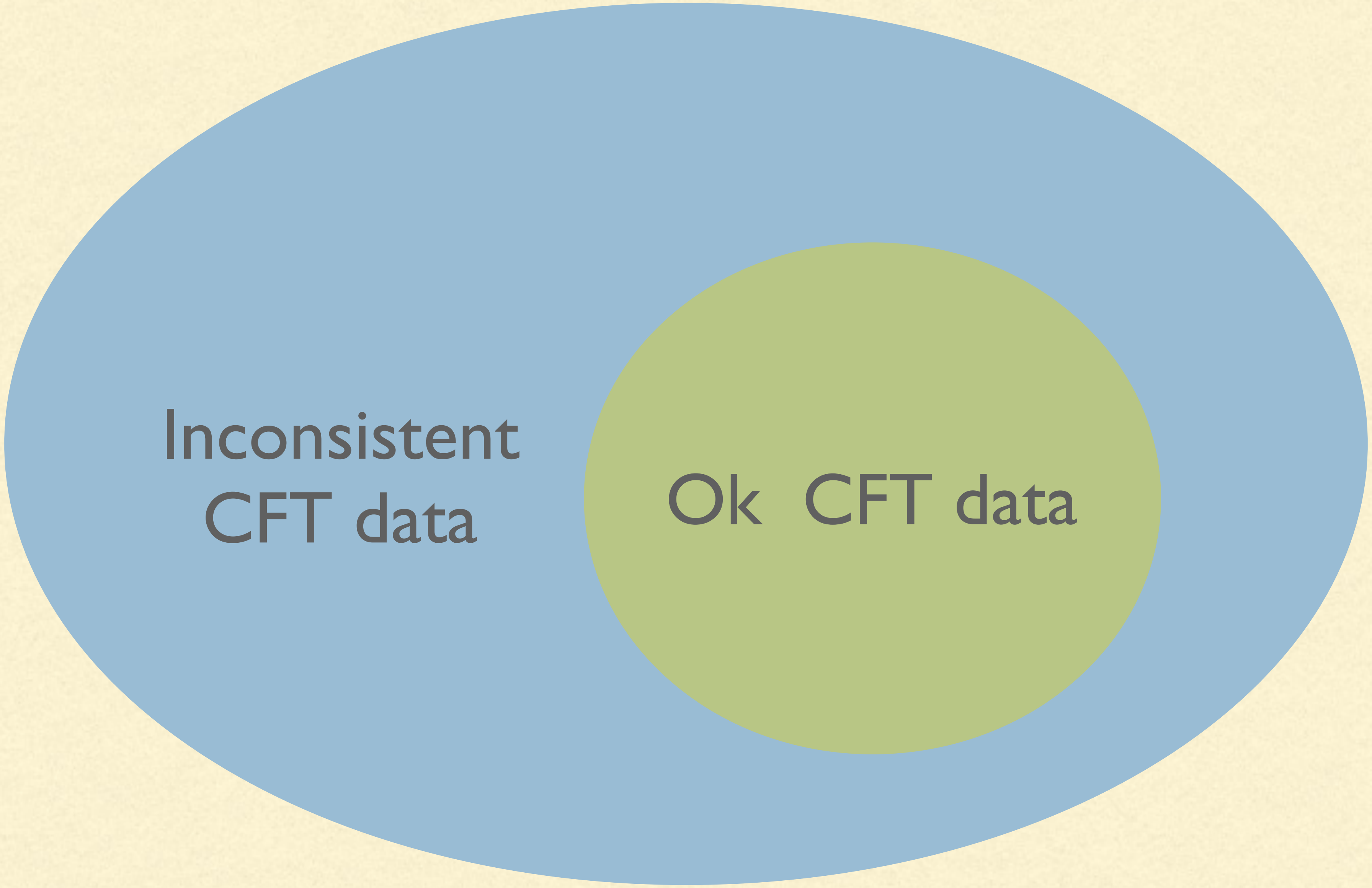

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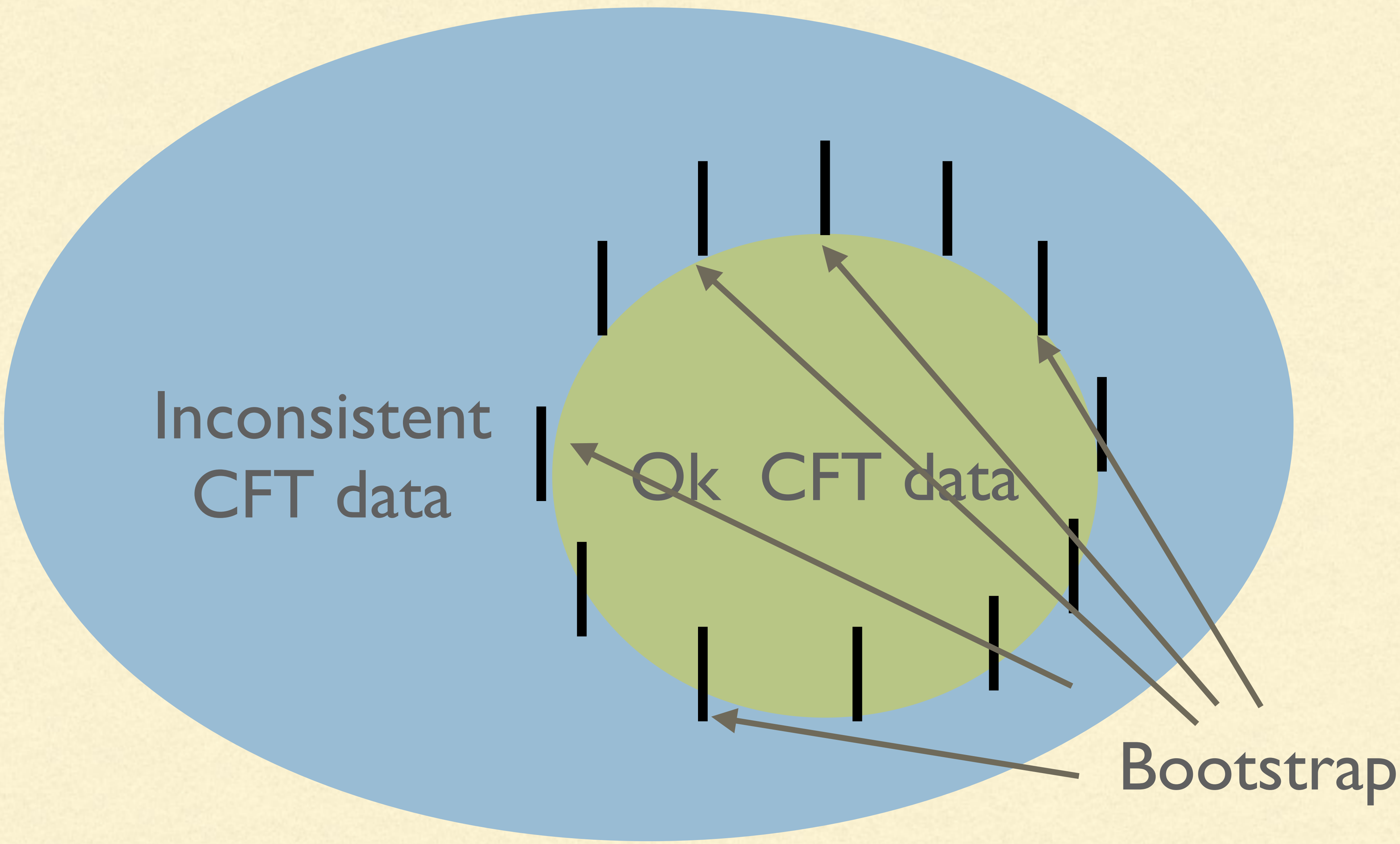

This **talk:**

Work in progress w. **Gary Shiu** (UW-Madison)

Photo: Matteo Lotito

Strategy and **preliminary results**
in using the modular bootstrap to prove
a strong version of the WGC in the worldsheet

- **Many versions** of WGC. "Universal" part of the statement:

$$\boxed{m \leq g M_P q}$$

Arkani-Hamed, Motl
Nicolis Vafa '06

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(Called "mild form" in Shiu-Cottrell-Soler '16)

- **More generally:**

Replace gM_P by Q/M of extremal black hole.
For several U(1)'s, apply in every direction of ch. lattice.

$$m \leq \sqrt{\langle \vec{Q}, \vec{Q} \rangle}$$

Cheung-Remmen '14

- **Comes in a lot of flavors!**

(Burati, Van de Heisteeg, Andriolo, Heidenreich, Bonnefoy, Gonzalo...)

Ibañez's talk

Axions, generalized gauge symmetries, scalar forces…

Grimm's talk

Arkani-Hamed Motl Nicolis Vafa '06, Heidenreich-Reece-Rudelius '16

Palti '17, Palti & Lust '17, Heidenreich-Reece-Rudelius '17, Grimm…

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Read Eran's review!

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Grimm's talk

- **Problem:** Mild WGC is **not** a constraint on the EFT!

(I've disappointed several PhD students disappointed here)

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(q can be as large as you want!)

S. Andriolo's talk

- Even **black holes themselves** can be the WGC particle!

(Via higher-derivative corrections that alter Q/M)

Some recent evidence suggests that **they must**

Cheung-Liu-Remmen '18, Andriolo-Junghans-Noumi-Shiu '18, Hamada-Noumi-Shiu '18, Mehrdad '19, Charles '19

- Also consistent with black hole heuristics and entanglement entropy arguments in AdS Arkani-Hamed, Motl Nicolis Vafa '06 MM '18

- Mild form understood — but what we really want is a **strong** form

$$\boxed{m \leq g M_P q, \quad \text{and} \quad q \leq a}$$

- This is really a constraint on the EFT!
- No "WGC" heuristics or EFT arguments. Just examples.

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Main idea: We think we can do better in the **worldsheet**

WORLDSHEET 101 AND WGC

Spectrum of string states ⟷ Level-matched, GSO-✓ primary operators of 2d CFT

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$$m^2 = \frac{2}{\alpha'}\Delta = \frac{2}{\alpha'}\left(h - \frac{c}{24} + \tilde{h} - \frac{\tilde{c}}{24}\right)$$

$$h = \tilde{h}$$

Spectrum of string states ⟷ Level-matched, GSO-✔ primary operators of 2d CFT

Fixed central charge

c=24, bosonic
c=12, superstring
both, heterotic

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Spectrum of string states ⟷ Level-matched, GSO-✔ primary operators of 2d CFT

Fixed central charge

c=24, bosonic
c=12, superstring
c=18, heterotic

A_μ ⟷ $: e^{ip^\nu X_\nu} : j\partial_\mu X$

$$Q = \int_{\text{Worldsheet}} j$$

For more than one U(1), spectrum of allowed charges forms a **lattice**

$$\vec{Q} \in \Lambda$$

On this lattice we have two inner products:

$$\langle j_a(z) j_b(z) \rangle = \frac{(\mathbf{N}_L)_{ab}}{z^2}, \quad \langle \bar{j}_a(z) \bar{j}_b(z) \rangle = \frac{(\mathbf{N}_R)_{ab}}{\bar{z}^2}$$

$$(\vec{Q}_1, \vec{Q}_2) = \frac{1}{2} \vec{Q}_L^T \mathbf{N}_L \vec{Q} - \frac{1}{2} \vec{Q}_R^T \mathbf{N}_R \vec{Q}$$

$$\langle \vec{Q}_1, \vec{Q}_2 \rangle = \frac{1}{2} \vec{Q}_L^T \mathbf{N}_L \vec{Q} + \frac{1}{2} \vec{Q}_R^T \mathbf{N}_R \vec{Q}$$

- Worldsheet global charge is spacetime electric charge.

- Beautiful **proof** of the WGC using **modular invariance**

Arkani-Hamed, Motl Nicolis Vafa '06

Heidenreich, Reece Rudelius '16, 18, 19

MM, Shiu Soler '16 ⟶ AdS_3

$$\vec{Q} \in \Lambda$$

Lattice of charges

$$Z(z,\tau) \equiv \mathrm{Tr}\left(q^{L_0}\bar{q}^{\tilde{L}_0}e^{2\pi i \vec{Q}\cdot\vec{z}}\right)$$

Extended chiral algebra

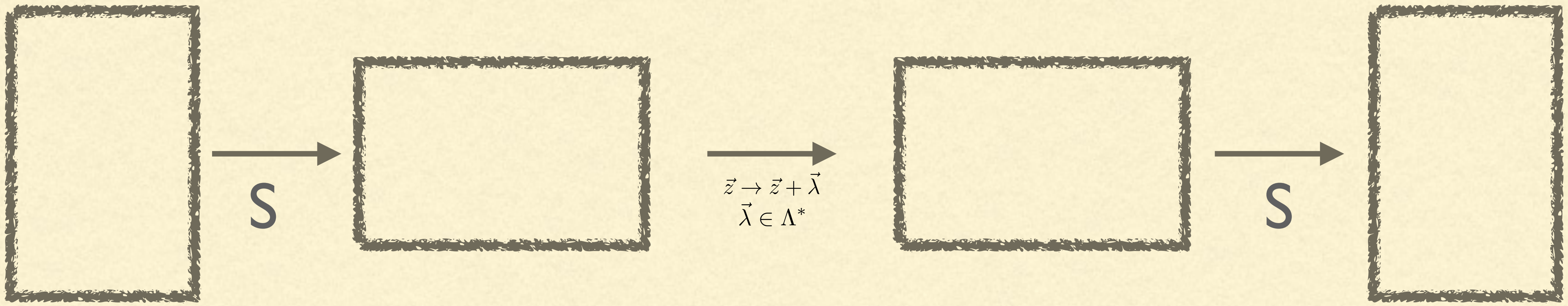
$$Z\left(\frac{z}{\tau}, -\frac{1}{\tau}\right) = e^{2\pi i(\vec{v},\vec{v})}Z(z,\tau) \qquad \vec{v} \equiv \left(\frac{z}{\tau}, \frac{z}{\bar{\tau}}\right)$$

Siegel modular form

(Similar story to Lerche-Lee-Weigand '18)

- Nice connection to SDC!

Lerche-Lee-Weigand '18,'19, Grimm-Palti-Valenzuela '18, Corvilain-Grimm-Valenzuela '18



Spectrum of mod-inv. 2d CFT is invariant under **spectral flow**

$$(\vec{Q}, \Delta) \to (\vec{Q} + \vec{\lambda}, \Delta + \langle \vec{\lambda}, \vec{\lambda} \rangle \qquad \Lambda^* \subset \Lambda$$

Spectral flow of graviton ops. produces states w.

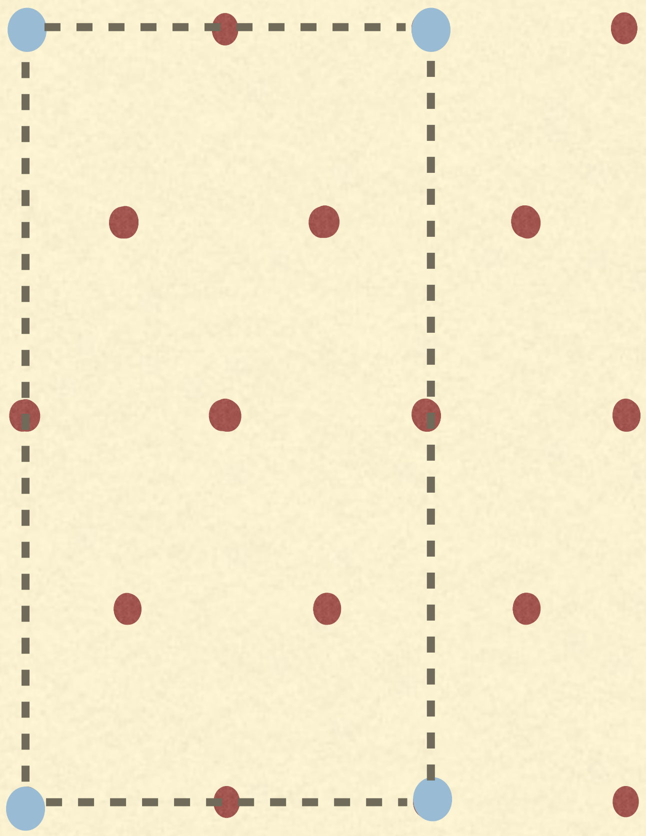
$$m = \frac{\sqrt{2}}{\alpha'} \sqrt{\langle \vec{\lambda}, \vec{\lambda} \rangle}$$

Heidenreich-Reece-Rudelius '16,'17,
Lerche-Lee-Weigand, Aalsma-Shiu '19

If the inner product coincides with notion of **extremality**,
this is a whole **lattice** of WGC particles!

- Goes under the name of **sublattice WGC:** There are extremal states for any

$$\vec{Q} \in \Lambda^* \subset \Lambda$$



- Missing WGC lattice sites form a **group**

$$\Gamma \equiv \Lambda/\Lambda^*$$

If Γ is "big", WGC particles are sparse

↓

This is a **mild** form, not constraining the EFT!

- Goes under the name of **sublattice WGC:** There are extremal states for any

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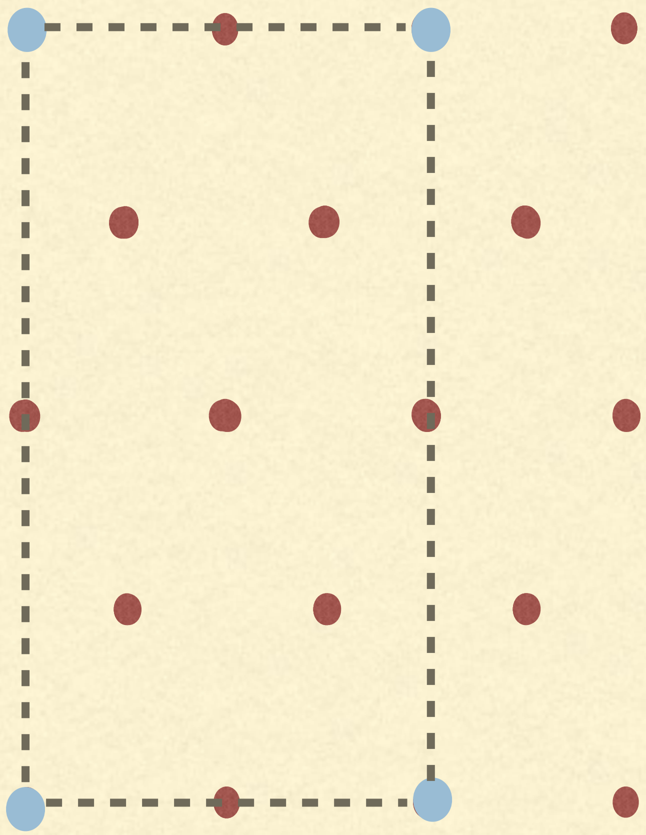

In **examples**, coarseness ("size" of Γ) is never big!

2,3 (Heidenreich-Reece-Rudelius '16)

6 (Lerche-Lee-Weigand '18)

6, possibly up to around 10 (Lerche-Lee-Weigand '19 + Lee, private communication)

Our job:

Upper-bound the **coarseness** of Γ using the **modular bootstrap**

HOW?

- Introduce Γ**-valued partition vector**

$$Z = \sum_{\gamma\in\Gamma} \mathcal{Z}_\gamma(\tau)\vartheta(\tau,\gamma), \quad \vec{\mathcal{Z}} = (\mathcal{Z}_{\gamma_1}, \mathcal{Z}_{\gamma_2}, \dots)$$

$$\vartheta(\tau,\gamma) \equiv \sum_{\vec{\lambda}\in\Lambda^*} e^{2\pi i(\tau_1(\vec{\gamma}+\vec{\lambda},\vec{\gamma}+\vec{\lambda})-i\tau_2\langle\vec{\gamma}+\vec{\lambda},\vec{\gamma}+\vec{\lambda}\rangle}$$

- Modular properties of the theta-functionsare well-known in mathematics, lead to **transformation law**

$$\vec{\mathcal{Z}}\left(\frac{a\tau+b}{c\tau+d}\right) = U_{a,b,c,d}\vec{\mathcal{Z}}(\tau)$$

Known finite-dim rep of SL(2,Z), that only depends on (; ·)

Parametrize τ = i e^s

then S maps s→-s

Hellerman '09, Lin-Shao '19, Yin '17, Dyer-Firzpatric-Yin '16, MM-Shiu-Soler '16, Qualls '14, Bae-Lee-Song '18…many more!

$$\vec{\mathcal{Z}}(-s) = \exp(-\frac{n_d}{2}s)U\vec{\mathcal{Z}}(s)$$

Number of linearly independent currents

Derivatives w.r.t β=-2π i τ are related to s-derivatives

$$\langle\Delta^k\rangle_a \mathcal{Z}_a = (-1)^k \frac{d^k \mathcal{Z}}{d\beta^k} = (-1)^k \sum_{l=0}^{k} \begin{bmatrix} k \\ l \end{bmatrix} (2\pi)^{-l} \mathcal{Z}_a^{(l)}$$

If k is even, this is positive!
If odd, bounded below by lightest op.

Derivative w.r.t. s

Can combine constraints into an **inequality**

$$\langle \Delta^k \rangle_a \mathcal{Z}_a - \left(\frac{1-(-1)^k}{2}\right)(\Delta_{0,a})^k \mathcal{Z}_a \geq \left(\frac{1+(-1)^k}{2}\right) e^{-2\pi \Delta_{0,a}} (\Delta_{0,a})^k$$

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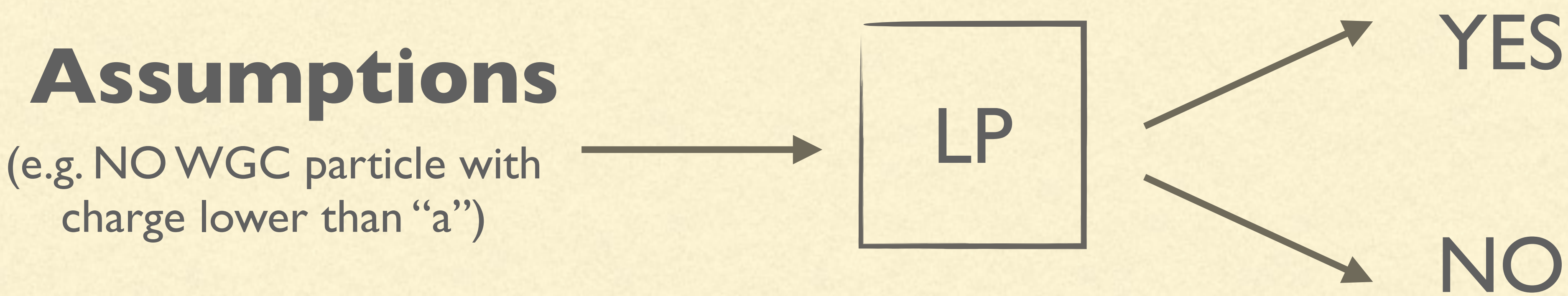


- If we want to get a **universal** constraint, must do this for **all** U i.e. all discrete groups Γ

$$U_{\gamma_1,\gamma_2} \propto \exp\left(2\pi i(\gamma_1,\gamma_2)\right)$$

Bilinear form on a discrete group

Wall '63, Miranda '84

- **Mathematicians** have proven that this can be reduced to one of **six cases**

$$\Gamma = \mathbb{Z}_{p_1}^{k_1} \oplus \mathbb{Z}_{p_1}^{k_1} \oplus \ldots$$

$$(\kappa,\kappa)_{A_{p^r}} = \frac{1}{p^r}, \quad (\kappa,\kappa)_{B_{p^r}} = \frac{\xi}{p^r}$$

$$A_{2^r}: \quad (\kappa,\kappa) = \frac{1}{2^r}, \quad B_{2^r}: \quad (\kappa,\kappa) = \frac{-1}{2^r},$$

$$C_{2^r}: \quad (\kappa,\kappa) = \frac{5}{2^r}, \quad D_{2^r}: \quad (\kappa,\kappa) = \frac{-5}{2^r}$$

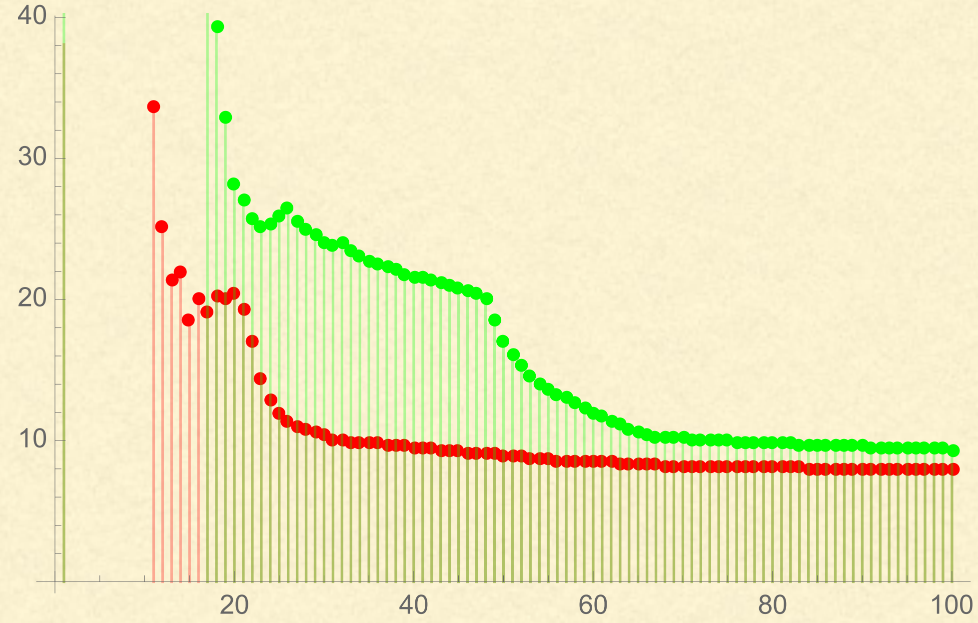
$$E_{2^r}: \quad (\kappa_i,\kappa_j) = \begin{pmatrix} 0 & \frac{1}{2^r} \\ \frac{1}{2^r} & 0 \end{pmatrix},$$

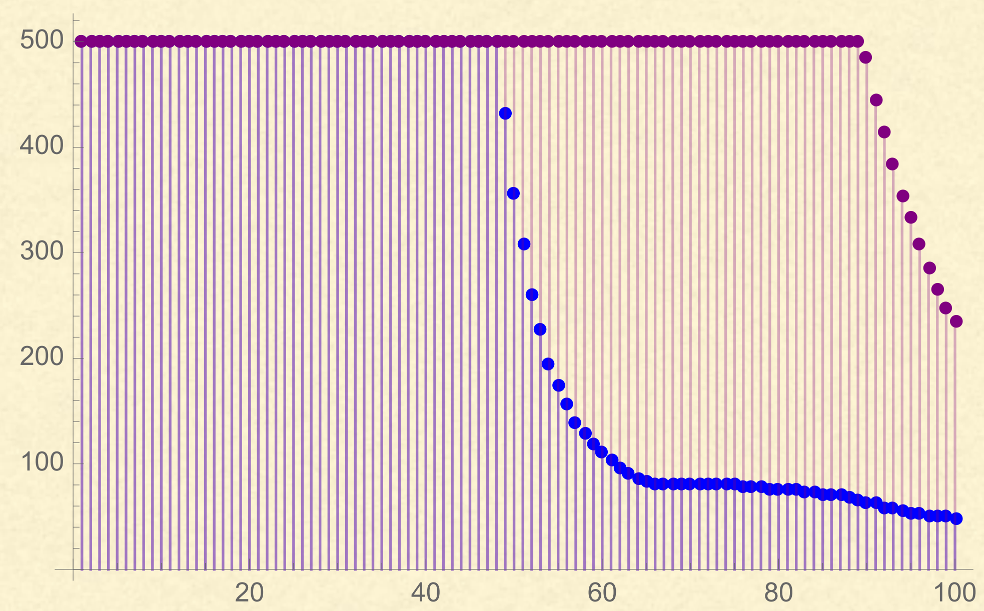
$$F_{2^r}: \quad (\kappa_i,\kappa_j) = \begin{pmatrix} \frac{1}{2^{r-1}} & \frac{1}{2^r} \\ \frac{1}{2^r} & \frac{1}{2^{r-1}} \end{pmatrix}$$


WORK IN
PROGRESS

Some **results**, with one **extra assumptions:**
-Lowest primary in each sector is a **worldsheet scalar**
-Only looking at **heterotic**
-Only looking at **A-class**

We are currently working on improving this/dropping the assumption, numerics takes time!





Numerics (**+ assumptions**) suggests WGC particle should have charge 7-8

CONCLUSIONS

- **Modular bootstrap** leads to a **strong version** of the WGC in the **worldsheet**
- Have some **assumptions** we are working to relax.
- We need to talk about fermions!
- Connection to SDC? F-theory/nonperturb. heterotic?

Lee, Lerche, Weigand '18, '19

Merci Beaucoup!