# A New Approach to Heterotic Moduli and Dualities

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# Heterotic Geometry and Phenomenology

- Heterotic phenomenology is closely tied to difficult questions in the geometry of manifolds and vector bundles
  - SUSY $\rightarrow$ bundle stability and holomorphy
  - Form of the 4D potential $\rightarrow$ Holomorphic Chern Simons theory, GKV Superpotential $W \sim \int_X H \wedge \Omega$ with $H \sim dB - \omega_{3YM} + \omega_{3L}$ (see James' talk)
  - Massless spectra/couplings $\rightarrow$ bundle valued cohomology, Yoneda products
- Finding realistic models and addressing moduli stabilization are both long-standing and difficult problems (though assorted recent progress).
- Better "control" of geometry of heterotic bundles/manifolds would be very helpful
- Here control = better dictionary linking EFT and geometry.

# Work in progress...

- This talk: a repackaging of heterotic geometry that may shed light on redundancies in the space of heterotic vacua and make it easier to find models with given spectra, moduli,etc. (Not yet String Pheno, but aiming that way...)
- Two hints of structure inspired this work:
- The heterotic moduli space naturally combines fluctuations of the background manifold and gauge fields (e.g. LA, Gray, Ovrut, Lukas, Sharpe, de la Ossa, Svanes, Hardy, Candelas, McOrist...).
- Heterotic dualities naturally mix moduli (and d.o.f) associated to manifolds and bundles (e.g. Distler, Kachru, Blumenhagen, Rahn, LA, Feng, etc)

# Holomorphic Vector bundles

- $V$ holomorphic if $F_{ab} = F_{\bar{a}\bar{b}} = 0$
- Suppose we begin with a holomorphic bundle and then vary the complex structure? Must a bundle stay holomorphic for any variation $\delta\mathfrak{z}^I v_I \in h^{2,1}(X)$? $\Rightarrow$ No
- $0 \to V \otimes V^\vee \to \mathcal{Q} \stackrel{q}{\to} TX \to 0$ is known as the Atiyah sequence.
- The long exact sequence in cohomology gives us

$$0 \to H^1(V \otimes V^\vee) \to H^1(\mathcal{Q}) \stackrel{dq}{\to} H^1(TX) \stackrel{\alpha}{\to} H^2(V \otimes V^\vee) \to \dots$$

- If the map $dq$ is surjective then $H^1(\mathcal{Q}) = H^1(V \otimes V^\vee) \oplus H^1(TX)$
- But $dq$ not surjective in general! $H^1(\mathcal{Q}) = H^1(V \otimes V^\vee) \oplus Im(dq)$
- $dq$ difficult to define, but by exactness, $Im(dq) = Ker(\alpha)$ where $\alpha = [F^{1,1}] \in H^1(V \otimes V^\vee \otimes TX^\vee)$ is the Atiyah Class

# Deformation Theory

There are three objects in deformation theory that we need

- $Def(X)$: Deformations of $X$ as a complex manifold. Infinitesimal defs parameterized by the vector space $H^1(TX) = H^{2,1}(X)$. These are the *complex structure* deformations of $X$.
- $Def(V)$: The deformation space of $V$ (changes in connection, $\delta A$) *for fixed* C.S. moduli. Infinitesimal defs measured by $H^1(End(V)) = H^1(V \otimes V^\vee)$. These define the *bundle moduli* of $V$.
- $Def(V, X)$: Simultaneous holomorphic deformations of $V$ and $X$. The tangent space is $H^1(X, \mathcal{Q})$ where

$$0 \to V \otimes V^\vee \to \mathcal{Q} \xrightarrow{\pi} TX \to 0$$

If $\mathcal{Z}$ is the (projectivized) total space of the bundle, $\mathcal{Q} = r_* T\mathcal{Z}$ (Donaldson).

# Heterotic redundancies

- Want to understand intriguing $(0,2)$ GLSM "Duality" from the '90s...
- Target Space Duality: Two $(0,2)$ GLSMs which share a non-geometric (i.e. LG) vacuum. In this case, the two large volume limits (i.e. $(X,V)$ and $(\tilde{X},\tilde{V})$ give same apparent effective 4D spectrum (Distler, Kachru, Blumenhagen...):

$$h^*(X,\wedge^k V) = h^*(\widetilde{X},\wedge^k \widetilde{V}), \qquad k = 1,2,\cdots,rk(V)$$
$$h^{2,1}(X) + h^{1,1}(X) + h^1_X(End_0(V)) = h^{2,1}(\widetilde{X}) + h^{1,1}(\widetilde{X}) + h^1_X(End_0(\widetilde{V}))$$

  and more recently shown to have same $4D$ potentials LA, Feng.
- Different manifolds and vector bundles, but same physics?
- Landscape study: Blumenhagen and Rahn created $\sim 83,000$ TSD pairs and nearly all produced same 4D spectra ($\sim 90\%$)

# Heterotic duals?

- E.g.

| $x_i$ | | | | | | | $\Gamma^j$ | | $\Lambda^a$ | | | | $p_l$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | −2 | −2 | 1 | 0 | 0 | 2 | −3 |
| 1 | 1 | 1 | 2 | 2 | 2 | 0 | −4 | −5 | 0 | 1 | 1 | 6 | −8 |

With $dim(\mathcal{M}_0) = h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = 2 + 68 + 322 = 392$

- $(0, 2)$ TSD dual

| $x_i$ | | | | | | $\Gamma^j$ | $\Lambda^a$ | | | | $p_l$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | −3 | 1 | 0 | 1 | 1 | −3 |
| 1 | 1 | 1 | 2 | 2 | 0 | −7 | 0 | 1 | 4 | 3 | −8 |

- $dim(\widetilde{\mathcal{M}}_0) = h^{1,1}(\widetilde{X}) + h^{2,1}(\widetilde{X}) + h^1(End_0(\widetilde{V})) = 2 + 95 + 295 = 392$

- Charged matter: $\#27$'s$= 120$,$\#\overline{27}$'s$= 0$

# More general possibilities

- $X = \mathbb{P}^5[2,4]$ with $0 \to V \to \mathcal{O}(1)^{\oplus 7} \to \mathcal{O}(3) \oplus \mathcal{O}(2)^{\oplus 2} \to 0$

  $dim(\mathcal{M}_0) = h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = 1 + 89 + 159 = 249$

- $\tilde{X} = \left[ \begin{array}{c} \mathbb{P}^1 \\ \mathbb{P}^5 \end{array} \middle| \begin{array}{ccc} 0 & 1 & 1 \\ 4 & 1 & 1 \end{array} \right]$ with

  $0 \to V \to \mathcal{O}(0,1)^{\oplus 5} \oplus \mathcal{O}(1,0) \oplus \to \mathcal{O}(0,3) \oplus \mathcal{O}(1,2) \to 0$

  $dim(\widetilde{\mathcal{M}}_0) = h^{1,1}(\widetilde{X}) + h^{2,1}(\widetilde{X}) + h^1(End_0(\widetilde{V})) = 2 + 86 + 161 = 249$

- Note: Base CY 3-folds related by conifold

- $(0,2)$ geometric transition?

# Questions

- Currently GLSM combinatorics lead to theories with the same spectra. Are they actually the same NLSM? Possibilities:
  - $(0,2)$ "Mirrors"? $\Leftrightarrow$ Same sigma models, different geometries.
  - $(0,2)$ Geometric transitions? (i.e. heterotic conifolds/flops). Branch structure in vacuum space?
  - Practically powerful tool (Might make it easier to find/characterize "interesting" heterotic vacua...)
  - Can this be understood purely in terms of geometry? $(X, V) \leftrightarrow (\tilde{X}, \tilde{V})$??

# $(0, 2)$ GLSMs as a playground

- Due to time, won't review here the mechanics/combinatorics of how $(0, 2)$ target space duality works here. Instead, will use $(0, 2)$ TS duals as a way of generating manifold bundle pairs leading to same heterotic EFT
- Question: In case of TSD or other heterotic "redundancies", can we produce Donaldson's (projective) total spaces of the bundle and can compare their properties?
- Hope is to extract "essential" features of manifold/bundle pair.

Inspired by GLSMs, let's begin by considering the case of $X$ a CY complete intersection manifold in a toric variety and $V$ defined via a monad

$$0 \to \mathcal{O}_{\mathcal{M}}^{\oplus r_{\mathcal{V}}} \xrightarrow{\otimes E_i^a} \oplus_{a=1}^{\delta} \mathcal{O}_{\mathcal{M}}(N_a) \xrightarrow{\otimes F_a^l} \oplus_{l=1}^{\gamma} \mathcal{O}_{\mathcal{M}}(M_l) \to 0$$

with $V = \frac{ker(F_a^l)}{im(E_i^a)}$

- Result: Let $V$ be a stable, holomorphic $SU(n)$ bundle.

  $V$ is defined via a monad over a toric CICY 3-fold iff its projectivized total space, $\mathcal{Z}$ is an $dim_{\mathbb{C}} = 3 + n - 1$ (Kahler) toric complete intersection manifold.

Let's consider an example of the total space for one of the previous examples:

- Given $X = \mathbb{P}^5[2,4]$ with $0 \to V \to \mathcal{O}(1)^{\oplus 7} \to \mathcal{O}(3) \oplus \mathcal{O}(2)^{\oplus 2} \to 0$
- $\mathcal{Z}$ defined by

$$\mathcal{Z} = \left[ \begin{array}{c|ccccc} \mathbb{P}^6 & 0 & 0 & 1 & 1 & 1 \\ \mathbb{P}^5 & 2 & 4 & 2 & 1 & 1 \end{array} \right]$$

  $\mathcal{Z}$ is a Kähler 6-fold with $h^{1,1} = 2$ and $h^1(T\mathcal{Z}) = 248$.

- Neat feature: The ambient space is determined by the ambient spaces of bundle/monad. If X is a CICY in $\mathcal{A}$ and if $0 \to V \to B \to C \to 0$ is a monad, can define fiber space of $V$ as CICY in $\mathcal{E} = \mathbb{P}(\pi : B \to X)$ (total ambient space not in general a product).

- Total space $\Leftrightarrow (X, V)$?

$$0 \rightarrow \mathcal{O}^{\oplus r} \rightarrow \bigoplus_{i=1} \mathcal{O}(\mathbf{D}_i) \rightarrow T\mathcal{A} \rightarrow 0$$
$$0 \rightarrow T\mathcal{Z} \rightarrow T\mathcal{A} \rightarrow \mathcal{N} \rightarrow 0$$

with $\mathbf{D_i}$ determined by GLSM charges and the $\mathcal{N} = \bigoplus_{j=1} \mathcal{O}(\mathbf{P}_j)$, with $\mathbf{P}_j$ is the multi-degree of the j-th hypersurface.

- How to construct $X, V$? If $0 \rightarrow A \xrightarrow{E} B \xrightarrow{F} C \rightarrow 0$ is a three-term monad, the "display" is useful:

$$\begin{array}{ccccccccc} & & 0 & & 0 & & 0 & & \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \rightarrow & A & \rightarrow & K & \rightarrow & V & \rightarrow & 0 \\ & & || & & \downarrow & & \downarrow & & \\ 0 & \rightarrow & A & \rightarrow & B & \rightarrow & Q & \rightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \rightarrow & 0 & \rightarrow & C & = & C & \rightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ & & 0 & & 0 & & 0 & & \end{array}$$

where $K = \ker(F)$ and $Q = \mathrm{coker}(E)$.

To reconstruct $(X, V)$ from $\mathcal{Z}$, consider the display and restrict to fiber and base e.g.

$$\begin{array}{ccccccccc} & & 0 & & 0 & & 0 & & \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \to & \mathcal{O} & \to & \pi^*(V) \otimes \xi_{\mathcal{Z}} & \to & T_{\mathcal{Z}|X} & \to & 0 \\ & & || & & \downarrow & & \downarrow & & \\ 0 & \to & \mathcal{O} & \to & \pi^*(B) \otimes \xi_{\mathcal{Z}} & \to & T_{\mathcal{E}|X}|_{\mathcal{Z}} & \to & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \to & 0 & \to & \pi^*(C) \otimes \xi_{\mathcal{Z}} & = & \mathcal{N}|_{\mathcal{Z}} & \to & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ & & 0 & & 0 & & 0 & & \end{array}$$

We have "reconstructed" $V$ from the $\mathbb{P}^{n-1}$ fiber of $\mathcal{Z}$.

**Useful for systematically classifying heterotic geometries**?

It is natural to ask where the degrees of freedom of the heterotic theory are realized in $\mathcal{Z}$?

- $h^1(\mathcal{Z}, T\mathcal{Z}) = h^1(X, \mathcal{Q})$ (i.e. the complex moduli of a heterotic theory)
- $h^{1,1}(\mathcal{Z}) = 1 + h^{1,1}(X)$ (one more than the number of Kahler moduli. Dilaton?)
- The *tautological line bundle* $\xi_{\mathcal{Z}}$ is uniquely defined by the properties that $\xi_{\mathcal{Z}}|_F = \mathcal{O}_F(1)$ and $\pi_*(\xi_{\mathcal{Z}}) = V$ (moreover $c_1(\mathcal{Z}) = n\xi_{\mathcal{Z}}$ for an $Su(n)$ bundle).

$$h^*(Z, \xi_{\mathcal{Z}}) = h^*(X, V) \text{ i.e. counts charged matter}$$

- Chern classes: $ch(\mathcal{Z}) = function(ch_2(X) = ch_2(V), ch_3(V))$, etc

# Summary

- We have begun a systematic rewriting of heterotic geometry in terms of the total space of the bundle
- In the case of $SU(n)$ monad bundles and toric CICY 3-folds $\Rightarrow$ explicit realization of Kähler $(n+2)$-fold as a toric CICY.
- In the simplest cases of $(0,2)$ "dualities", Hodge numbers, Chern classes and cohomology of the tautological line bundle are identical $\Rightarrow$ Sigma model automorphism?
- In other dualities, can in principle track geometric transitions in total space $(1 \leftrightarrow 1?)$, properties of 4D EFT preserved.
- Utility for new $(0,2)$ dualities? String Pheno? Systematic constructions?
- Further study underway...