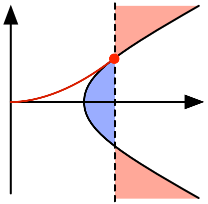
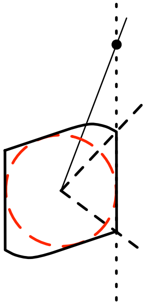
The Weak Gravity Conjecture and Repulsive Forces



Ben Heidenreich
(UMass Amherst)



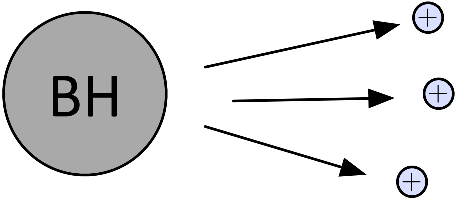
BH, Reece, Rudelius, 1906.02206
BH, to appear

Alim, BH, Rudelius, to appear
BH, Lotito, work in progress

String Pheno 2019, 25/06/19

The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06



Subextremal black holes can decay

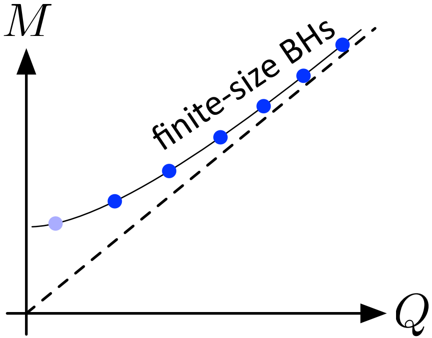
$\exists$ particle w/ $\quad |q|/m \geq |q|/m \Big|_{\text{ext BH}}$

The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06

WGC violation:

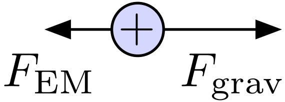
Lightest BHs of fixed charge are stable!

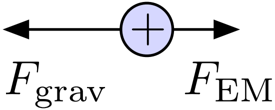


c.f., Kats, Motl, Padi '06
many recent works

The Weak Gravity Conjecture

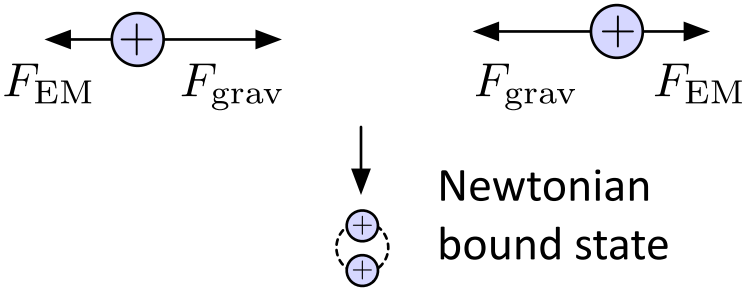
Arkani-Hamed, Motl, Nicolis, Vafa '06





The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa '06



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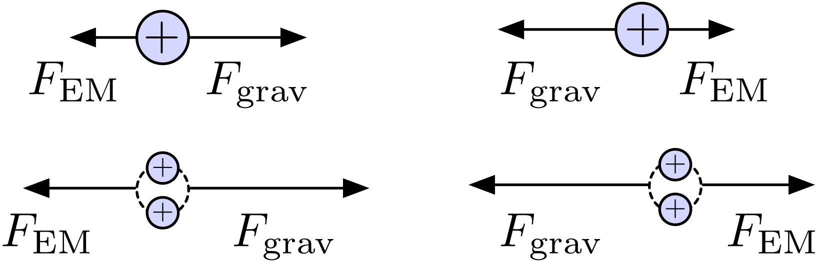

The Weak Gravity Conjecture

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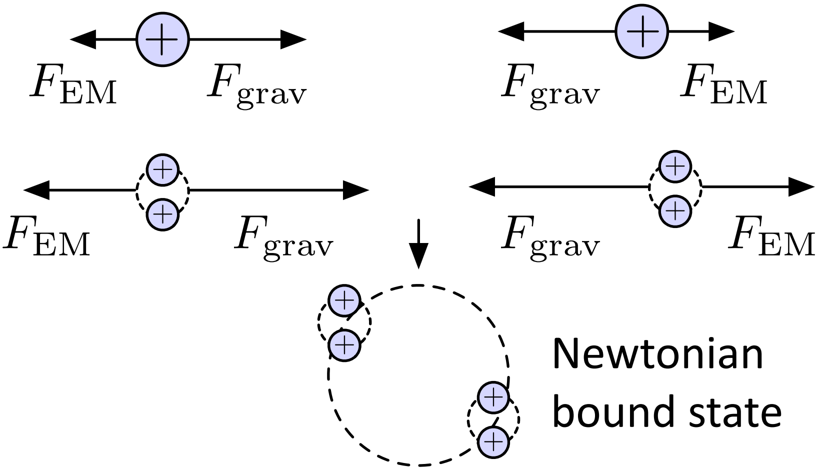
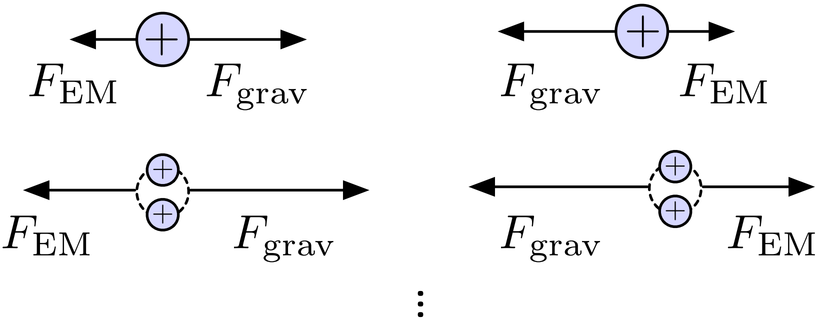


Newtonian bound state

The Weak Gravity Conjecture

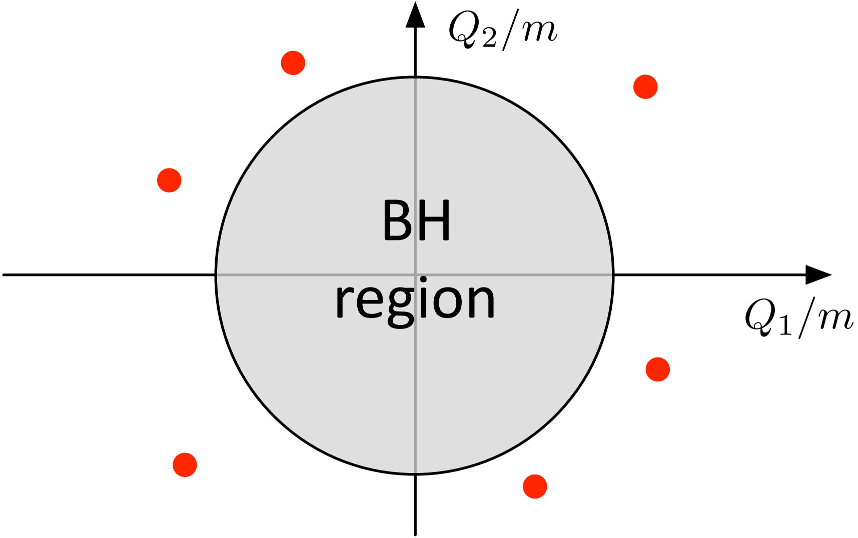
Arkani-Hamed, Motl, Nicolis, Vafa '06



WGC violation implies infinite tower of Newtonian bound states (in $D = 4$)

The WGC – Multiple γs

Cheung
Remmen '14



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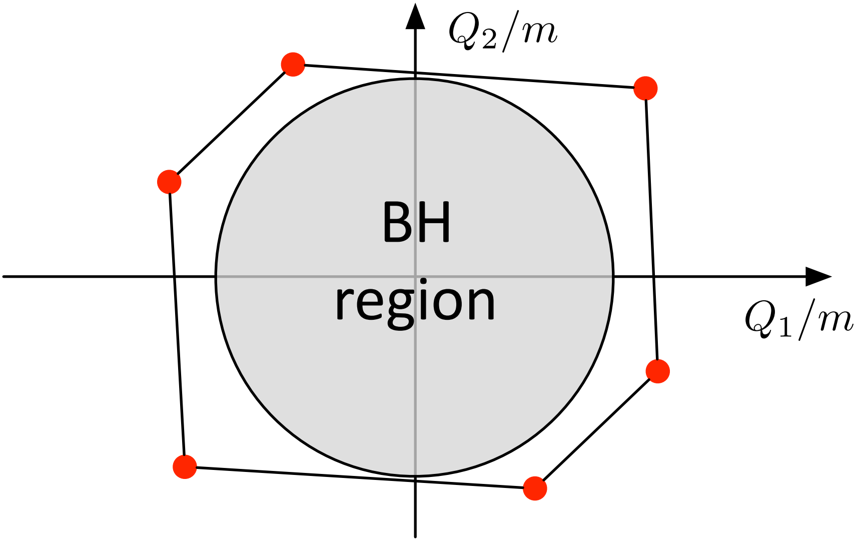

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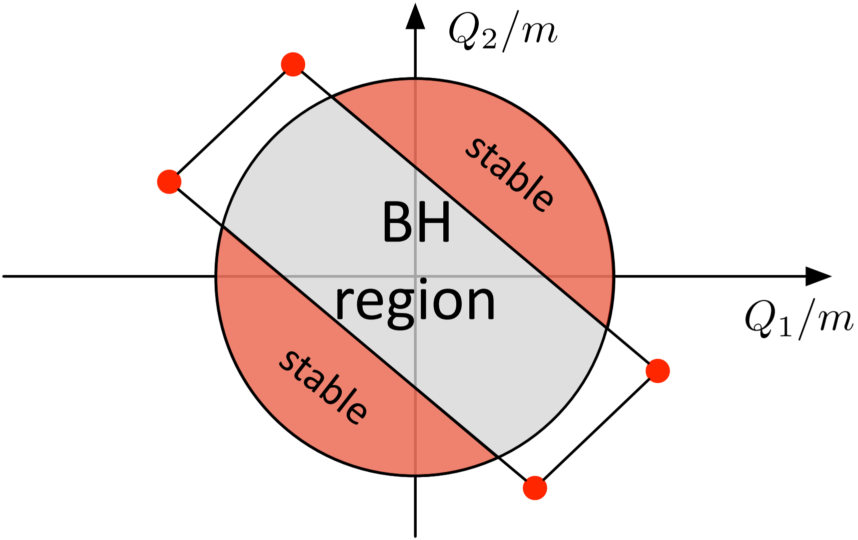
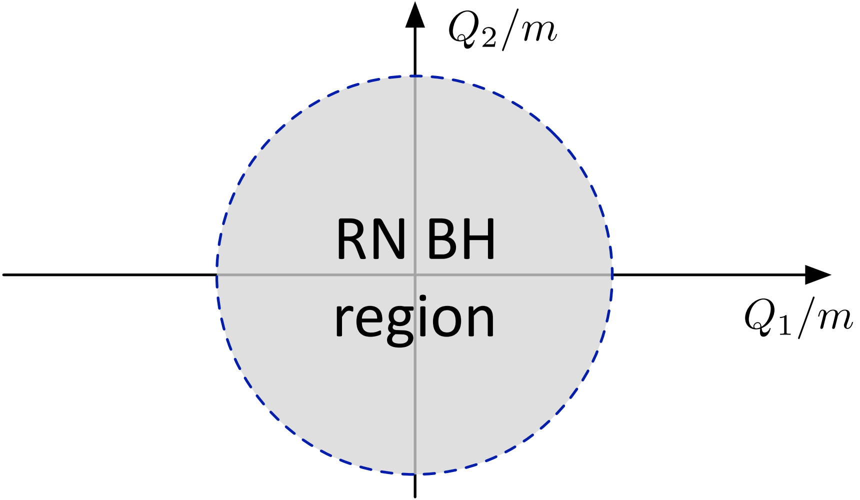


The WGC with Moduli

BH, Reece
Rudelius '15



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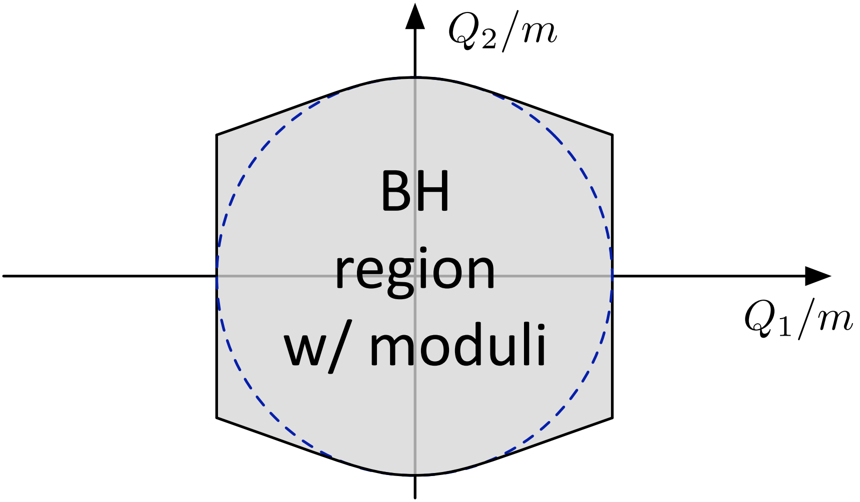

Ex. from Alim, BH, Rudelius, to appear

How to compute BH ext. bound?

BH, to appear
older works

Use the low-energy EFT:

$$S=\int d^Dx\left(\frac{1}{2\kappa_D^2}\sqrt{|g|}R-\frac{1}{2}G_{\phi\phi}(\phi)(\nabla\phi)^2-\frac{1}{4e^2(\phi)}F_{\mu\nu}F^{\mu\nu}\right)$$

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Extremality bound is solution to diff. eqn:

$$\frac{D-3}{D-2}\kappa_D^2 M^2(\phi) + G_{\phi\phi}^{-1}(\phi)\left(\frac{dM}{d\phi}\right)^2 = e^2(\phi)Q^2$$

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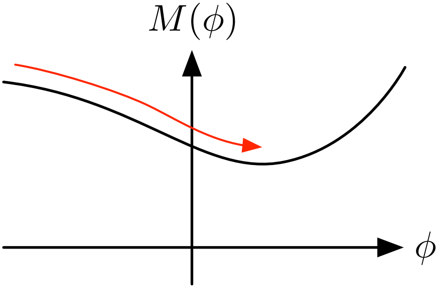
…such that $M(\phi) > 0$ everywhere along descending gradient flow path.

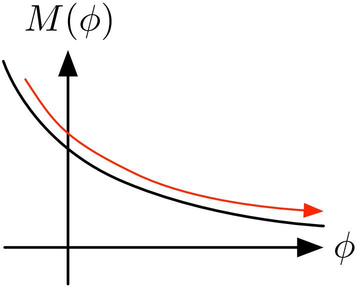
How to compute BH ext. bound?

BH, to appear
older works

Characteristic Exs:



“Attractor”



“Dilaton-like”

Extremal BH solutions

Extremal solns given by gradient flow:

$$\frac{d\phi}{dz} = -e^{\psi} G^{-1}_{\phi\phi} \frac{dM}{d\phi} \qquad \frac{d\psi}{dz} = -\frac{D-3}{D-2} e^{\psi} \kappa_D^2 M(\phi)$$

where
$$ds^2 = -e^{2\psi} dt^2 + e^{-\frac{2\psi}{D-3}} (dr^2 + r^2 d\Omega^2)$$
$$z = \frac{1}{(D-3)(\mathrm{Vol}\, S^{D-2})\, r^{D-3}}$$

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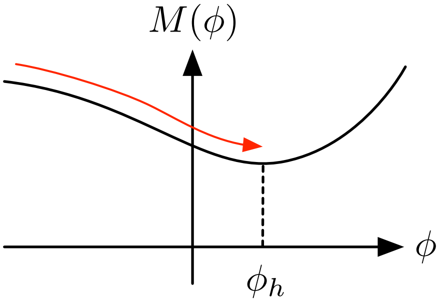
$$z = \frac{1}{(D-3)(\mathrm{Vol}\, S^{D-2})\, r^{D-3}}$$

$M(\phi)$ is “fake superpotential”

Ceresole, Dall’Agata '07
Andrianopoli, D’Auria, Orazi, Trigiante '07

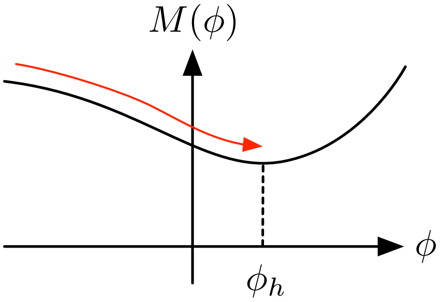
Extremal BH solutions



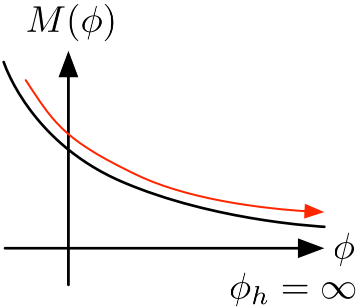
Attractor mechanism

Ferrara, Kallosh, Strominger '95
Ferrara, Kallosh '96 (x2)

Extremal BH solutions



vs.



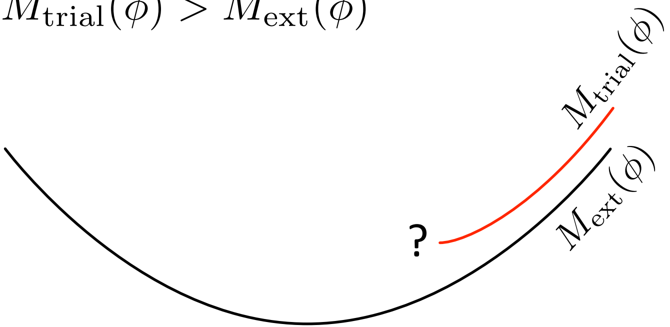
Attractor mechanism

Ferrara, Kallosh, Strominger '95
Ferrara, Kallosh '96 (x2)

Dilaton-like;
singular in ext. limit

Initial conditions: locally unique

$$M_{\mathrm{trial}}(\phi) > M_{\mathrm{ext}}(\phi)$$



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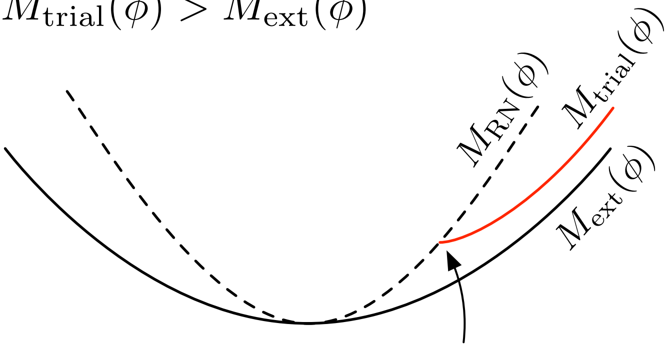

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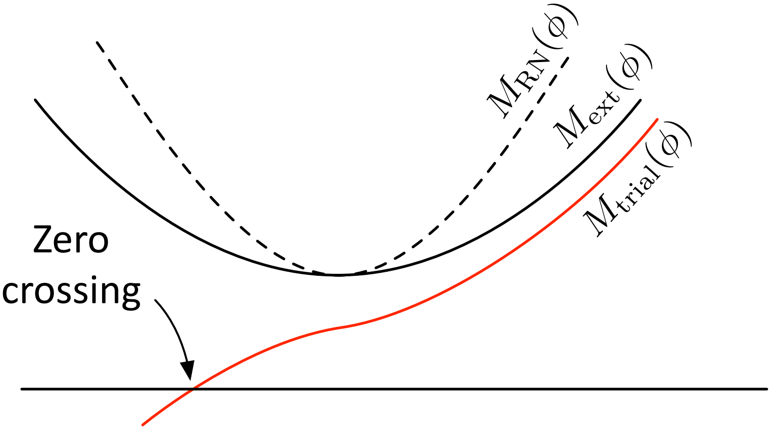
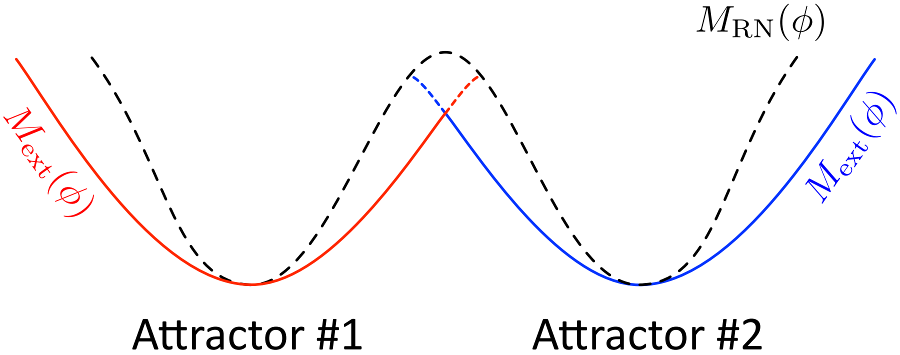


…but globally ambiguous



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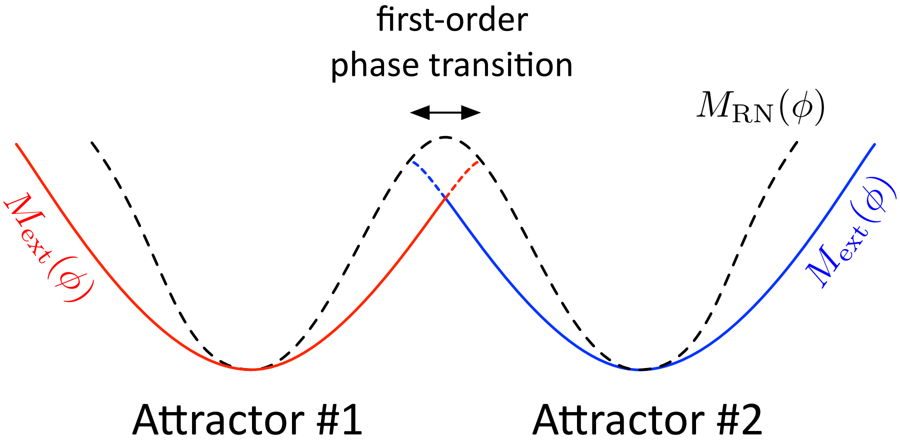
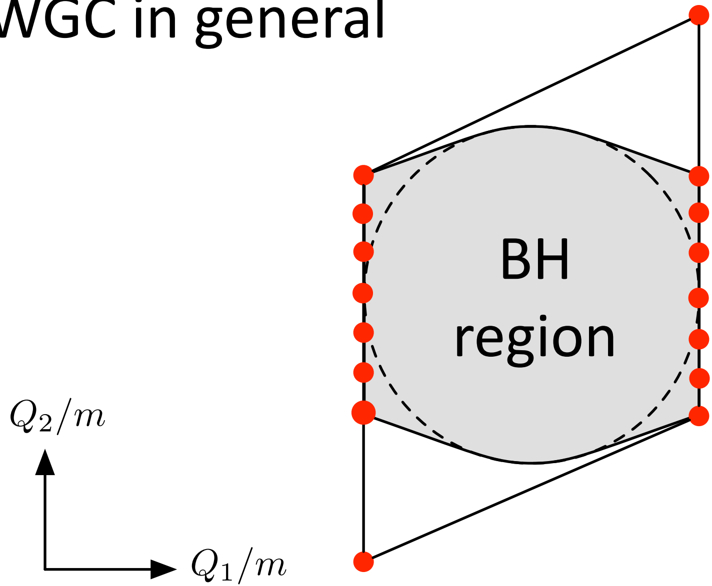


WGC in general



WGC in general

Equivalently:

$\forall$ rational charge direction $\hat{Q}$ ($\hat{Q} \propto \vec{q} \in \Gamma$),

$\exists$ a superextremal multipart. state with charge $\vec{Q} \propto \hat{Q}$

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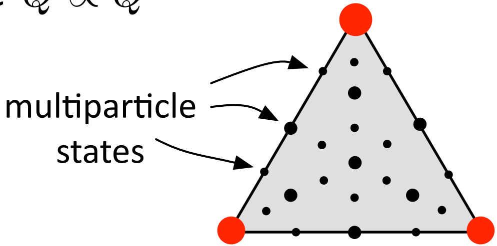

Bound states w/ moduli

$$F_{12} = \frac{e^2 Q_1 Q_2 - \frac{D-3}{D-2}\kappa_D^2 m_1 m_2 - G^{-1}_{\phi\phi}\frac{dm_1}{d\phi}\frac{dm_2}{d\phi}}{(\mathrm{Vol}\, S^{D-2}) r^{D-2}} + \dots$$

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gauge

gravity

modulus

short-range

Bound states w/ moduli

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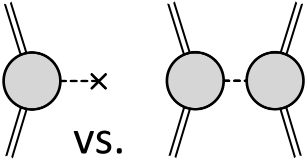
gauge | gravity | modulus | short-range

Scalar "charge" is $\frac{dm}{d\phi}$:

$$S_{\rm pp} = -\int m(\phi) ds - q \int A$$
$$+(\text{deriv. interactions})$$



Bound states w/ moduli

Particle attracts identical copy if

$$\mathcal{F} \equiv e^2 q^2 - \frac{D-3}{D-2}\kappa_D^2 m^2 - G^{-1}_{\phi\phi}\left(\frac{dm}{d\phi}\right)^2 < 0$$

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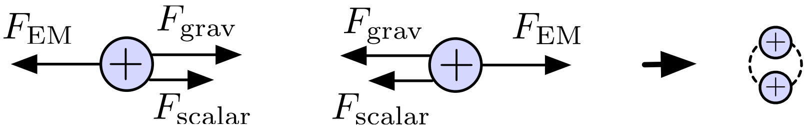
→ Newtonian/scalar bound state (D=4)



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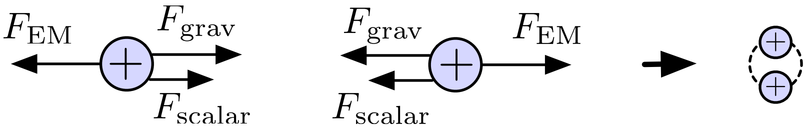

Depends on $\frac{|q|}{m}$ **and** $\frac{1}{m}\frac{dm}{d\phi}$.

Repulsive Force Conjecture

Palti '17

$\exists$ a self-repulsive ($\mathcal{F} \geq 0$) charged particle.

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What about ext. BHs?

BH, to appear

$$\frac{D-3}{D-2}\kappa_D^2 M^2(\phi) + G_{\phi\phi}^{-1}(\phi)\left(\frac{dM}{d\phi}\right)^2 = e^2(\phi)Q^2$$

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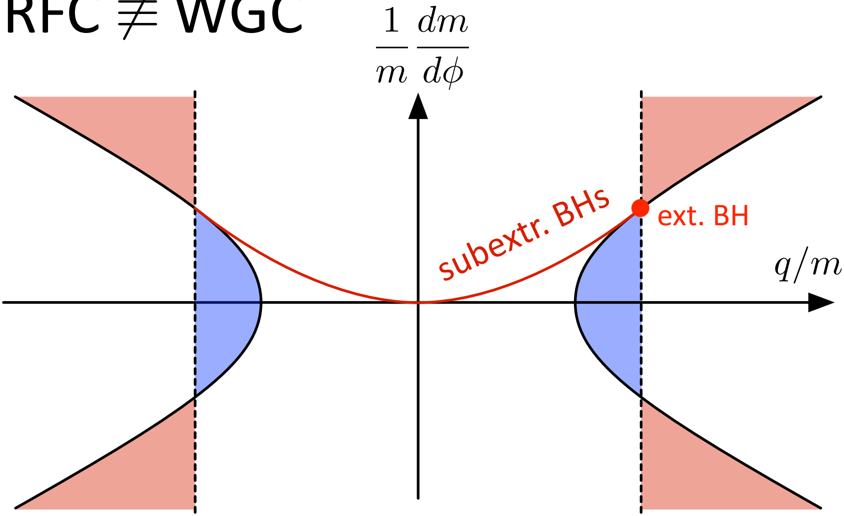
$$\Longrightarrow \mathcal{F} = 0 \ !$$

So RFC $\equiv$ WGC??

RFC $\neq$ WGC



Lee, Lerche, Weigand '18
BH, Reece, Rudelius '19

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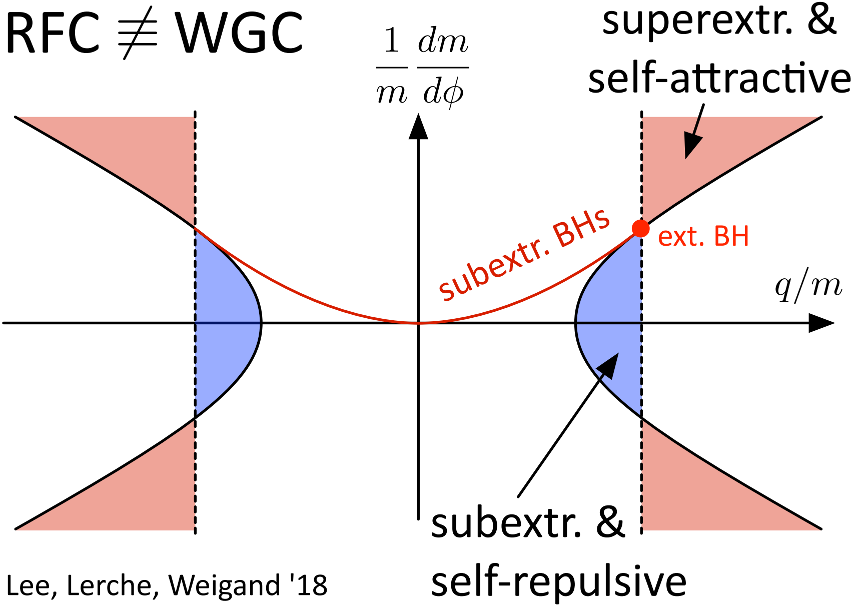

Lee, Lerche, Weigand '18
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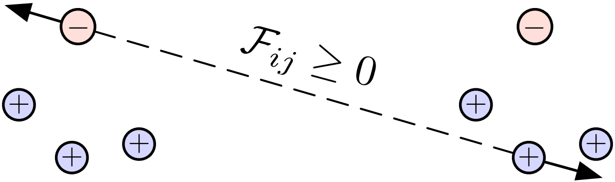
CHC for the RFC

BH, Reece, Rudelius '19

Strongly self-repulsive m.p. state:



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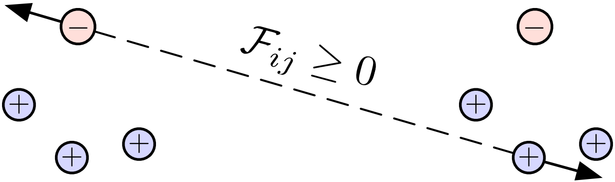

$\forall$ rational charge direction $\hat{Q}$,

$\exists$ a strongly self-repulsive m.p. state with charge $\vec{Q} \propto \hat{Q}$

RFC examples: 10d Heterotic ST

Lightest particle of charge Q:

$$m^2 = \frac{2}{\alpha'}(Q^2 - 2) = e^2 M_{10}^2 (Q^2 - 2)$$

RFC examples: 10d Heterotic ST

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WGC: $e^2 Q^2 M_{10}^2 \geq \left[\frac{\alpha^2}{2} + \frac{7}{8}\right] m^2 = m^2 \quad \left(\alpha = \frac{1}{2}\right)$

RFC: $e^2 Q^2 M_{10}^2 \geq 2\left(\frac{dm}{d\Phi}\right)^2 + \frac{7}{8} m^2 = m^2$

$\left(\frac{dm}{d\Phi} = \frac{m}{4}\right)$

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Bounds are equivalent!

RFC examples: Het. ST on T^{10-D}

$$\frac{\alpha'}{4}m^2 = \frac{1}{2}Q_L^2 + N - 1 = \frac{1}{2}Q_R^2 + \tilde{N}, \qquad N, \tilde{N} \geq 0$$

Ext. bound (Sen '94): $\frac{\alpha'}{4}m^2 \geq \frac{1}{2}\max(Q_L^2, Q_R^2)$

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RFC:

$$\mathcal{F} = \frac{2\kappa_D^2}{\alpha'}(Q_L^2 + Q_R^2) - \kappa_D^2 m^2 - \kappa_D^2 \frac{Q_L^2 Q_R^2}{\frac{(\alpha')^2}{4}m^2}$$

$\mathcal{F}_{\text{gauge}}$ $\quad$ $\mathcal{F}_{\text{grav+dil.}}$ $\quad$ $\mathcal{F}_{\text{Wilson line}}$

($\Gamma \to \Lambda\Gamma$)

RFC examples: Het. ST on T^{10-D}

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RFC:

$$\mathcal{F} = -\frac{4\kappa_D^2}{(\alpha' m)^2}\left(\frac{\alpha'}{2}m^2 - Q_L^2\right)\left(\frac{\alpha'}{2}m^2 - Q_R^2\right)$$

RFC examples: Het. ST on T^{10-D}

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$\mathcal{F} \geq 0$: *either* $\frac{\alpha'}{2}m^2 = Q_R^2$ (BPS, $\mathcal{F} = 0$)

or $\frac{\alpha'}{2}m^2 \leq Q_L^2$

RFC examples: Het. ST on T^{10-D}

$$\frac{\alpha'}{4}m^2 = \frac{1}{2}Q_L^2 + N - 1 = \frac{1}{2}Q_R^2 + \tilde{N}, \qquad N, \tilde{N} \geq 0$$

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RFC & WGC bounds again equivalent!

F-theory exs: (Lee, Lerche, Weigand '18, '19)

BH/self-force theorems

BH, to appear

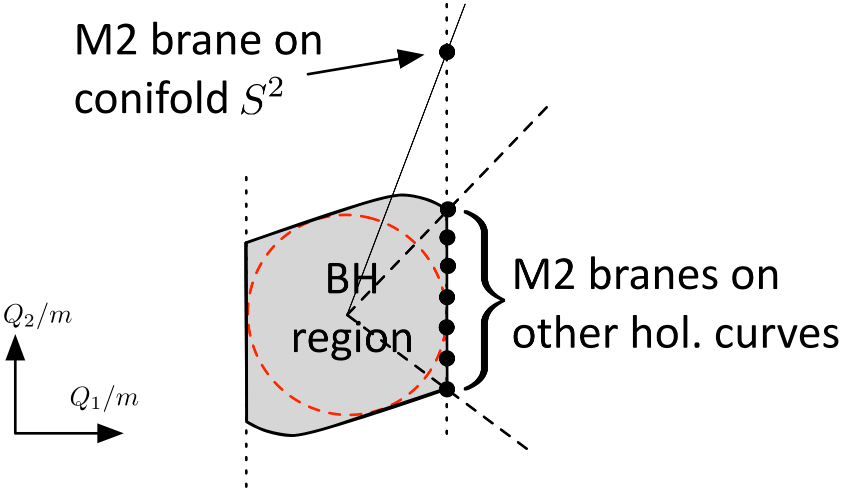
I. A particle that is self-repulsive throughout moduli space is superextremal

II. A particle that has vanishing self-force and non-vanishing mass throughout moduli space is extremal

$$\frac{D-3}{D-2}\kappa_D^2 M^2(\phi) + G_{\phi\phi}^{-1}(\phi)\left(\frac{dM}{d\phi}\right)^2 = e^2(\phi)Q^2$$

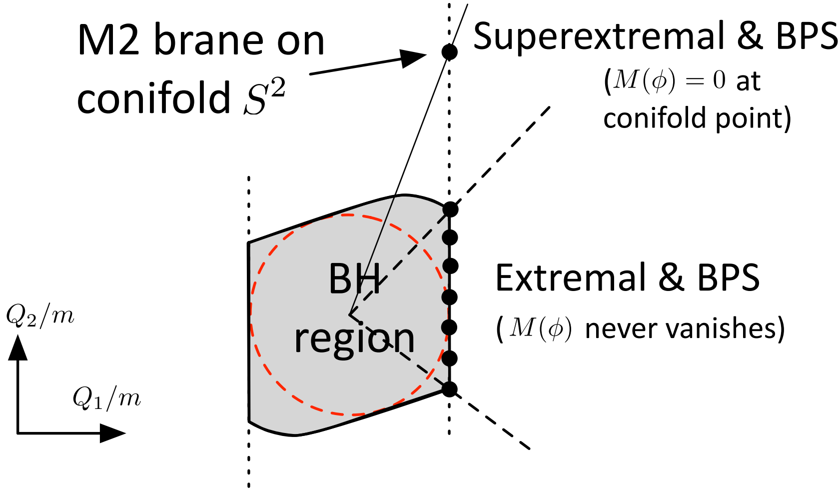
s.t. $M(\phi) > 0$ everywhere

Ex: BPS versus extremal



Ex. from Alim, BH, Rudelius, to appear

Ex: BPS versus extremal



Ex. from Alim, BH, Rudelius, to appear

Are RFC/WGC truly independent?

Assume 4d theory, one γ, WGC, ~~RFC~~

$\Longrightarrow \exists$ superext., self-attractive particle

BH, Reece, Rudelius '19

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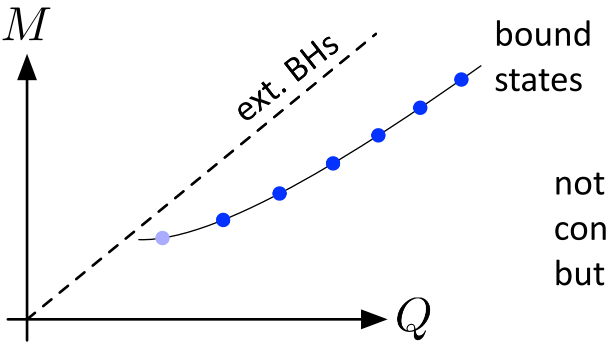


(not BHs!)

not a sharp contradiction, but bizzare

BH, Reece, Rudelius '19

Are RFC/WGC truly independent?

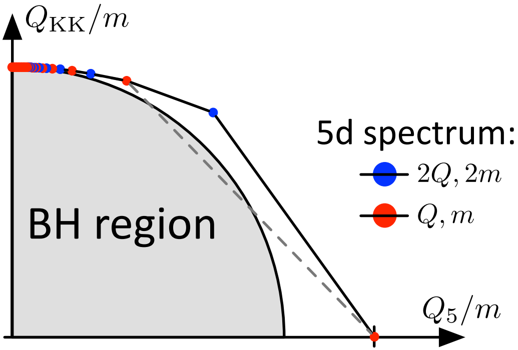
Lee, Lerche, Weigand '18:

WGC/RFC agree in asymptotic regions of moduli space

(Under certain assumptions; excluding bizarre scenarios)

Are RFC/WGC equivalent in ST?

More generally, consider "strong forms":



BH, Reece
Rudelius '15

Are RFC/WGC equivalent in ST?

More generally, consider "strong forms":

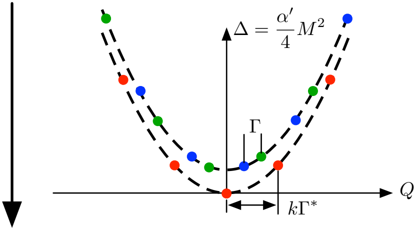
Tower WGC: $\forall \vec{Q} \in \Gamma,\ \exists n \in \mathbb{Z}_{>0}$ s.t. there is a superextr. particle of charge $n\vec{Q}$.

Andriolo, Junghans, Noumi, Shiu '18

BH, Reece, Rudelius '19

Are RFC/WGC equivalent in ST?

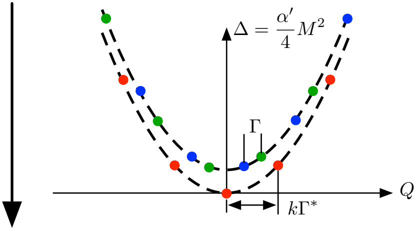
String modular invariance



BH, Reece, Rudelius '16
Montero, Shiu, Soler '16

Are RFC/WGC equivalent in ST?

String modular invariance

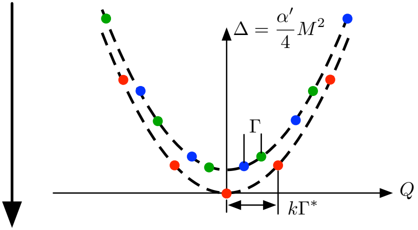


BH, Reece, Rudelius '16
Montero, Shiu, Soler '16

Sublattice WGC: $\exists n \in \mathbb{Z}_{>0}$ s.t. $\forall \vec{Q} \in \Gamma$, there is a superextr. particle of charge $n\vec{Q}$.

Are RFC/WGC equivalent in ST?

String modular invariance



BH, Reece, Rudelius '16
Montero, Shiu, Soler '16

Sublattice WGC: $\exists n \in \mathbb{Z}_{>0}$ s.t. $\forall \vec{Q} \in \Gamma$,
there is a superextr. particle of charge $n\vec{Q}$.

Caveat: $\frac{\alpha'}{4} m^2 = \frac{1}{2} \max(Q_L^2, Q_R^2)$ is extremal?

The sublattice RFC

Sublattice RFC: $\exists n \in \mathbb{Z}_{>0}$ s.t. $\forall \vec{Q} \in \Gamma$, there is a self-repulsive particle of charge $n\vec{Q}$.

Similar KK theory motivation (for TRFC)

Towards a proof of the sLRFC

BH, Lotito, work in progress

Show that particles with mass

$$\frac{\alpha'}{4}m(\phi)^2 = \frac{1}{2}\max(Q_L^2, Q_R^2)$$

have vanishing self force.

Towards a proof of the sLRFC

BH, Lotito, work in progress

Show that particles with mass

$$\frac{\alpha'}{4} m(\phi)^2 = \frac{1}{2} \max(Q_L^2, Q_R^2)$$

have vanishing self force.

These couple to Wilson line moduli ($\Gamma' = \Lambda\Gamma$) and the dilaton

WL moduli: $\lambda^{a\tilde{b}}(z,\bar{z}) = J^a(z)\tilde{J}^{\tilde{b}}(\bar{z})$

Towards a proof of the sLRFC

BH, Lotito, work in progress

Calculate κ_D^2, f_{ab}, $G_{\lambda\lambda'}$, $G_{\Phi\Phi}$, $G_{\lambda\Phi}$, $G_{\lambda i}$, $G_{\Phi i}$
in terms of CFT 0,1,2-pt. functions

Towards a proof of the sLRFC

BH, Lotito, work in progress

Calculate κ_D^2, f_{ab}, $G_{\lambda\lambda'}$, $G_{\Phi\Phi}$, $G_{\lambda\Phi}$, $G_{\lambda i}$, $G_{\Phi i}$
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other moduli

Towards a proof of the sLRFC

BH, Lotito, work in progress

Calculate κ_D^2, f_{ab}, $G_{\lambda\lambda'}$, $G_{\Phi\Phi}$, $G_{\lambda\Phi}$, $G_{\lambda i}$, $G_{\Phi i}$ in terms of CFT 0,1,2-pt. functions

$\rightarrow 0$

Towards a proof of the sLRFC

BH, Lotito, work in progress

Calculate κ_D^2, f_{ab}, $G_{\lambda\lambda'}$, $G_{\Phi\Phi}$, $G_{\lambda\Phi}$, $G_{\lambda i}$, $G_{\Phi i}$ in terms of CFT 0,1,2-pt. functions

0

All of these are *universal*

$\Longrightarrow$ by same calc. as Het. on T^{10-D},

$\mathcal{F} = 0$!

(Checked so far for *bosonic* string)

A Weak Gravity Theorem

BH, Lotito, work in progress

Actually, this fixes the normalization issue in the sLWGC proof!

Recall: a particle with vanishing self-force and non-vanishing mass throughout moduli space is extremal BH, to appear

A Weak Gravity Theorem

BH, Lotito, work in progress

Actually, this fixes the normalization issue in the sLWGC proof!

So the extremality bound is always

$$\frac{\alpha'}{4} m(\phi)^2 \geq \frac{1}{2} \max(Q_L^2, Q_R^2)$$

in tree-level ST,

A Weak Gravity Theorem

BH, Lotito, work in progress

Actually, this fixes the normalization issue in the sLWGC proof!

So the extremality bound is always

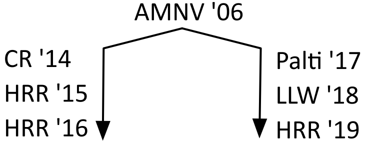
$$\frac{\alpha'}{4}m(\phi)^2 \geq \frac{1}{2}\max(Q_L^2, Q_R^2)$$

in tree-level ST,

…provided above argument succeeds.

Summary

A priori, WGC & RFC are indep. conjectures.



…and many others

Summary

A priori, WGC & RFC are indep. conjectures.

However, they are closely related:

Ext. BHs have vanishing self-force

WGC w/o RFC $\Longrightarrow$ large, superext. bound states

sLWGC & sLRFC have unified ST proof
(at tree-level in NSNS sector)

Summary

A priori, WGC & RFC are indep. conjectures.

However, they are closely related:

Ext. BHs have vanishing self-force, …

Important distinctions remain:

BPS states have vanishing self-force
but are not always extremal!