Generic matter & discrete gauge group structure in 6D and 4D supergravity and F-theory models

(and a general construction of F-theory models with SM gauge group and generic matter reps)

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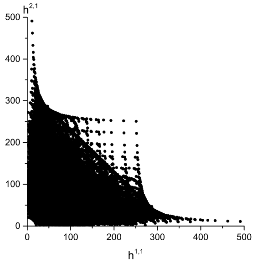
Washington (Wati) Taylor, MIT

Based in part on

arXiv:1901.0212, 1906.11092, 19mm.xxxx: WT, A. Turner

Lessons from F-theory:

6D: Global picture of connected moduli space of 6D SUGRA



(6D: ECY3's)

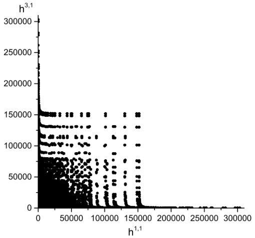
— Some gauge groups are generic in the sense that they are present everywhere in certain branches of moduli space: $SU(3), G_2, SO(8), F_4, E_6, E_7, E_8$

— Other gauge groups (e.g. $SU(5)$) require tuning

— Some gauge groups cannot be realized (e.g. $SU(500)$)

— Most branches of vacua have large generic (non-Higgsable) G (e.g. $E_8^n \times F_4^m \times (G_2 \times SU(2))^k$, natural hidden sector dark matter

4D: similar story at geometric level



(4D: ECY4's)

Geometric non-Higgsable products include $SU(3) \times SU(2)$

But:

— Moduli lifted by superpotential

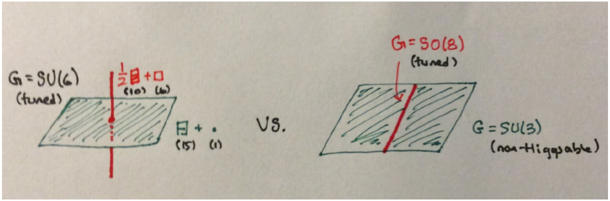
— Geometrically generic gauge groups affected by fluxes, superpotential

This talk: focus on “generic” matter representations

Fix gauge group G (generally tuned/Higggsable)

6D SUGRA: generic matter (for fixed, not large anomaly coefficients a, b)

Defined as matter on moduli branch of greatest dimension



Note: non-generic gauge groups lie on subspaces,
non-generic matter lies on distinct branches reached by “matter transitions”

[Anderson/Gray/Raghuram/WT: 1512.05791]

Generic matter: examples

U(1): $\{q = 1, 2\}$ generic

SU(N): $\{\square, \begin{smallmatrix}\square\\\square\end{smallmatrix}$, adjoint$\}$ generic (no $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ for SU(2), $\begin{smallmatrix}\square\\\square\end{smallmatrix} = \bar{\square}$ for SU(3))

In these cases,

generic representations = # anomaly constraints on charged matter

e.g. U(1) anomaly conditions ($a, \tilde{b}$ anomaly coefficients for BRR, BFF)

$$
\begin{aligned}
-a \cdot \tilde{b} &= \frac{1}{6} \sum q_i^2 \\
\tilde{b} \cdot \tilde{b} &= \frac{1}{3} \sum q_i^4
\end{aligned}
$$

Anomaly equivalence: connects matter w/same RHS, e.g.

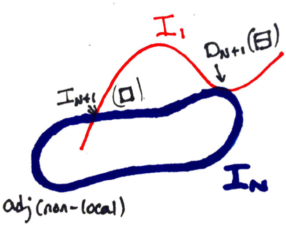
$$6 \times (2) + 10 \times (0) \leftrightarrow (3) + 15 \times (1)$$

lose 10 moduli to get $q = 3$ (and 15 $q = 1$'s) from 6 $q = 2$'s

Generic matter in F-theory

Generic matter matches simplest F-theory singularities
& Weierstrass model constructions

SU(N): standard Tate models $y^2 + a_1 yx + a_3 y = x^3 + a_2 x^2 + a_4 x + a_6$
tune $a_i \sim z^{k_i} \tilde{a}_i \Rightarrow$ generic matter [n.b. SU(6) exception]



U(1): Morrison-Park model

$$y^2 = x^3 + (e_1 e_3 - b^2 e_0 - \frac{1}{3} e_2^2) x + (-e_0 e_3^2 + \frac{1}{3} e_1 e_2 e_3 - \frac{2}{27} e_2^3 + \frac{2}{3} b^2 e_0 e_2 - \frac{1}{4} b^2 e_1^2) \,.$$

Exotic matter: realized in conventional F-theory, requires more tuning

SU(N) exotic matter:

$\yng(1,1,1)$: $SU(6), SU(7), SU(8)$ $\quad g = 0$

$\yng(2)$: $SU(N)$ $\quad g = 1$

$\yng(3)$: $SU(2)$ $\quad g = 3$

$g > 0$ realized by singularities over singular 7-branes
[Kumar/Park/WT, Klevers/Morrison/Raghuram/WT]

Some possibility beyond conventional F-theory: $SU(2)\yng(4)$?
T-branes? [Cvetič/Heckman/Lin]

Larger representations seem impossible in F-theory

U(1) exotic matter: $q > 2$; implicit constructions to $q = 21$
[$q = 3$: Klevers/Mayorga Peña/Oehlmann/Piragua/Reuter, $q = 3, 4$: Raghuram, $q \leq 21$: Raghuram/WT, $q \leq 6$: Collinucci/Fazzi/Morrison/Valandro]

Possibly no local bound, only global constraint

Generic matter in 4D

— Dimensionality definition, anomaly matching inadequate in 4D SUGRA

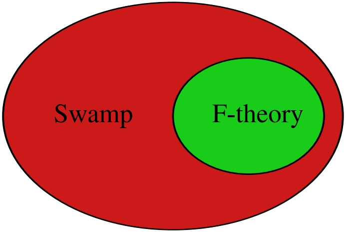
— Underlying CY4 geometry moduli dimension, singularity/construction complexity still valid for 4D F-theory

Suggests generic matter for 4D F-theory $\sim$ 6D SUGRA/F-theory

In 4D, working concept: "generic F-theory matter" given G

Connect with other string descriptions?

6D supergravity and the swampland



(Inconsistent theories)

Many “swampland” questions for 6D SUGRA related to exotic matter

For generic matter, particularly at $T = 0$, small or empty swamp

Example: $T = 0, G = U(1)$

$\{U(1)\ q = 1, 2$ F-theory models$\} = \{U(1)\ q = 1, 2$ anomaly-free models$\}$

$$\tilde{b}\left(24 - \tilde{b}\right) \times (\pm 1) + \frac{1}{4}\tilde{b}\left(\tilde{b} - 6\right) \times (\pm 2),\ 6 \leq \tilde{b}\ (\text{even})\ \leq 24\,.$$

[n.b. subtlety at $\tilde{b} = 24$; Turner/WT: 1803.04447]

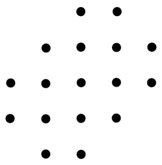
Generic matter with multiple factors, U(1)'s

The story becomes more interesting with U(1)'s and multiple gauge factors

U(1) $\times$ U(1) generic matter:

Sign combinations of $(2, \pm 1), (\pm 1)$ fixed by anomaly coefficients

Generic matter compatible with Higgsing $SU(3) \times SU(2) \times SU(2)$



Fits with general $U(1) \times U(1)$ construction of [Cvetič/Klevers/Piragua/WT]

Corresponds to unHiggsing U(1)'s to divisors AB, AC

Generic matter for $U(1)^k$ more complex, non-unique

Suggests e.g. structure from $SU(2)^3 \times SU(3)^3$ for $U(1)^3$

$SU(N) \times U(1)$ and discrete quotients

(cf. also [Cvetič/Lin, Grimm/Kapfer/Klevers])

Generic matter for $SU(N) \times U(1)$: (no $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ for SU(2))

$\text{adj}_0, \begin{smallmatrix}\square\\\square\end{smallmatrix}_0, \square_0, \square_{\pm 1}, \cdot_1, \cdot_2$

Only bi-charged matter is $\square_1$

Similar for $G_{\text{SM}} = SU(3) \times SU(2) \times U(1)$:

bi-charged matter is $(3,1)_1, (3,2)_0, (1,2)_1$

standard model matter not generic for G_{SM}

(though generic chiral matter sol'ns in 4D)

But: generic matter for $(SU(N) \times U(1))/\mathbb{Z}_N$ is (e.g.)

$\text{adj}_0, \begin{smallmatrix}\square\\\square\end{smallmatrix}_{2/N}, \square_{1/N}, \square_{1/N \pm 1}, \cdot_1, \cdot_2$

Similarly, SM matter is generic for $G'_{\text{SM}} = (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$

$G_{N1} = SU(N) \times U(1)$ vs. $G'_{N1} = (SU(N) \times U(1))/\mathbb{Z}_N$
(generator $(\omega, \omega, \omega, \omega, \omega) \times \omega^{-1}, \omega = e^{2\pi i/5}$)

Explore 6D SUGRA anomaly solutions at $T = 0$ (F-theory on $\mathbb{P}^2$)

Group $G_{5\ 1}$:

b	$\tilde{b}$	$\cdot_0$	adj_0	$\begin{smallmatrix}\square\\\square\end{smallmatrix}_0$	$\square_0$	$\square_1$	$\square_{-1}$	$\cdot_1$	$\cdot_2$
1	6	95	0	3	13	3	3	78	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
3	8	37	1	9	3	12	12	8	4

14 solutions, no swamp
Match Higgsing $SU(5) \times SU(2)$ on $SU(2)$ adjoint, ac's $(b, B_2 = \tilde{b}/2)$

$G'_{5\ 1}$:

b	$\tilde{b}$	$\cdot_0$	adj_0	$\begin{smallmatrix}\square\\\square\end{smallmatrix}_{1/5}$	$\square_{1/5}$	$\square_{6/5}$	$\square_{-4/5}$	$\cdot_1$	$\cdot_2$
1	5.2	103	0	3	14	0	5	70	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
4	4.8	26	3	12	0	4	12	0	0

20 solutions, match Higgsing $SU(6) \times SU(2)$ on bif or SU(6) on adj $(B_6 = b)$
One exceptional case in swamp: $b = 4, \tilde{b} = 14/5 \sim B_2 = -1$

What does this tell us?

- Both $SU(N) \times U(1)$ and $(SU(N) \times U(1))/\mathbb{Z}_N$ have families of solutions
  (also solutions (15) for ω^2 type quotient)

- Neither group seems particularly favored; anomaly coefficients different

- Small or empty swampland, at least at $T = 0$

- Models classified by Higgsing $SU(6) \times SU(2)$ models; $\sim U(1) \to B_6 B_2$ suggests universal construction, w/natural generalization to 4D

Comparison to toric examples, other analyses

Various toric constructions give $(SU(5) \times U(1))/\mathbb{Z}_5$
[Cvetič/Grimm/Klevers, Mayrhofer/Palti/Weigand, Braun/Grimm/Keitel, Cvetič/Lin/Lieu/Zhang/Zoccorato]

- $\rightarrow$ various special cases of 6D anomaly-free models
- $U(1)$ from $\sim a_6 = 0$ corresponds to $[U(1)] \sim -K$ in MP
  ($\sim B_6 + B_2 = -K$ in SU(6) $\times$ SU(2) Higgsed models)
- Some toric constructions of ω^2 (SO(10) type) quotient
- Toric constructions may give insight for construction of universal Weierstrass model, general ω^2 models

General models with generic $SU(5) \times U(1)$, $(SU(5) \times U(1))/\mathbb{Z}_5$ compatible with local analyses [Lawrie/Schäfer-Nameki/Wong, Cvetič/Lin]

- All respect shift symmetry: $\forall$ representations $R_x, R_y, y - x \in \mathbb{Z}$
- Generic matter structure provides natural charge combinations

Standard Model gauge group and generic matter

As for $SU(5) \times U(1)$, 6D $T = 0$ anomaly-free models

For $G_{\text{SM}} = SU(3) \times SU(2) \times U(1)$: 66 solutions

For $G'_{\text{SM}} = G_{\text{SM}}/\mathbb{Z}_6$: 98 solutions

G'_{SM} generic matter:

Generic Matter	MSSM Multiplet
$(\mathbf{1}, \mathbf{1})_0$	N^c
$(\mathbf{1}, \mathbf{1})_1$	E^c
$(\mathbf{1}, \mathbf{1})_2$	
$(\square, \mathbf{1})_{2/3}$	$\overline{U^c}$
$(\square, \mathbf{1})_{-1/3}$	$\overline{D^c}$
$(\square, \mathbf{1})_{-4/3}$	
$(\mathbf{1}, \square)_{1/2}$	$\overline{L} = \begin{pmatrix} \overline{N} & \overline{E} \end{pmatrix}, H_u, \overline{H_d}$
$(\mathbf{1}, \square)_{3/2}$	
$(\text{Adj}, \mathbf{1})_0$	
$(\mathbf{1}, \text{Adj})_0$	
$(\square, \square)_{1/6}$	$Q = \begin{pmatrix} U \\ D \end{pmatrix}$

Explicitly solve 10 anomaly equations for G'_{SM} generic matter

Define β, X, Y:

$$\tilde{b} = 2\beta + \frac{3}{2}b_2 + \frac{4}{3}b_3$$

$$X = -8a - 4b_3 - 3b_2 - 2\beta$$

$$Y = b_3 + b_2 + \beta + a\,.$$

Generic Matter	Multiplicity	MSSM Multiplet
$(\square, \square)_{1/6}$	$b_3 \cdot b_2$	$\boldsymbol{Q}$
$(\square, \mathbf{1})_{2/3}$	$b_3 \cdot (\beta - 2a)$	$\overline{\boldsymbol{U}^c}$
$(\square, \mathbf{1})_{-1/3}$	$b_3 \cdot X$	$\overline{\boldsymbol{D}^c}$
$(\square, \mathbf{1})_{-4/3}$	$b_3 \cdot Y$	
$(\mathbf{1}, \square)_{1/2}$	$b_2 \cdot (X + \beta - a)$	$\overline{\boldsymbol{L}}, \boldsymbol{H}_u, \overline{\boldsymbol{H}_d}$
$(\mathbf{1}, \square)_{3/2}$	$b_2 \cdot Y$	
$(\mathbf{1}, \mathbf{1})_1$	$(b_3 + b_2 + 2\beta) \cdot X - a \cdot b_2$	$\boldsymbol{E}^c$
$(\mathbf{1}, \mathbf{1})_2$	$\beta \cdot Y$	
$(\text{Adj}, \mathbf{1})_0$	$1 + b_3 \cdot (b_3 + a)/2$	
$(\mathbf{1}, \text{Adj})_0$	$1 + b_2 \cdot (b_2 + a)/2$	

Specialize to $T = 0$ ($a = -3$)

Need $b_2, b_3 > 0$ for good gauge theory kinetic terms, $\Rightarrow X, Y \geq 0$.

Two types of solutions:

A) $\beta \geq 0$

B) $Y = 0 \Rightarrow \beta + b_2 + b_3 = -K$

(Note: some solutions fit both criteria)

Two classes of $(SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$ models:

All 98 $T = 0$ models fit one or both of these types (no swamp!)

A) $\beta \geq 0$: $SU(4) \times SU(3) \times SU(2)$ type models

- 71 $T = 0$ models of this type, three parameters $B_4 = b_3, B_3 = b_2, B_2 = \beta \geq 0$
- F-theory construction starting with (Tate) $SU(4) \times SU(3) \times SU(2)$,
  Higgs on $(\square, 1, \square) + (\square, \square, 1) + (1, \square, \square)$
  Expect universal 3-parameter Weierstrass model a la Morrison-Park for U(1)

B) $Y = 0$

- 30 $T = 0$ models of this type, two parameters $S_9 = b_3, S_7 = b_2 + b_3 + a$
- All models of this type realized using "F11" toric fiber
  [Klevers/Mayorga Peña/Oehlmann/Piragua/Reuter]
- All but 3 realized as Pati-Salam $(SU(4) \times SU(2) \times SU(2))/\mathbb{Z}_2$ models:
  $B_4 = b_3, B_2 = b_2, B_2' = -4a - b_2 - 2b_3$
- One model $(b_3, b_2) = (3, 3)$ is F11 + Pati-Salam + $SU(5)$ type model
- Fewer "extra" representations than type A

4D G'_{SM} models (same principle for $(SU(5) \times U(1))/\mathbb{Z}_5$)

Generalization to 4D:

Type A: Construct as deformed $SU(4) \times SU(3) \times SU(2)$ models.

- Bounds on b_i from Tate/Kodaira bounds
- Matter curves determined in terms of intersections of b_i's
  e.g. $(\Box, \Box)_{1/6}$ multiplicity in 6D G'_{SM} model given by $\#((\Box, \Box)_{1/6}) = b_3 \cdot b_2$
  generalizes to matter curve $C_{(\Box,\Box)_{1/6}} = b_3 \cap b_2$ for 4D models
- Note: no chiral non-MSSM generic matter needed; flux-dependent

Type B: Construct either as F11 fiber model
or (in most cases) deformed Pati-Salam

- Used to construct 3-generation models: [Cvetič/Lin/Liu/Oehlmann]
- Special $(SU(5))$ case $b_2 = b_3 = -K$
  recently studied in detail (w/ 3-generation SM) over weak Fano bases
  [Cvetič/Halverson/Lin/Liu/Tian]

Gives general construction of (all?) tuned $G'_{\mathrm{SM}} = (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$ 4D F-theory models over any given base B_3

- e.g. over $\mathbb{P}^3$ gives 181 type A +54 B G'_{SM} models (6 A + B) (A: $\sum b_n \geq -K, \sum n b_n \leq -8K, b_4, b_3 > 0, b_2 \geq 0$, B: F11)
- Generalizes base-independent toric construction of SM [Cvetič/Halverson/Lin/Liu/Tian]
- Works over all weak Fano bases, + many (type A) w/non-Higgsable groups ($\sim 10^{3000}$ bases [WT/Wang², Long/Halverson/Sung]), but tuning difficult on large bases due to NHC's
- Other SM constructions:
  use non-Higgsable groups in SM [Grassi/Halverson/Shaneson/WT],
  fluxes in Higgsable (e.g. SU(5)) or non-Higgsable (e.g. E_8) unification groups [Donagi/Wijnholt, Beasley/Heckman/Vafa, etc.]; [Tian/Wang]
- Tuned models: interesting range of SM-like constructions to explore

Conclusions

1. 6D provides a rigorous definition of "generic" matter representations, generalizes naturally to 4D

2. General SUGRA + F-theory models with SM group + matter reps
$G = (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$+ generic matter:

A) Deformed $SU(4) \times SU(3) \times SU(2)$; (3-parameter set)
— Larger set of models
— Compatible with NHC dark matter

B) F11 (mostly Pati-Salam); (2-parameter set)
— More explicit toric description
— Fewer non-MSSM matter multiplets

Suggests increased motivation to study these F-theory models for string pheno