Landscape of F-theory Standard Models

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Progress on the program:
Globally consistent F-theory compactifications with the
gauge symmetry and matter spectrum of the Standard Model

First globally consistent three-family Standard Models:


with $Z_2$ matter parity:


Landscape of three-family Standard Models

Outline

I. Particle physics model building in F-theory:
   Building blocks (via toric techniques)
   First globally consistent three family Standard Models

II. Landscape of three family Standard Models:
   Globally consistent models via toric techniques

III. Outlook: work in progress & open issues
I.a F-theory basic ingredients

Type IIB string perspective
**F-theory compactification to 4D**

[Vafa’96], [Morrison,Vafa’96],...
c.f., review [Weigand 1806.01854]

Singular elliptically fibered Calabi-Yau manifold $X$  

**CY four-fold**

Modular parameter of two-torus (elliptic curve)

[SL(2,Z) of Type IIB]

$$\tau \equiv C_0 + ig^{-1}_s$$

Weierstrass normal form for elliptic fibration of $X$

$$y^2 = x^3 + f x z^4 + g z^6$$

$(x, y, z) \simeq (\lambda^2 x, \lambda^3 y, \lambda z)$

$[z:x:y]$ - homogeneous coordinates on $\mathbb{P}^2[2,3,1]$

Calabi-Yau conditions:

$f, g$ - sections of $\overline{K}_B^6$ and $\overline{K}_B^4$ on $B$

$[\overline{K}_B$ - anti-canonical bundle on $B$]

$[x:y:z]$- sections of specific line-bundles on $B$
F-theory compactification

Singular elliptically fibered Calabi-Yau manifold $X$

Modular parameter of two-torus (elliptic curve)
$$\tau \equiv C_0 + ig_s^{-1}$$

- Matter (co-dim 2)
  - Chirality: $G_4$-flux w/ fiber resolutions crucial
  - c.f., [Lin, Mayrhofer, Till, Weigand, 1508.00162]
  - [M.C., Grassi, Klevers, Piragua, 1306.3987]

- Non-Abelian gauge symmetry (co-dim 1) – ADE singularities
  - c.f., [M.C. Lin, 1809.00012] review

- Yukawa couplings (co-dim 3)

- Abelian symmetries different

- Divisor- singular elliptic-fibration

- $g_s \rightarrow \infty$ location of (p,q) 7-branes
1.b Particle physics in F-theory

Globally consistent models
Construction of elliptically fibered Calabi-Yau manifold

i. Elliptic curve $E$

Examples of constructions via toric techniques:

$E_{F_i}$ as a hypersurface in the two-dimensional toric variety $\mathbb{P}_{F_i}$

(associated with 16 reflexive polytopes $F_i$):

c.f., [Klevers, Pena, Oehlmann, Piragua, Reuter '14]

$$E_{F_i} = \{p_{F_i} = 0\} \text{ in } \mathbb{P}_{F_i}$$

ii. Elliptically fibered Calabi-Yau space: $X_{F_i}$

Impose Calabi-Yau condition:

coordinates in $\mathbb{P}_{F_i}$ and coeffs. of $E_{F_i}$ lifted to sections of specific line-bundles on $B$

Fibration depends only on the anti-canonical divisor $\overline{K}$

& two additional $S_7$ and $S_9$ divisor classes
iii. Chiral index for D=4 matter:

\[ \chi(R) = \int_{C_R} G_4 \]

\[ c_{\text{mat}} \rightarrow C_R \]

\[ \sum_R \]

a) construct \( G_4 \) flux by computing \( H^{(2,2)}_V(\hat{X}) \)

[so-called vertical fluxes – do not induce Gukov-Vafa-Witten potential]

b) determine matter surface \( C_R \) (via resultant techniques)

diagram:

iv. Global consistency – D3 tadpole cancellation:

\[ \frac{\chi(X)}{24} = n_{D3} + \frac{1}{2} \int_X G_4 \wedge G_4 \]

a) satisfied for integer and positive \( n_{D3} \)

b) constraint on integer valued flux \( G_4 \)

\[ G_4 + \frac{1}{2} c_2(X) \in H^4(\mathbb{Z}, \hat{X}) \]
in the toric ambient space

gauge symmetry, including the U(1) generator, as well as the matter representations. While

2.

Natural

In this section, we analyze the elliptically fibered Calabi-Yau manifold

3.5.2 Polyhedron

F

polytope

Elliptic curve:

\[ p_{F_{11}} = s_1 e_1^2 e_2^2 e_3 e_4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 w^2 + s_5 e_1^2 e_2^3 u^2 w + s_6 e_1 e_2 e_3 e_4 u v w + s_9 e_1 v w^2 \]

(hypersurface constraint in \( \mathbb{P}^2 \) [u:v:w] with four blow-ups [e_1:e_2:e_3:e_4])

Global [geometric origin of U(1)]

Gauge Symmetry: \( [\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)] / \mathbb{Z}_6 \)

Matter:

\[ [s_3]=0 \quad [s_9]=0 \quad [e_4]=0 \] rational section

<table>
<thead>
<tr>
<th>Representation</th>
<th>((3, 2)_{1/6})</th>
<th>((\bar{3}, 1)_{-2/3})</th>
<th>((\bar{3}, 1)_{1/3})</th>
<th>((1, 2)_{-1/2})</th>
<th>((1, 1)_{-1})</th>
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Construct G_4 for chiral index & D3-tadpole constraint
### Standard Model:

**Base** $B = \mathbb{P}^3$  
**Divisors in the base:**  

$S_7 = n_7 H$  
$S_9 = n_9 H$  

$n_7, n_9 \in \mathbb{Z}$

#### Solutions (#(families);$n_{D3}$) for allowed ($n_7, n_9$):

<table>
<thead>
<tr>
<th>$n_7 \setminus n_9$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td>7</td>
<td>-</td>
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<td>(12; 81)</td>
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<td>5</td>
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<td>(12; 57)</td>
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<td>4</td>
<td>(42; 4)</td>
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<tr>
<td>3</td>
<td>-</td>
<td>(21; 72)</td>
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<td>(15; 30)</td>
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<tr>
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<td>(24; 79)</td>
<td>(21; 66)</td>
<td>(24; 44)</td>
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**Hyperplane divisor class**

$H = 4 \overline{K}$
II. Landscape of Standard Models

Toric analysis
a) Take the same toric elliptic fibration as before: hyperplane constraint in 2D reflexive polytope $F_{11}$.

Gauge symmetry: $\frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}$

Global gauge symmetry

b) Take bases $B$, associated with 3D reflexive polytopes.

E.g., $\mathbb{P}^3$, $\mathbb{P}^2 \times \mathbb{P}^1$

For each reflexive polytope, different bases $B$ are associated with different fine-star-regular triangulations of a chosen polytope. [Triangulations determine intersections of divisors.]

Triangulations grow exponentially with the complexity of a polytope.
c) Specific choice of divisors: \( S_{7,9} = \bar{\mathcal{K}} \)

[anti-canonical divisor of the base B – fixed by the polytope]

SU (3) and SU (2) divisors \( S_9 \) and \( S_3 \) with class \( \bar{\mathcal{K}} \) →

\[
g_{3,2}^2 = \frac{2}{\text{vol}(\bar{\mathcal{K}})}
\]

U(1) - (height-pairing) divisor volume \( \frac{5}{6} \bar{\mathcal{K}} \) →

\[
\frac{5}{3} g_Y^2 = \frac{2}{\text{vol}(\bar{\mathcal{K}})}
\]

\[
g_3^2 = g_2^2 = \frac{5}{3} g_Y^2
\]

Standard Model with gauge coupling unification!

\[\text{c.f., [M.C., Klevers, Peña, Oehlmann, Reuter, 1503.02068]}\]

Connected torically to Pati-Salam Model \( \text{SU(4)}_C \times \text{SU(2)}_L \times \text{SU(2)}_R \)

Non-torically connected to SU(5) GUT[Taylor, Turner, 1906.11092]

\[\text{c.f., W. Taylor’s talk}\]
d) Remaining conditions:
  iii. 3-families of quarks and leptons (chiral index)
  iv. D3-tadpole constraints

- Construct $G_4$ flux in terms of $(1,1)$-forms, Poincaré dual to divisor classes. 
  c.f., [Lin, Mayrhofer, Till, Weigand, 1508.00162]  
  [M.C., Grassi, Klevers, Piragua, 1306.3987]

- Chirality, D3 tadpole and $G_4$ integrality expressed in terms of intersection numbers of divisors in the base $B$ → Geometric conditions!

- In the case $S_{7,9} = \overline{K}$ and $n_F$ - families D3 tadpole:

\[
   n_{D3}(n_F, K^3) = 12 + \frac{5K^3}{8} - \frac{5n_F^2}{2K^3} \in \mathbb{Z}_{\geq 0}
\]

Geometrized D3-tadpole condition

Depends only on the polytope and not on triangulation. → Universality of the Standard Model
Landscape count for $n_F=3$ families:

$$\left(12 + \frac{5}{8}K^3 - \frac{45}{2K^3}\right) \in \mathbb{Z}_{\geq 0}$$

satisfied for $K^3 \in \{2, 6, 10, 18, 30, 90\}$

- Out of 4319 3D reflective polytopes $\rightarrow$ 708 satisfy the constraint. (many of them with a large number of lattice points)

- Triangulation of polytopes can be handled combinatorially. (each corresponds to a different bases $B$)

  It can be implemented on computer, e.g., in SageMath:
  i) for 237 polytopes w/ $< 15$ lattice points $\rightarrow$ 414310 MSSM models
  ii) for 471 polytopes w/ $\geq 15$ lattice points -- exp. growing comp. time $\rightarrow$

  counting via fine-regular triangulation of facets & estimate regular fine-star triang.

  c.f., [Halverson, Tian, 1610.08864]

- Provide bound: $7.6 \times 10^{13} \lesssim N_{SM}^{\text{toric}} \lesssim 1.6 \times 10^{16}$
III. Summary, Work in Progress, Outlook
Summary

Globally consistent F-theory Standard Models
(Toric techniques w/elliptic fibration: hypersurface in F11)

First three family Standard Models
Anticipated: tip of the iceberg

Indeed, geometric advances

Landscape of globally consistent Standard Models w/ exact chiral spectrum of three-families of quarks & leptons & gauge coupling unification > quadrillion models!

6D analysis: [Taylor, Turner 1906.11092], c.f., W. Taylor’s talk
• Branch B: SM & SM matter \leftrightarrow precisely the one presented
• Branch A: SM w/ additional matter [Higgsed SU(4)xSU(3)xSU(2)]

Note, 4D: resolutions, fluxes & D3 tadpoles \rightarrow major complications already achieved for B branch
Work in progress

- **Number of vector pairs**
  \[ C_3 \text{ w/ } G_4 = dC_3 = 0, \text{ encoded in the intermediate Jacobian of } X \]
  
  c.f., [Bies, Mayrhofer, Weigand, 1706.04616]

  For higher genus matter curves \( \rightarrow \) technically difficult

  SM’s Higgs doublets on matter curves w/  
  \[ g = 1 + \frac{9}{2K^3} > 0 \]
  
  work in progress M.C., Bies, Lin, Liu

- **Yukawa couplings**
  First calculation for a global (toy) SU(5)xU(1)model:
  explicit complex structure dependence of holomorphic couplings w/ rank >1 and large hierarchies!

  [M.C., Lin, Liu, Zoccarato, Zhang 1906.10119]  
  c.f., L. Lin’s talk

  SM’s Yukawa couplings: multiple triple intersections \( \rightarrow \) rank > 2
  matter curves w/ large genera \( \rightarrow \) quantitative results difficult
As SM’s can be torically un-Higgsed to Pati-Salam models $[SU(4)_C \times SU(2)_L \times SU(2)_R] / \mathbb{Z}_2 \Rightarrow$
eq expect some key-features of SM’s inherited from there:

- Some R-parity violating terms suppressed in SM, in particular lepton violating tri-linear terms.

- Constructed globally consistent Pati-Salam Models w/ gauge coupling unification, $n_F$ - families & w/
  $$n_{D3}(n_F, \bar{K}^3) = 12 + \frac{-2n_F^2}{\bar{K}^3} \in \mathbb{Z}_{\geq 0}$$

- All matter curves w/ lower genera: $g=1+\bar{K}^3/2$
  including $(1,2,2)$ (SM Higgs doublets)

- $n_F = 3$, $\bar{K}^3 = 6$ and $n_{D3} = 12$ - solution for SM & Pati-Salam model!
  Possibly, easier to calculate Higgs doublet pairs
Further outstanding issues

Usual suspects:
moduli stabilization...supersymmetry breaking...
dark matter...cosmological implications...
(not-unrelated)

Horizontal $G_4$ Fluxes & non-pert. effects
(difficult, but depend on triangulation);
D3 brane-sector (more promising $\rightarrow$ dark photon matter)

Report on progress in the future
Thank you!