String axiverse and γ-ray spectral modulation of galactic pulsars & SN remnants

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String Pheno 2019, CERN

KC, S. Lee, H. Seong, S. Yun, 1806.09508

and work in progress

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Outline

- Axions or axion-like particles (ALPs) in particle physics, cosmology and string theory.

  It is quite well motivated that there exist multiple light axions with hierarchically different decay constants and masses, and string theory provides an attractive theoretical framework to realize such a scenario.

- A specific scenario involving two axions with hierarchically different decay constants and masses, which can explain the recently noticed γ-ray spectral modulation of some galactic pulsars and SN remnants, and also the TeV γ-ray transparency.

Axions or Axion-like Particles (ALPs)

Pseudo-Nambu-Goldstone boson associated with an approximate $U(1)_{PQ}$ shift symmetry:

$$U(1)_{\rm PQ}:\ a \to a + \text{constant}$$

In most cases, axions are compact field:

$$a \equiv a + 2\pi f_a \quad (f_a = \text{axion scale or axion decay constant})$$

➔ $\dfrac{a}{f_a}$ is an angular field, suggesting that axion mass and couplings $\propto \dfrac{1}{f_a}$.

Low energy effective lagrangian at scales below f_a :

$$\mathcal{L}_{\rm axion} = \frac{1}{2}(\partial_\mu a)^2 + \frac{\partial_\mu a}{f_a} J^\mu + \Delta\mathcal{L} + \text{higher derivative couplings}$$

PQ-invariant derivative couplings

with $J^\mu = c_\psi \bar{\psi}\gamma^\mu\psi + c_\phi \phi^* D^\mu \phi + \ldots$

PQ-breaking non-derivative couplings:

$$V\left(\frac{a}{f_a}\right) + \frac{c_A}{32\pi^2}\frac{a}{f_a}F^{A\mu\nu}\tilde{F}^A_{\mu\nu} + \ldots = \frac{1}{2}m_a^2 a^2 + \frac{c_\gamma}{8\pi^2}\frac{a}{f_a}\vec{E}\cdot\vec{B} + \ldots$$

Axions in particle physics & cosmology

➢ Axions to solve the naturalness problems

1) QCD axion to solve the strong CP problem

Peccei, Quinn '77; Weinberg; Wilczek '78

$\mathrm{U(1)}_{\mathrm{PQ}}$ dominantly broken by the QCD anomaly:

$$\Delta\mathcal{L} = \frac{1}{32\pi^2}\frac{a}{f_a}G^{a\mu\nu}\tilde{G}^a_{\mu\nu} + \ldots \quad \Rightarrow \quad V \simeq -m_u\Lambda^3_{\mathrm{QCD}}\cos\left(\frac{a}{f_a}\right)$$

$$\Rightarrow \quad \theta_{\mathrm{QCD}} = \frac{\langle a\rangle}{f_a} = 0, \quad m_a \sim 5\times 10^{-6}\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)\mathrm{eV}$$

Astrophysical and cosmological considerations imply

$$10^8\,\mathrm{GeV} \lesssim f_a \lesssim 10^{12}\,\mathrm{GeV}$$

2) Inflaton for natural inflation to avoid the fine tuning problems

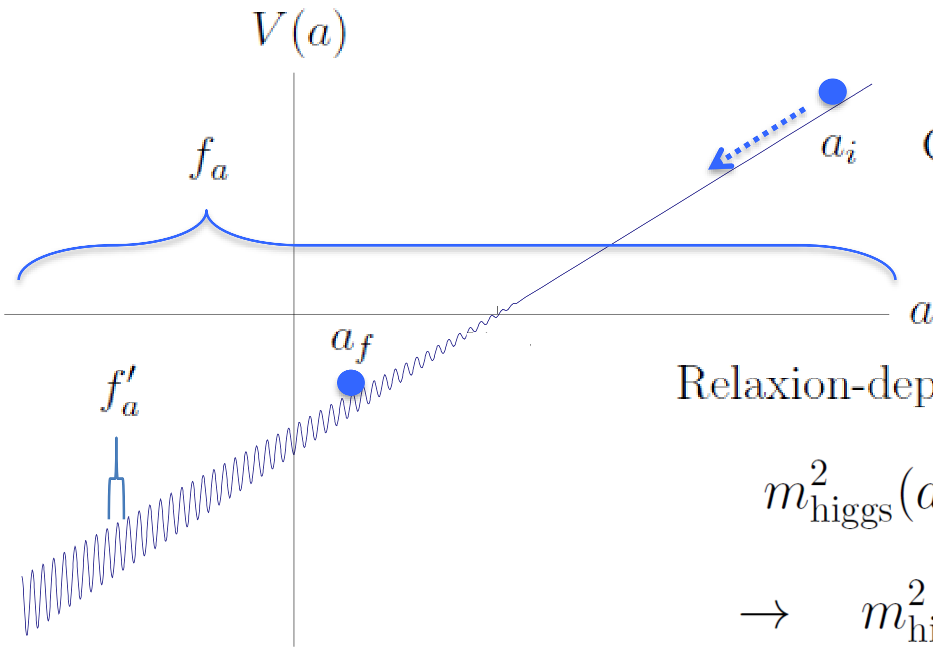
Freese, Frieman, Olinto, '90

$$V(a) = \Lambda^4_{\mathrm{inf}}\left(1-\cos\left(\frac{a}{f_a}\right)\right)$$

$$\Rightarrow \quad f_a \sim \sqrt{N_e}\,M_{\mathrm{Pl}} \quad \left(N_e = \text{Number of e-foldings} \sim 60\right)$$

3) Relaxion to solve the weak scale hierarchy problem

Graham, Kaplan, Rajendran '15



Cosmological relaxion evolution:

$$a = a_i \quad \rightarrow \quad a = a_f$$

Relaxion-dependent Higgs mass: $m^2_{\text{higgs}}(a)$

$$m^2_{\text{higgs}}(a_i) = \Lambda^2_{\text{higgs}} \gg (100\,\text{GeV})^2$$

$$\rightarrow \quad m^2_{\text{higgs}}(a_f) \simeq -(100\,\text{GeV})^2$$

Three new mass scales in the relaxion model:

$$f_a \gg f'_a \gtrsim \Lambda_{\text{higgs}} = \text{Higgs mass cutoff} \gg 100\,\text{GeV} \quad \left(\frac{f_a}{f'_a} \gtrsim \left(\frac{\Lambda_{\text{higgs}}}{100\,\text{GeV}}\right)^4\right)$$

$$\Rightarrow \quad f_a \gtrsim 10^{12}\,\text{GeV} \quad \text{for} \quad \Lambda_{\text{higgs}} \sim 10\,\text{TeV} \quad (\text{In some cases,} \quad f_a \gg M_{\text{Pl}})$$

➢ Axions constituting the dark sector

1) Axion dark matter Preskill, Wise, Wilczek; Abott, Sikivie; Dine, Fischler '83

$$\text{Initial axion field misalignment } \delta a_i \sim f_a$$

$$\Rightarrow \quad \Omega_a h^2 \sim 0.2 \left(\frac{m_a}{10^{-22}\,\mathrm{eV}}\right)^{1/2} \left(\frac{f_a}{10^{17}\,\mathrm{GeV}}\right)^{\frac{3n+8}{2n+4}}$$

$$(\, m_a(T) \propto T^{-n} \text{ when the axion field begins to oscillate}\,)$$

* QCD axion dark matter: $f_a \sim 10^{12}\,\mathrm{GeV}, \quad m_a \sim 5 \times 10^{-6}\,\mathrm{eV} \quad (n \simeq 4)$

* Fuzzy dark matter: $f_a \sim 10^{17}\,\mathrm{GeV}, \quad m_a \sim 10^{-22}\,\mathrm{eV} \quad (n = 0)$

2) Quintessence axion for dark energy KC '99; Kim, Nilles '09

$$V_{\mathrm{DE}} \sim (10^{-3}\,\mathrm{eV})^4 \cos\left(\frac{a}{f_a}\right) \quad \text{with } f_a \sim M_{\mathrm{Pl}}, \quad m_a \sim H_0 \sim 10^{-33}\,\mathrm{eV}$$

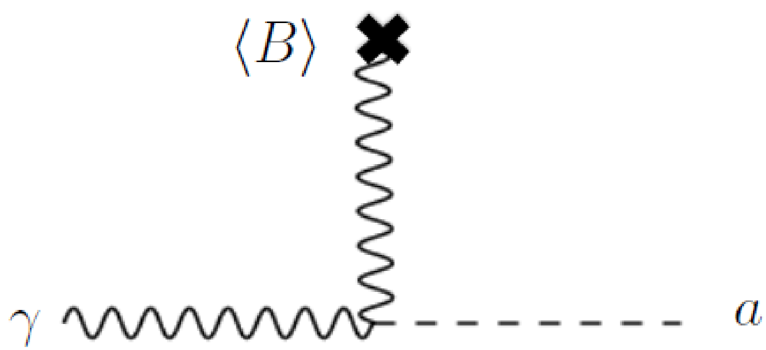
- Axions to explain some astrophysical puzzles

* Transparency of TeV γ-rays in extragalactic background lights,

* Spectral modulation of GeV γ-rays from galactic pulsars and SN remnants,

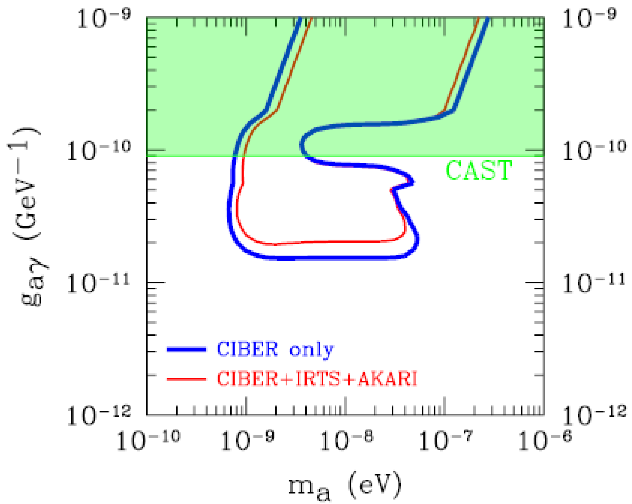
which may be due to the conversion of photons to axions induced by the axion-photon coupling in background B-field: Raffelt & Stodolsky '88

$$\Delta\mathcal{L} = \frac{c_\gamma}{32\pi^2}\frac{a}{f_a}F^{\mu\nu}\tilde{F}_{\mu\nu} = \frac{c_\gamma}{8\pi^2}\frac{a}{f_a}\vec{E}\cdot\vec{B}$$



TeV gamma-ray transparency

Meyer, Horns, Raue '13; Kohri, Kodama '17

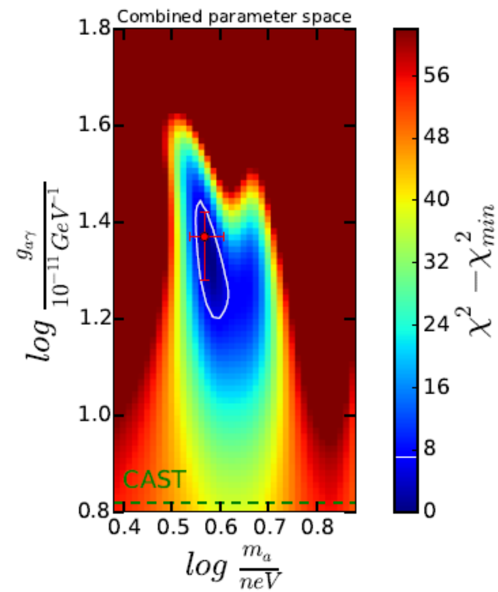


$$f_a \sim 10^8 - 10^9\,\mathrm{GeV}, \quad m_a \sim 10^{-9} - 10^{-7}\,\mathrm{eV}$$

$$g_{a\gamma} = \frac{c_\gamma}{8\pi^2}\frac{1}{f_a} \quad (c_\gamma = \mathcal{O}(1))$$

GeV gamma-ray spectral modulations of galactic pulsars

Majumdar, Calore, Horns '18



$$f_a \sim 2 \times 10^8\,\mathrm{GeV}, \quad m_a \sim 4 \times 10^{-9}\,\mathrm{eV}$$

➔ Studies of various issues in particle physics and cosmology imply that there may exist multiple light axions having hierarchically different decay constants and masses.

Axions in string theory

String theory involves a variety of anti-symmetric tensor gauge fields which couple to the extended objects in the theory:

p-form gauge field C_p which couples to $(p-1)$-brane,

with the associated gauge symmetry $\Gamma_p: \;\; C_p \to C_p + d\Lambda_{p-1}$

Upon compactification, these anti-symmetric tensor fields generically have multiple zero modes which can be identified as axions in 4D effective theory:

Witten '84

$$C_p = \sum_I a_I(x)\omega_p^I \qquad (\omega_p^I = \text{harmonic } p\text{-forms on internal space})$$

The shift symmetry "U(1)$_{PQ}$: $a_I(x) \to a_I(x) + \text{constant}$ " is locally equivalent to the gauge symmetry Γ_p because ω_p^I is locally an exact form.

U(1)$_{PQ}$ can be explicitly broken only by non-local effects in the internal space, which would allow some of those shift symmetries explicitly broken only by exponentially small non-perturbative effects.

So it is quite plausible that there can be multiple light axions with exponentially suppressed and thus hierarchical masses in string theory.

Can these stringy axions have hierarchically different decay constants f_a?

Ideas to generate the scale hierarchy $m_{\rm higgs}/M_{\rm Planck} \sim 10^{-16}$, which may have a good chance to be realized in string theory:

* Low scale SUSY:

Higgs mass protected by SUSY. $\Rightarrow \quad \dfrac{m_{\rm higgs}^2}{M_{\rm Planck}^2} \sim \dfrac{m_{\rm SUSY}^2}{M_{\rm Planck}^2}$

* Large volume extra dimension: Arkani-Hamed, Dimopoulos, Dvali '98

Gravity lives in a large volumw extra dim, while the Higgs boson is localized on small cycle (or on the boundary). $\Rightarrow \quad \dfrac{m_{\rm higgs}^2}{M_{\rm Planck}^2} \propto \dfrac{1}{\text{large volume}}$

* Warped extra dimension: Randall, Sundrum '99

Higgs boson localized at the IR side of warped extra dimension. $\Rightarrow \quad \dfrac{m_{\rm higgs}^2}{M_{\rm Planck}^2} \propto \text{small warp factor}$

* Reference axion scale in string theory:

Gravity-axion-YM unification without involving large compactification volume or significant warping.

$$\Rightarrow \quad f_a \sim \frac{g_{\rm YM}^2}{8\pi^2} M_{\rm Planck} \sim \frac{M_{\rm Planck}}{S_{\rm ins}} \sim 10^{16} - 10^{17}\,{\rm GeV}$$ KC, Kim '85

(e.g. Model-independent axion in heterotic string)

* Axion scale hierarchy from large volume extra dimension:

Some axions live in a large extra dim, while the other axions are localized on small cycle or boundary.

Burgess, Ibanez, Quevedo '99; Conlon '06; Cicoli, Goodsell, Ringwald '12

$$\Rightarrow \quad \frac{f_a(\text{boundary})}{f_a(\text{bulk})} \propto \frac{1}{\text{large volume}}$$

* Axion scale hierarchy from warped extra dimension:

Some axions are localized near the UV side of warped extra dim, while the other axions are localized near the IR side.

$$\Rightarrow \quad \frac{f_a(\text{IR})}{f_a(\text{UV})} \propto \text{small warp factor}$$

KC '04;
Flacke, Gripaios, March-Russell, Maybury '07

* Axion scale hierarchy with low scale SUSY:

String models which admit vanishing FI term of an anomalous $U(1)_A$ gauge symmetry in the SUSY limit. Poppitz '98; Ibanez, Quevedo '99; ...

The p-form axion required for anomaly cancellation is eaten by the $U(1)_A$ gauge boson, while leaving an anomalous global symmetry $\tilde{U}(1)_A$ in the low limit:

$$\text{Global } U(1)_{\rm shift}: \quad a \to a + \text{constant}$$

$$\text{Local } U(1)_A: \quad A_\mu \to A_\mu + \partial_\mu \alpha(x), \quad \phi_i \to e^{iq_i\alpha(x)}, \quad a \to a + f_a \delta_{\rm GS} \alpha(x)$$

$$\frac{1}{2}(\partial_\mu a - f_a \delta_{\rm GS} A_\mu)^2 \quad \Rightarrow \quad M_{A_\mu} = \delta_{\rm GS} f_a \sim \frac{g^2}{8\pi^2} M_{\rm Planck} \text{ with unbroken SUSY}$$

$$\Rightarrow \quad \text{Anomalous global } \tilde{U}(1)_A: \quad \phi_i \to e^{iq_i\beta}$$

This global $\tilde{U}(1)_A$ is spontaneously broken by the VEVs of $U(1)_A$ charged matter fields, which are developed by SUSY-breaking tachyonic mass:

$$V(|\phi_i|^2) = m_i^2 |\phi_i|^2 + ... \quad (m_i^2 = q_i \langle D \rangle + ... < 0 \quad \text{for some } \phi_i)$$

$$\Rightarrow \quad f_a(\tilde{U}(1)_A) \sim \langle \phi_i \rangle \quad \sim m_{\rm SUSY} \text{ or } \sqrt{m_{\rm SUSY} M_{\rm Planck}} \ll \frac{g^2}{8\pi^2} M_{\rm Planck}$$

KC, Jeong, Okumura, Yamaguchi '01; Buchbinder, Constantin, Lukas '15

* Axion scale enhanced by the alignment, monodromy, or clockwork

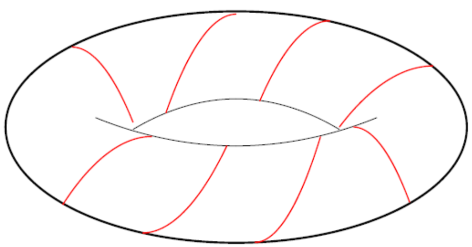
Kim, Nilles, Peloso '04; Silverstein, Westphal '08;
KC, Im '15; Kaplan, Rattazzi '15

p-form axion which couples to a hidden SUSY YM sector:

$$\mathcal{L}_{\rm tree} = \int d^2\theta\, S W^{a\alpha} W^a_\alpha \quad \left({\rm Im}(S) = \frac{a}{f_a} = p\text{-form axion} \right)$$

After the formation of gaugino condensation, we have two axions with

$$V_{\rm axion} \simeq \Lambda^2 \left| \frac{a}{f_a} + N \frac{\eta}{f_\eta} \right|^2 \quad \left(\langle \lambda^a \lambda^a \rangle = \Lambda^3 \exp\left(i \frac{\eta}{f_\eta} \right) \right)$$



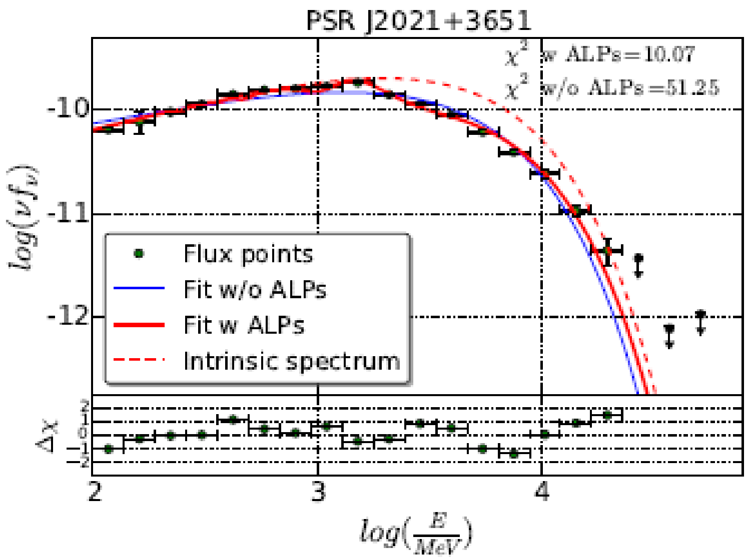
Decay constant (field range) of the massless axion component is enhanced as

$$f_{\rm eff} = \sqrt{N^2 f_a^2 + f_\eta^2} \simeq N f_a \quad (N = \text{dual Coxeter number})$$

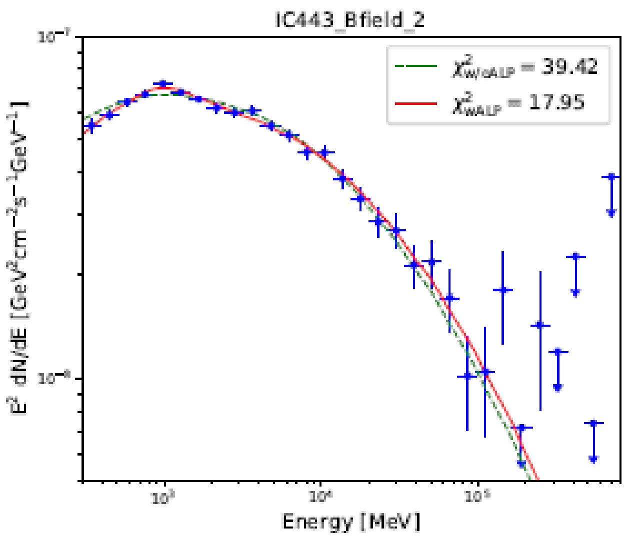
➔ String theory offers a good theoretical framework for multiple light axions with hierarchically different masses and decay constants.

Astrophysical puzzles which may find an explanation involving two axions with very different decay constants and masses:

Majumdar et al. '18



Xia et al. '18



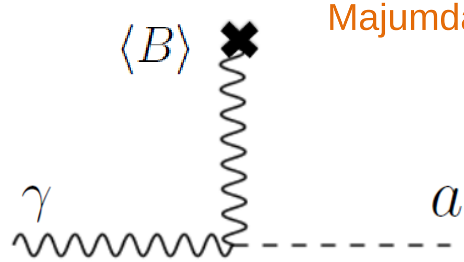
Fermi-LAT data of certain galactic pulsars and supernova remnants indicate a depletion of gamma rays at E > 1 GeV, which might be due to the conversion of gamma rays to some invisible particles, which becomes efficient for E > 1 GeV.

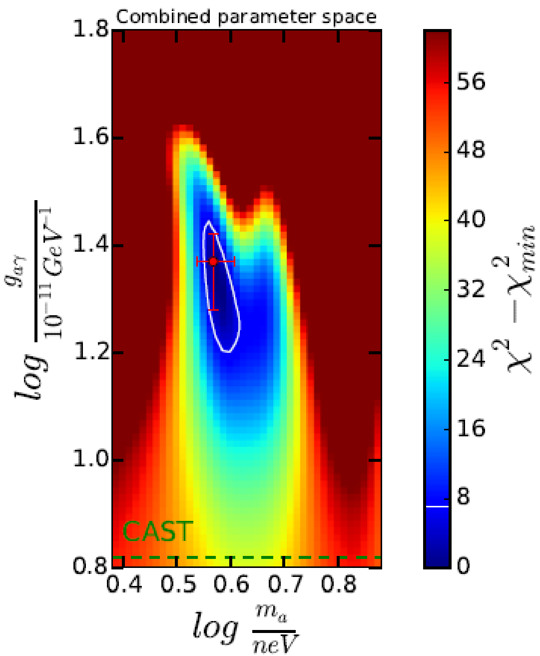
Within the simple scenario based on the usual ALP to photon coupling, the data favor some particular region in the ALP parameter space:

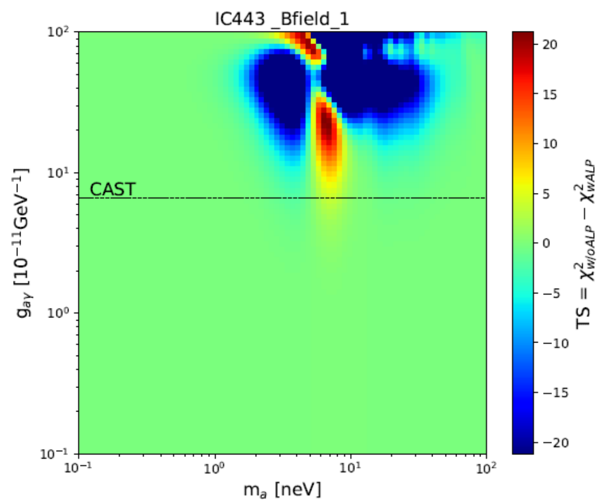
Majumdar et al. '18; Xia et al. '18

$$\frac{1}{4} g_{a\gamma\gamma} a F^{\mu\nu} \tilde{F}_{\mu\nu} = g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

$$g_{a\gamma\gamma} = \frac{c_\gamma}{8\pi^2} \frac{1}{f_a} \quad \Big(c_\gamma = \mathcal{O}(1) \Big)$$



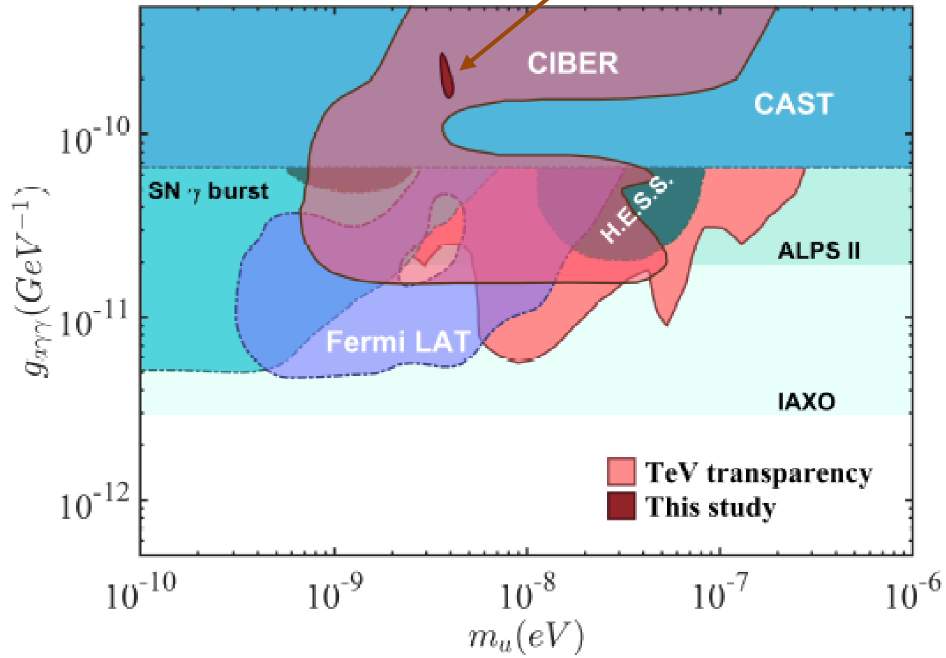




$$\Rightarrow \quad f_a \sim 2 \times 10^8\,\mathrm{GeV}, \quad m_a \sim 4 \times 10^{-9}\,\mathrm{eV}$$

But this scenario is in conflict with the constraints from CAST & SN1987A:

- γ-ray spectral modulation of galactic pulsars and SN remnants
- TeV γ-ray transparency



Majumdar et al. ‘18

More complicate but viable scenario

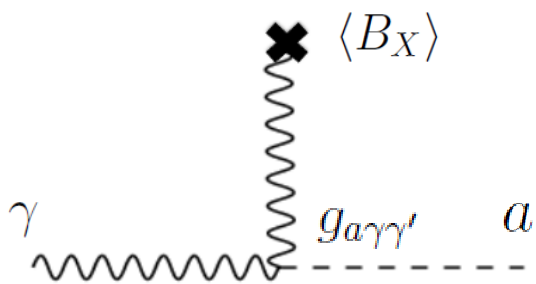
KC, S. Lee, H. Seong, S. Yun, 1806.09508

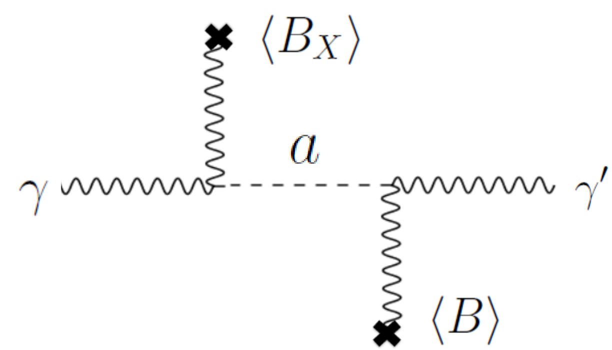
Introduce a massless dark photon X_μ together with an ALP which couples to the ordinary photon & dark photon as

$$\frac{1}{4} g_{a\gamma\gamma'} a F^{\mu\nu} \tilde{X}_{\mu\nu} = \frac{1}{2} g_{a\gamma\gamma'} a \left(\vec{E}_X \cdot \vec{B} + \vec{B}_X \cdot \vec{E} \right)$$

Note that additional U(1)'s are ubiquitous in string models, and one of those U(1)'s in the hidden sector might be unbroken.

$g_{a\gamma\gamma'}$ is significantly less constrained than $g_{a\gamma\gamma}$, yet it can convert photons to either ALPs or dark photons in the presence of nonzero background dark photon fields (B_X,E_X):





Conversion probabilities in background (B_X,E_X) and B:

$$P_{\gamma\to a} = P_{a\to\gamma} \;=\; \left(\frac{B_{XT}^2}{B_{XT}^2+B_T^2}\right)\left(\frac{\omega^2}{\omega^2+\omega_c^2}\right)\sin^2\frac{\triangle_{\rm osc}d}{2},$$

$$P_{\gamma\to\gamma'} = P_{\gamma'\to\gamma} \;=\; \frac{2B_{XT}^2B_T^2}{(B_{XT}^2+B_T^2)^2}\left(1-\cos\frac{\triangle_a d}{2}\cos\frac{\triangle_{\rm osc}d}{2}\right.$$
$$\left.-\frac{\omega_c}{\sqrt{\omega^2+\omega_c^2}}\sin\frac{\triangle_a d}{2}\sin\frac{\triangle_{\rm osc}d}{2}-\frac{\omega^2}{2(\omega^2+\omega_c^2)}\sin^2\frac{\Delta_{\rm osc}d}{2}\right)$$

$$\omega_c = \frac{m_a^2}{2g_{a\gamma\gamma'}B_{\rm eff}},\quad \Delta_a = \frac{m_a^2}{2\omega},\quad \Delta_{\rm osc} = \frac{g_{a\gamma\gamma'}B_{\rm eff}\sqrt{\omega^2+\omega_c^2}}{\omega}$$

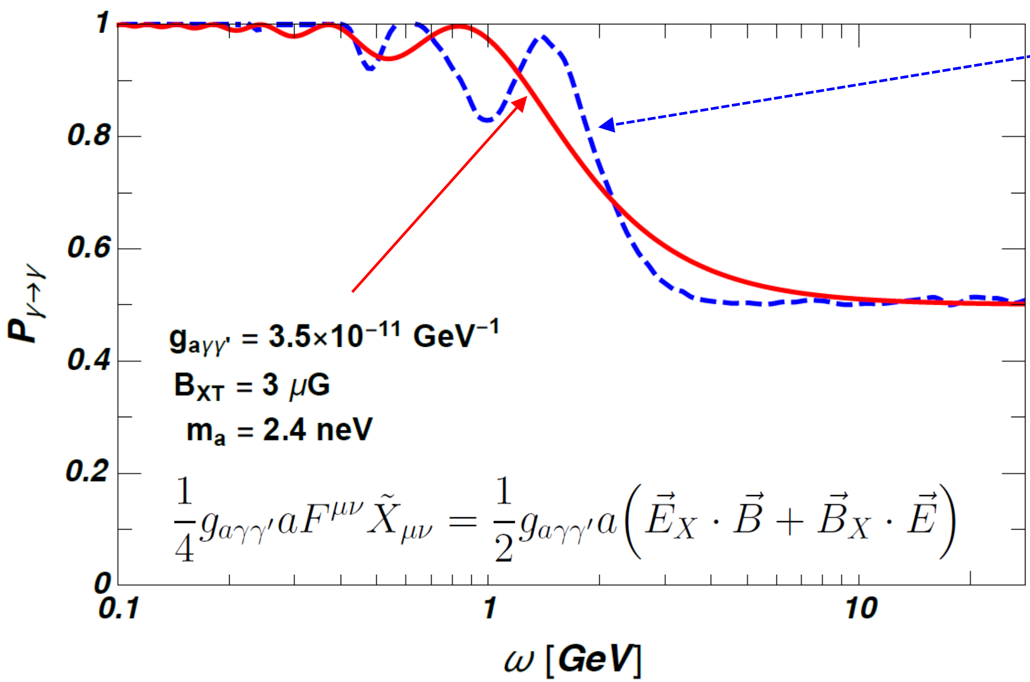
$$B_{\rm eff}\equiv\sqrt{B_{XT}^2+B_T^2}$$

$$\vec{B}_T \;=\; \langle\vec{B}\rangle-\hat{k}(\hat{k}\cdot\langle\vec{B}\rangle)$$
$$\vec{B}_{XT} \;=\; \left(\langle\vec{B}_X\rangle-\hat{k}(\hat{k}\cdot\langle\vec{B}_X\rangle)\right)-\hat{k}\times\langle\vec{E}_X\rangle$$

ALP parameters for sizable depletion of gamma-rays with E > 1 GeV at galactic distance scales:

$$m_a = \mathcal{O}(10^{-9})\ \mathrm{eV}, \quad g_{a\gamma\gamma'} = \mathcal{O}(10^{-10} - 10^{-11})\,\mathrm{GeV}^{-1}, \quad B_X = \mathcal{O}(1)\ \mu\mathrm{G}$$

Photon survival probability for PSR J2021+3651



$$\frac{1}{4} g_{a\gamma\gamma} a F^{\mu\nu} \tilde{F}_{\mu\nu} = g_{a\gamma\gamma} a \vec{E} \cdot \vec{B}$$

$$g_{a\gamma\gamma} = 3.5 \times 10^{-10}\,\mathrm{GeV}^{-1}$$

$$m_a = 4.4\,\mathrm{neV}$$

Constraints on $\frac{1}{4} g_{a\gamma\gamma'} a F^{\mu\nu} \tilde{X}_{\mu\nu} = \frac{1}{2} g_{a\gamma\gamma'} a \left(\vec{E}_X \cdot \vec{B} + \vec{B}_X \cdot \vec{E} \right)$

1) Stellar emission of ALP or dark photon:

Plasmon decays: $\gamma(\text{plasmon}) \to a + \gamma' \quad \Rightarrow \quad g_{a\gamma\gamma'} < 5 \times 10^{-10}\,\text{GeV}^{-1}$

2) Gamma-ray bursts associated with SN1987A, resulting from

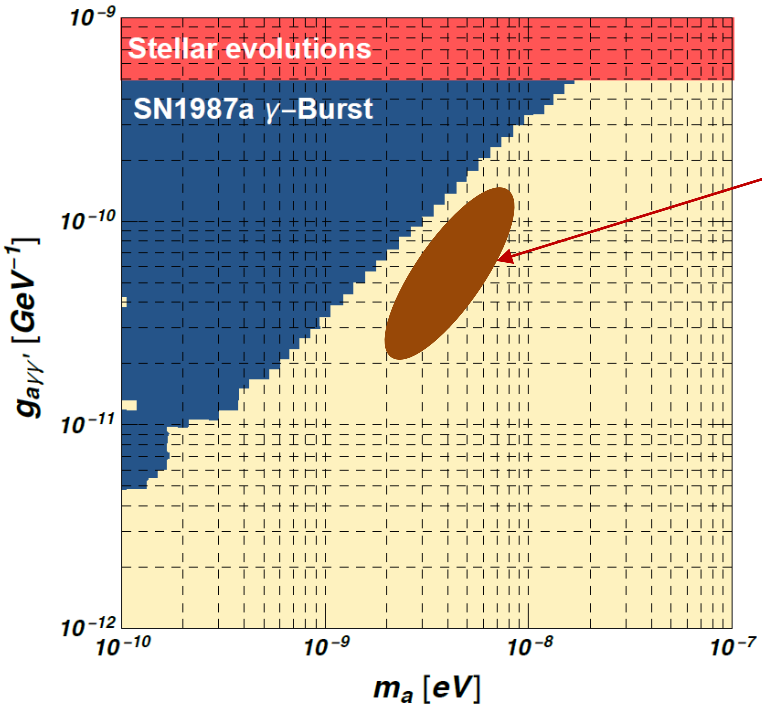
$$a \text{ or } \gamma' \text{ emitted from SN1987A} \quad \to \quad \gamma$$
$$(\text{in background } B_X \text{ or } E_X)$$

Compared to $g_{a\gamma\gamma}$, the bound on $g_{a\gamma\gamma'}$ is weaker as $g_{a\gamma\gamma'}$ is less efficient in producing ALP from SN1987A.
(Plasmon decay vs Primakov process)

[* No constraint from CAST as there is no $a \to \gamma$ induced by background $\langle B \rangle$
* Constraint from the CMB distortion on the mechanism to generate the background B_X or E_X.]

$$\frac{1}{4}g_{a\gamma\gamma'}aF^{\mu\nu}\tilde{X}_{\mu\nu} = \frac{1}{2}g_{a\gamma\gamma'}a\left(\vec{E}_X\cdot\vec{B}+\vec{B}_X\cdot\vec{E}\right)$$

$$B_X = 3\,\mu\mathrm{G}$$



ALP parameter region for sizable depletion of γ-rays with E > 1 GeV at galactic distance scales

A key ingredient of this scenario is the background dark photon gauge field which should be generated at late time (z< 1000) to avoid a too large distortion of CMB.

This can be achieved for instance by an additional ultra-light ALP ϕ with

KC, H. Kim, T. Sekiguchi '18

$$f_\phi \sim 10^{17}\,\mathrm{GeV}, \quad m_\phi \sim 10^{-28}\,\mathrm{eV}$$

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{4}g_{\phi\gamma'\gamma'}\phi X^{\mu\nu}\tilde{X}_{\mu\nu}$$

Coherent oscillation of ϕ beginning when $3H(\tau_{\rm osc}) \simeq m_\phi$:

$$\phi(\tau) \sim f_\phi \left(\frac{R(t)}{R(t_{\rm osc})}\right)^{-3/2} \cos\left(m_\phi(t - t_{\rm osc})\right) \qquad \left(\phi_{\rm initial} \sim f_\phi\right)$$

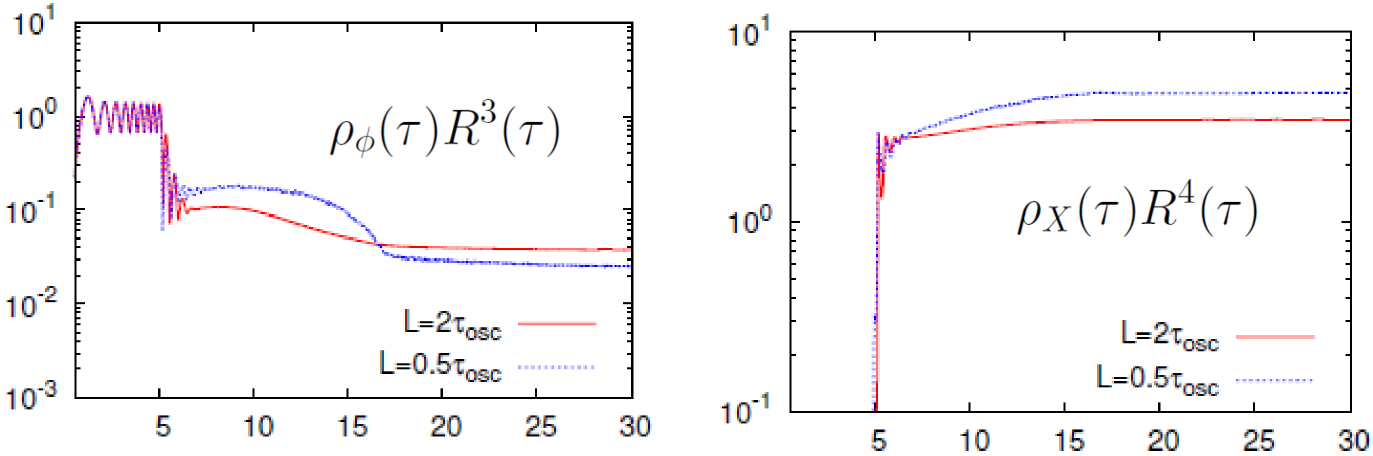
$$\left(ds^2 = dt^2 - R^2(t)dx^2 = R^2(\tau)(d\tau^2 - dx^2)\right)$$

Tachyonic instability of X_μ caused by oscillating ϕ, resulting in exponential amplification of the vacuum fluctuations of X_μ:

$$\ddot{\mathbf{X}}_{k\pm} + k(k \pm g_{\phi\gamma'\gamma'}\dot{\phi})\mathbf{X}_{\mathbf{k}\pm} = \mathbf{0} \quad \left(\text{tachyonic instability for } k \sim g_{\phi\gamma'\gamma'}\dot{\phi} \sim g_{\phi\gamma'\gamma'}f_\phi m_\phi\right)$$

Evolution of $\rho_\phi(\tau)R^3(\tau)$ and $\rho_X(\tau)R^4(\tau)$ for $g_{\phi\gamma'\gamma'}f_\phi \gtrsim \mathcal{O}(10)$

KC, H. Kim, T. Sekiguchi '18



$$\frac{\tau}{\tau_{\rm osc}} = \frac{R(\tau)}{R(\tau_{\rm osc})}$$

$$B_X \sim E_X \sim 0.6\,\mu{\rm G}\left(\frac{10^{-28}\,{\rm eV}}{m_\phi}\right)^{1/3}\left(\frac{f_\phi}{10^{17}\,{\rm GeV}}\right)$$

$$\text{produced at } z_{\rm prod} \equiv \frac{R(\tau_0)}{R(\tau_{\rm prod})} \sim 300\left(\frac{m_\phi}{10^{-28}\,{\rm eV}}\right)^{1/2}$$

$$\Omega_\phi h^2 \equiv \frac{\rho_\phi h^2}{\rho_c} \sim 3.6\times 10^{-6}\left(\frac{m_\phi}{10^{-28}\,{\rm eV}}\right)^{1/2}\left(\frac{f_\phi}{10^{17}\,{\rm GeV}}\right)^2$$

* $z_{\rm prod} < 1000$ to avoid a too large distortion of CMB.

* $\Omega_\phi h^2 < 10^{-4}$ to be consistent with other CMB constraints.

Conclusion

- Various issues in particle physics and cosmology suggest that there may exist multiple light axions with hierarchically different masses and decay constants, and string theory offers an attractive theoretical framework to realize such a scenario.

- Recently noticed gamma-ray spectral modulations of certain galactic pulsars and supernova remnants, as well as the TeV gamma-ray transparency, can be explained by the $a - \gamma - \gamma'$ oscillations in background dark photon gauge fields, while satisfying all the available observational constraints.

  The underlying scheme involves two axions with

$$f_a = \mathcal{O}(10^8 - 10^9)\,\mathrm{GeV}, \quad m_a = \mathcal{O}(10^{-9})\,\mathrm{eV}$$

$$f_\phi = \mathcal{O}(10^{17})\,\mathrm{GeV}, \quad m_\phi = \mathcal{O}(10^{-28} - 10^{-29})\,\mathrm{eV}$$