StringPheno 2019 CERN 25 June 2019

Moduli Stabilisation and Inflation in Type IIB/F-theory

George K. Leontaris

University of Ioannina
$I\omega\alpha\nu\nu\iota\nu\alpha$
GREECE

based on work with Ignatios Antoniadis and Yifan Chen

arXiv: 1803.08941 ; 1809.05060 & to appear

$\mathcal{A}$im of our $\mathcal{W}$ork

- Propose a solution to the Moduli Stabilisation problem
- Examine whether a dS vacuum exists in String Theory

($\cdots$ based on **perturbative** quantum corrections only!)

- If yes,

examine if slow roll inflation can be accommodated.

Outline of the Talk

- II-B/$\mathcal{F}$-Theory bosonic spectrum and Moduli fields
- *Effective Supergravity from type II-B & effective potential*
- D7 branes and logarithmic corrections
- Inflation
- Concluding Remarks

★ **Type II-B/F-theory**

★ **Bosonic** Spectrum - **Moduli** Space

▲ Graviton, dilaton and Kalb-Ramond (KR)-field

$$g_{\mu\nu},\ \phi,\ B_{\mu\nu} \to B_2$$

▲ Scalar, 2- and 4-index fields (*p-form potentials*)

$$\mathbf{C_0}, C_{\mu\nu}, C_{\kappa\lambda\mu\nu} \to C_p,\ p = 0, 2, 4$$

1. ▲ $C_0,\ \phi$ $\to$ *combined to* **axion-dilaton** *modulus:*

$$S = C_0 + i\, e^{\phi} \to C_0 + \frac{i}{g_s}$$

2. z_a : *Complex Structure (CS)* moduli (*shape*)

3. T_i : Kähler (*size*) moduli ($\tau_i = \mathrm{Re}T$).

▲ $\exists$ **plethora** *of* **moduli fields** *in* *CY* *compactifications*$\rightarrow$
... if massless $\rightarrow$ problems with fifth forces and other cosmological issues...

▲ Important Task ▲

▲ Generate a potential and assure positive mass-squared for all moduli fields $\Rightarrow$

$\Rightarrow$ $\mathcal{M}oduli$ $\mathcal{S}tabilisation$ $\Leftarrow$

Type II-B effective Supergravity

Basic 'ingredients':

Superpotential $\mathcal{W}$ *and Kähler potential* $\mathcal{K}$

▲ The Superpotential $\mathcal{W}$ ▲

▲ *Field strengths:*

$$F_p := d\,C_{p-1},\ H_3 := d\,B_2,\ \ \Rightarrow \mathbf{G_3} := F_3 - S H_3$$

▲ *Holomorphic* $(3,0)$*-form :* $\Omega(z_a)$

Flux-induced superpotential (*G. V. W. hep-th/9906070*):

$$\mathcal{W}_0 = \int \mathbf{G_3} \wedge \Omega(z_a)$$

▲ Supersymmetric conditions:

$$\mathcal{D}_{z_a}\mathcal{W} = 0, \quad \mathcal{D}_S \mathcal{W} = 0\ :$$

$\Rightarrow$ z_a *and* S **stabilised** $\Leftarrow$

but!

▲ Kähler moduli $\notin \mathcal{W}_0 \ \Rightarrow$ remain unfixed! ▲

▲ The Kähler potential ▲

$$\mathcal{K}_0 = -\sum_{i=1}^{3} \ln(-i(T_i - \bar{T}_i)) - \ln(-i(S - \bar{S})) - \ln(i \int \Omega \wedge \bar{\Omega}) \cdot$$

▲ The scalar potential ▲

$$\begin{aligned} V &= e^{\mathcal{K}} (\sum_{I,J} \mathcal{D}_I \mathcal{W}_0 \mathcal{K}^{I\bar{J}} \mathcal{D}_{\bar{J}} \mathcal{W}_0 - 3|\mathcal{W}_0|^2) \\ &= e^{\mathcal{K}} \sum_{I,J=z_a,\neq T_i} D_I \mathcal{W}_0 \mathcal{K}^{-1}_{I\bar{J}} D_{\bar{J}} \mathcal{W}_0 \quad (D_I \mathcal{W}_0 = 0, \text{ flatness}) \\ &\quad + e^{\mathcal{K}} \left(\sum_{I,J=T_i} \mathcal{K}_0^{I\overline{J}} D_I \mathcal{W}_0 D_{\overline{J}} \mathcal{W}_0 - 3|\mathcal{W}_0|^2 \right) \quad (= 0, \text{ no scale}) \end{aligned}$$

Kähler moduli completely **undetermined**!

$\Rightarrow$ ...need to include **Quantum corrections ...** $f(\tau)$ ...

"breaking" no-scale structure:

$$\mathcal{K} = -2\log\left(\tau^{\frac{3}{2}} + \gamma f(\tau)\right), \quad \mathcal{V} = \tau^{\frac{3}{2}}$$

Resulting F-term potential (γ-expansion):

$$V_F \propto \gamma\tau^{-\frac{9}{2}} \left(3f(\tau) - 4\tau f'(\tau) + 4\tau^2 f''(\tau)\right)$$

<u>*Some possible $f(\tau)$ functions*</u>:

▲▲α) power-law corrections (V_F: *homogeneous*) $f(\lambda\tau) = \lambda^n f(\tau)$)

$$f(\tau) \propto \tau^n \;\Rightarrow\; \boxed{V_F \propto \tau^{n-\frac{9}{2}}} \Rightarrow \nexists\; (V_F)_{min}$$

▲▲β) **logarithmic** $f(\tau) \propto \log \tau$:

$$\boxed{V_F \propto \gamma \tau^{-\frac{9}{2}} \left(\log(\tau) - \tfrac{8}{3}\right) + \cdots} \Rightarrow \ \exists \ (V_F)_{min} \ \forall \ \gamma < 0$$

▲ Moreover, adding a constant $\xi = \gamma \log(\mu)$:

$$\begin{aligned} \mathcal{K} &= -2 \log \left(\mathcal{V} + \gamma \log(\mathcal{V}) + \xi + \mathcal{O}(\frac{1}{\mathcal{V}}) \right) \\ &= -2 \log \left(\mathcal{V} + \gamma \log(\mu \mathcal{V}) + \mathcal{O}(\frac{1}{\mathcal{V}}) \right) \end{aligned}$$

At $(V_F)_{min}$ volume size controlled by μ:

$$\mathcal{V}_{min} = \frac{1}{\mu} e^{\frac{13}{3}}$$

$\Rightarrow$ large volume expansion for $\mu = e^{-\frac{\xi}{|\gamma|}} \ll 1$

Origin of ξ and $f(\tau)$

from

$\mathcal{PERTURBATIVE}$

String Loop Corrections

Quantum Corrections

1. α'-corrections

   shifting volume by a constant (Becker et al, hep-th:0204254)

$$\mathcal{V} \to \mathcal{V} + \xi, \quad \xi = -\frac{\zeta(3)}{4(2\pi)^3}\chi$$

2. Perturbative string loop-corrections

   important in the presence of D-branes

   (Antoniadis, Chen, G.K.L.: hep-th/1803.08941 & to appear)

▲▲ String Coupling Loop Corrections ▲▲

In String Theory:

Multigraviton scattering generates higher derivative couplings in curvature

Leading term in type II-B:

$$\propto R^4$$

Reduction on $\mathcal{M}_4 \times \mathcal{X}_6$, (with $\mathcal{M}_4$ 4-d Minkowski) induces:

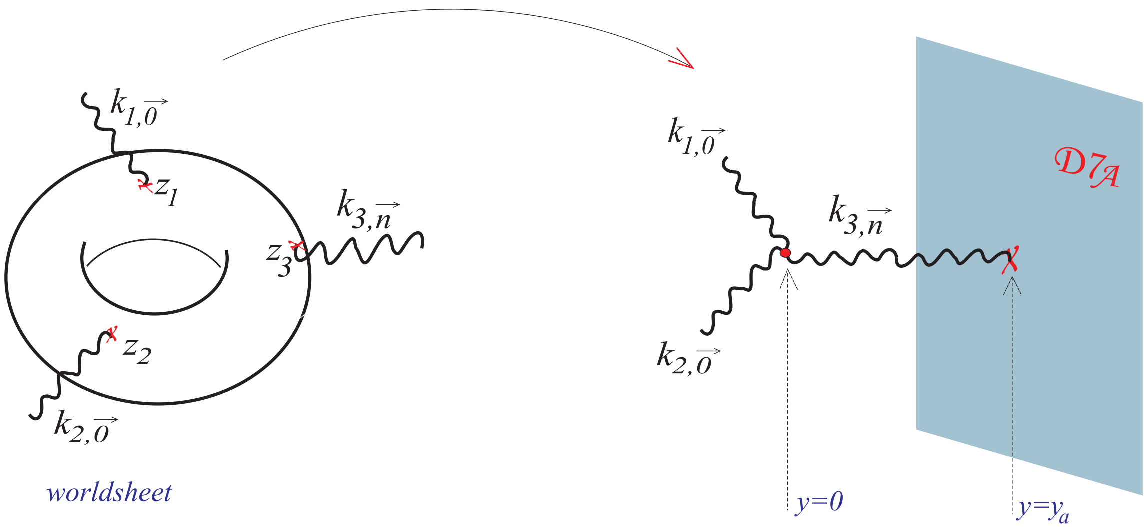
localised Einstein Hilbert ($\mathcal{EH}$) term $\propto$ Euler characteristic

$$\frac{1}{3!(2\pi)^3}\,\chi = \int R \wedge R \wedge R$$

▲▲ *this $\mathcal{EH}$ term possible in 4-dimensions only*!

Localised vertices can emit gravitons and KK-excitations in 6d

⇒ KK-exchange between graviton vertices and $D7$-branes



Non-zero contribution from 1-loop; 3-graviton scattering amplitude 2 massless 1 KK:

$$\text{Correction} \propto \chi \int_{M_4} \left(1 - \sum_{i=1,2,3} e^{\phi} T_i \log(R_\perp^i)\right) \mathcal{R}_{(4)}$$

⇑⇑

▲▲ Analogously, the corrections to the Kähler potential involve the corresponding Kähler moduli and have the form:

$$\delta = \sum_{i=1}^{3} \gamma_i \log(\tau_i)$$

$$\begin{aligned}\mathcal{K} &= -2\ln\left[e^{-2\phi}(\mathcal{V}+\xi)+\delta\right] \\ &= -\ln[-i(S-\bar{S})] - 2\ln\left(\hat{\mathcal{V}}+\hat{\xi}+\hat{\delta}\right)\end{aligned}$$

with $S = b + ie^{-\phi}$, $\hat{\delta} = \delta g_s^{1/2}$

▲▼ ξ and δ break **no-scale** structure of Kähler potential

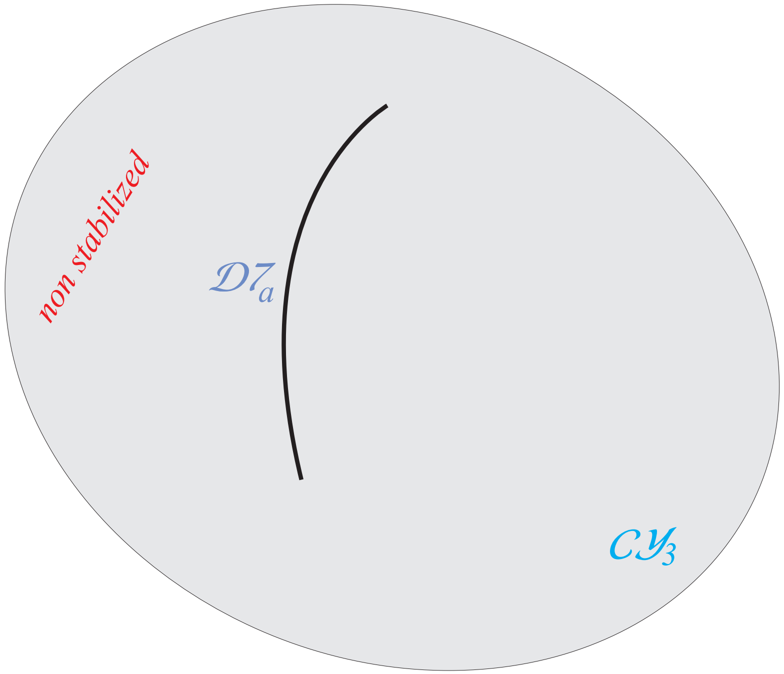
Stabilisation and $D7$ Branes

Looking for the minimum number of $D7$ branes required to stabilise the T_i-fields and lead to a dS minimum.

Quantum corrections from a single $D7$ Brane

▲ *volume parametrised in terms of τ, u 4-cycle moduli:* $\mathcal{V} = \tau\sqrt{u}$

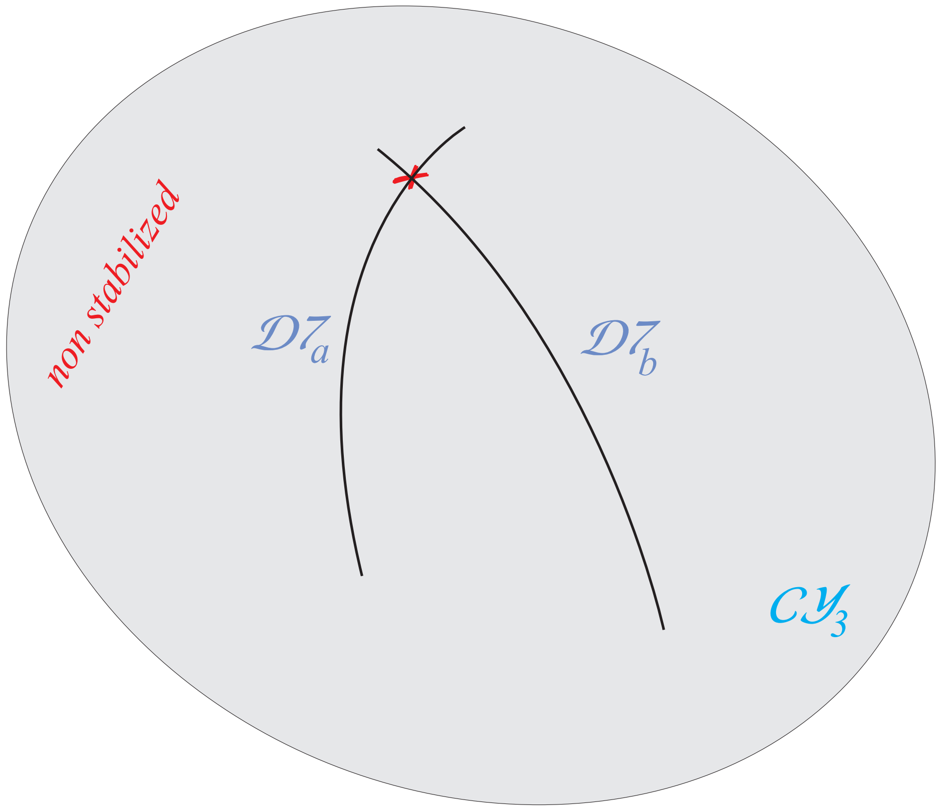
$$\mathcal{K} = -2\ln\left(\tau\sqrt{u} + \xi + \eta \ln u\right)$$



▲ $\exists$ minimum for u-direction iff $\eta < 0$, however ...

▲ stabilisation of τ-direction not possible with only one $D7$!

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \xi + \gamma_1 \ln u_1 + \gamma_2 \ln u_2\right)$$



▲ *... not even for two intersecting D7s!*

Stabilisation requires three intersecting $D7$s! (at least)

▲ Kähler potential including loop corrections: from $3 \times D7$:

$$\mathcal{K} = -2\ln\{\mathcal{V} + \sum_{k=1}^{3} \gamma_k \ln(\mathcal{V}/\tau_k)\}$$

F-term potential (*assuming* $\gamma_k \to \gamma, \xi = 0$):

$$V_F \approx 3\gamma \frac{\ln \mathcal{V} - 4}{\mathcal{V}^3} + \mathcal{O}(\gamma^3)$$

▲ **Inclusion of $\mathcal{D}$-terms**:

$$V_{\mathcal{D}} = \sum_a \frac{d_a}{\tau_a} \left(\frac{\partial \mathcal{K}}{\partial \tau_a} \right)^2 \approx \sum_a \frac{d_a}{\tau_a^3} + \cdots$$

Minimisation of $V_{\text{eff}} = V_F + V_{\mathcal{D}}$ w.r.t.:

$$\tau_1, \tau_2 \; and \; \mathcal{V} = \sqrt{\tau_1 \tau_2 \tau_3} \Rightarrow$$

$$V_{\text{eff}} \propto \gamma \frac{\ln \mathcal{V} - 4}{\mathcal{V}^3} + \frac{d}{\mathcal{V}^2}$$

▲ *focusing on* $\frac{dV_{eff}}{d\mathcal{V}} = 0$ *condition:*▲

$$\boxed{w\, e^{w} \; = \; z}$$

where: $w = \frac{13}{3} - \ln \mathcal{V},\;\; z \propto \frac{d}{\gamma}$

▲ *Solution* → *multivalued Lambert W-function:* $w \Rightarrow W(z)$

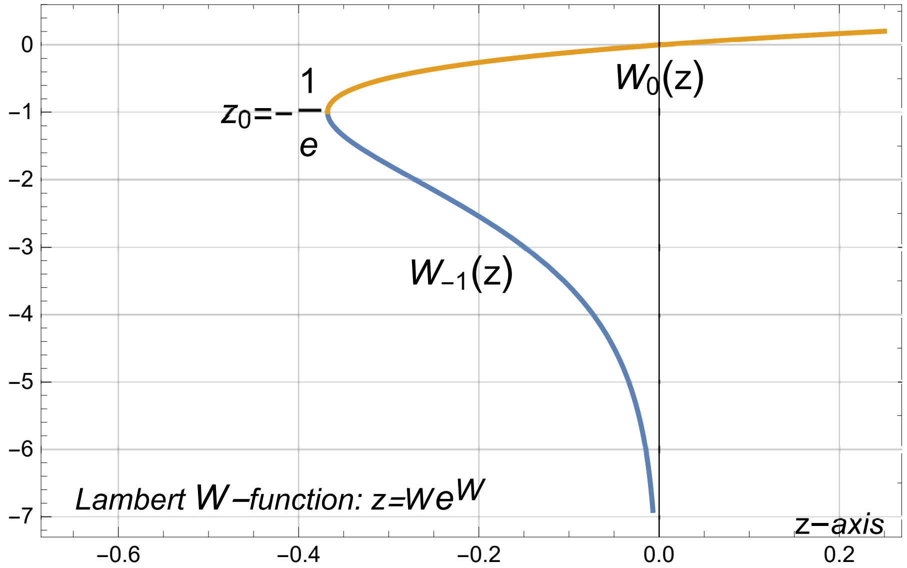
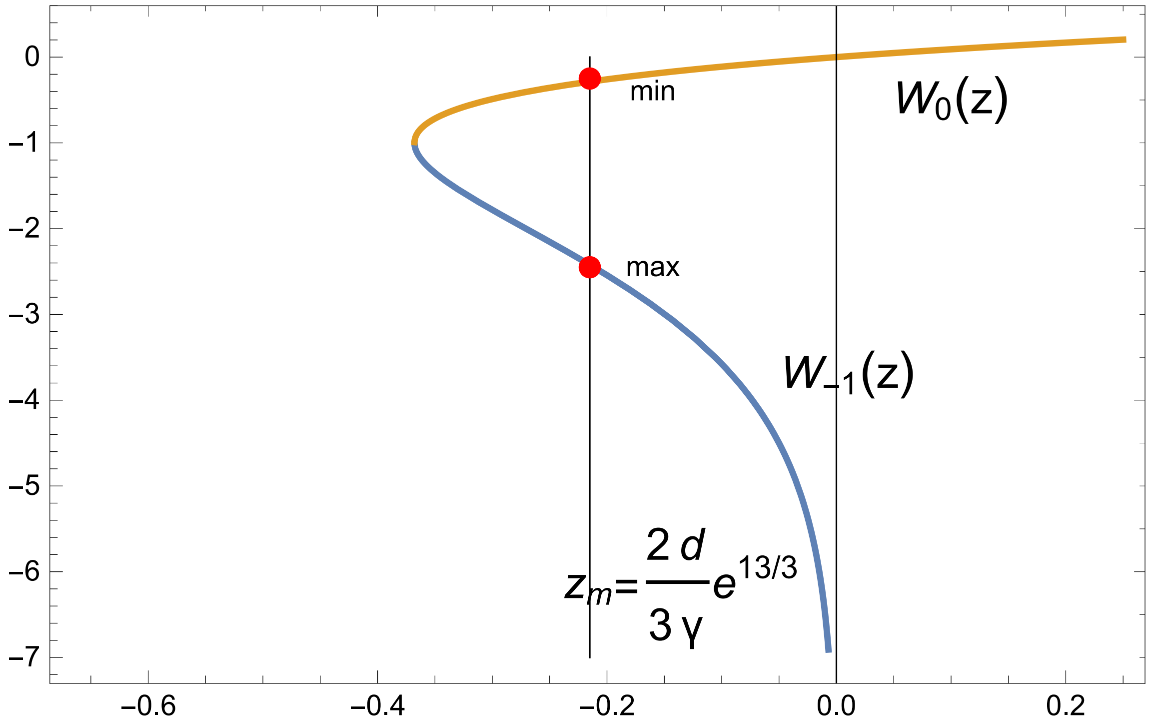


Figure 1: ▲▲ The two branches of the Lambert function. ▲▲ ⇒ Real, double valued for $-e^{-1} < z < 0 \Rightarrow V_{max}$ and V_{min}

vertical line represents any value of $z_m = \frac{2d}{3\gamma} e^{\frac{13}{3}}$ *between* $(-e^{-1}, 0)$ *where min and max can coexist.*

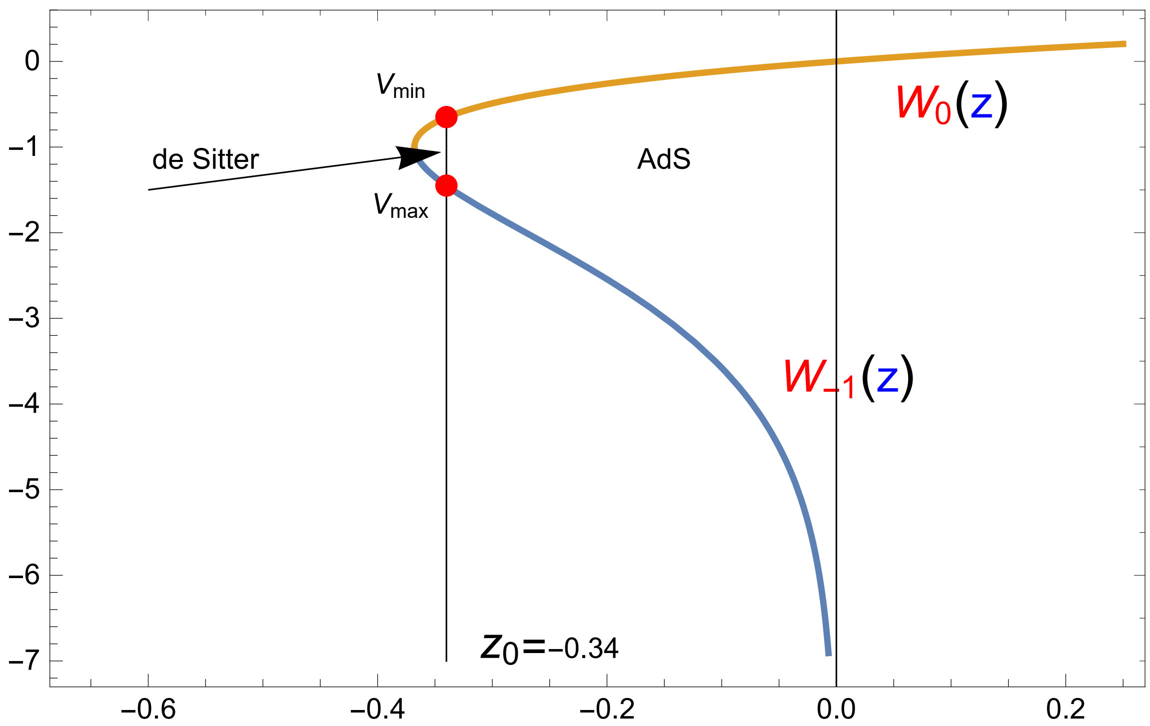


but! *requirement for de Sitter vacua puts additional restrictions*

▲ de Sitter vacua ▲

minimum $V_{\rm eff} = V_F + V_{\mathcal{D}}$ *at* $\mathcal{V}_0$ *must be positive:*

$$V_{\rm eff}^{min} = \frac{\gamma}{\mathcal{V}_0^3} + \frac{d}{\mathcal{V}_0^2} > 0 \ \to \ -7.24 < 10^3\frac{d}{\gamma} < -6.74$$



INFLATION

I. Antoniadis, Yifan Chen, GKL

(*hep-th/1809.05060*)

Although a dS minimum exists, allowed region too restrictive.
Additional requirements for slow-roll inflation hard to be met.

two ways out of this impasse

$\mathcal{A}$) Fayet-Iliopoulos (FI) contributions → uplift of $V_{\rm eff}$

▲ *Novel ways to construct FI-terms suggested in 1712.08601*

$\mathcal{B}$) introducing a nilpotent field X, $X^2 = 0$:

$$\Delta\mathcal{W} = fX, \ \Delta\mathcal{K} = \frac{X\bar{X}}{\tau_1^{n_1}\tau_2^{n_2}\tau_3^{n_3}},$$

$$for \ n_i = n \to V_{up} = \frac{f^2}{\mathcal{V}^{2-2n}}$$

▲ Total scalar potential

$$V_{total} = V_F + V_{\mathcal{D}} + V_{up}$$

normalised kinetic terms require redefinition of moduli fields :

$$t_i = \frac{1}{\sqrt{2}} \ln \tau_i$$

Inflaton is associated with volume $\mathcal{V}$:

$$t = \frac{t_1 + t_2 + t_3}{\sqrt{3}} = \sqrt{\frac{2}{3}} \ln(\mathcal{V}),$$

minimum and maximum w.r.t t:

$$t_{min/max} = \sqrt{\frac{2}{3}} \left(\frac{13}{3} - \mathcal{W}_{0/-1}\left(\frac{2}{3}\frac{d}{\gamma} e^{\frac{13}{3}}\right) \right) \qquad (1)$$

Redefining remaining two moduli fields to be "orthogonal" to t:

$$u = \frac{t_1 - t_2}{\sqrt{2}}, \; v = \frac{t_1 + t_2 - 2t_3}{\sqrt{6}}$$

Inflation period should be constrained between t_{max}, t_{min}

Constraints from slow-roll conditions

Ending point of inflation $t_{min} < t_{end} < t_{max}$:

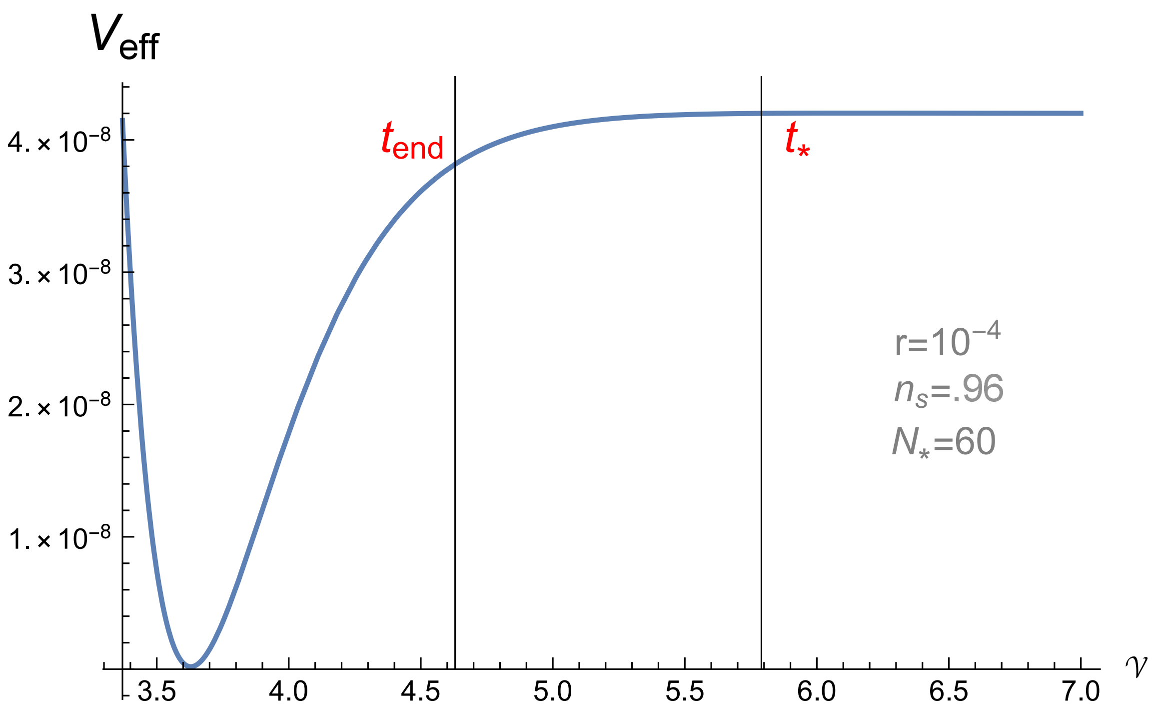
$$t_{end} = max\{t|_{\frac{1}{2}(\frac{V_t}{V})^2 \simeq 1}, t|_{\frac{|V_{tt}|}{V} \simeq 1}\},$$

Finding point t_* such that $t_{min} < t_{end} < t_* < t_{max}$ where e-folds:

$$N_* = \int_{t_{end}}^{t_*} \frac{V}{V_t} dt \sim 50 - 60$$

Spectral index at t_*:

$$n_s = 1 - 3\left(\frac{V_t}{V}\right)^2 + 2\frac{V_{tt}}{V}\bigg|_{t_*} \approx 0.96$$



Plot of $V_{\rm eff}$ *vs* $\mathcal{V}$. *Inflation occurs between* t_* *and* t_{end}

★ Conclusions ★

★ *IIB/F-theory:*

- Stabilisation of Kähler MF possible with Perturbative Corrections:

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \xi + \gamma_i \ln \tau_i\right)$$

Origin of log-corrections:

Induced Einstein-Hilbert terms from R^4-couplings in 10-d theory.

This $\mathcal{EH}$-term $\exists$ in $4d$ only!

$\Downarrow$

In the present context, *induced* EH-term ... indispensable element for a $4d$ de Sitter Universe

★ **Thank you for your attention** ★

APPENDIX

Type II 10-d effective action with $\mathcal{EH}$ & R^4 terms:

(see hep-th/9704145; hep-th/9707013, hep-th/9707018)

$$\mathcal{S} \supset \frac{c}{l_s^8}\int_{M_{10}} e^{-2\phi}\mathcal{R}_{(10)} + \frac{d}{l_s^2}\int_{M_{10}} (-2\zeta(3)e^{-2\phi} + 4\zeta(2))R^4 \wedge e^2$$

Compactifying on $\mathcal{CY}$ they give rise to localised $\mathcal{EH}$-term in 4-d:

$$\Rightarrow \frac{c}{l_s^8}\int_{M_4\times\mathcal{X}_6} e^{-2\phi}\mathcal{R}_{(10)} + \underbrace{2d\frac{\chi}{l_s^2}\int_{M_4}(\zeta(2) - \zeta(3)e^{-2\phi})\mathcal{R}_{(4)}}_{induced\ \mathcal{EH}\ term},$$

$$\Downarrow$$

$\chi \neq 0 \Rightarrow$ localised graviton kinetic terms: $\cdots(\mathcal{V} + \beta\chi)\mathcal{R}\cdots \Rightarrow$

A few details on localised graviton vertices

• Tree-level contribution to $\mathcal{EH}$-term vanishes in orbifold background $CY = T^6/Z_N$.

• 1-loop contribution from SUSY preserving $\mathcal{N} = (1,1)$ odd-odd spin structure of partition function

$$Z^{(1,1)}_{odd} \to \sum_{f=0,\ldots,n_f} \chi_f \equiv \chi$$

$(0,\ldots,n_f$ =fixed points)

1-loop amplitude: two massless gravitons and one KK-excitation. (in odd-odd spin structure $\to$ one graviton vertex must be taken in $(-1,-1)$-ghost picture.)

localisation property of w/f of $R_{(4)}$.

Amplitude:

$$\langle V_{(0,0)}^2 V_{(-1,-1)} \rangle = -\mathcal{C}_{\mathcal{R}} \frac{1}{N^2} \sum_{\substack{f=0,\ldots,n_f \\ k=0,\ldots,N-1}} e^{i\gamma^k q\cdot x_f} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} \int \prod_{i=1,2,3} \frac{d^2 z_i}{\tau_2} {\sum_{(h,g)}}' e^{\alpha' q^2 F_{(h,g)}(\tau, z_i)}$$

- $\mathcal{C}_{\mathcal{R}}$ tensor structure,
- $F_{(h,g)}(\tau, z_i) \to$ twisted sectors $(f,g) = (l,m)\frac{v}{N}$

Fourier transform $\Rightarrow$ Gaussian profile:

$$\to \frac{N}{w^6} e^{-\frac{y^2}{2w^2}}, \quad w^2 \sim \alpha' F_{(f,g)} \sim \frac{\ell_s^2}{N} \to \text{eff.width}$$