Two-field Cosmological α-attractors with Noether Symmetry

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Motivation

Cosmological Inflation:

Needed to solve several problems, chief among them being homogeneity and isotropy of the Universe on large scales

Standard description:

Inflationary expansion driven by the potential energy of a single scalar field φ (inflaton) with action:

$$S = \int d^4x \sqrt{-\det g} \left[\frac{R}{2} - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \varphi - V(\varphi) \right]$$

BUT: Many reasons to consider non-standard models

- Embedding in a fundamental theory:
  - In string compactifications 4d scalars arise in pairs

    (chiral superfields)
  - Compatibility with quantum gravity

    ('swampland' conjectures, in particular, constraints on $V(\varphi)$;

    very restrictive for a single scalar)
- Richer phenomenology:
  - Decoupling the generation of curvature perturbations (curvaton) from the inflaton
  - Non-Gaussianity of primordial fluctuations

Two-field α-attractor Models

Action:

$$S = \int d^4x\,\sqrt{-\det g}\left[\frac{R}{2} - \frac{1}{2}G_{ij}(\varphi)\,g^{\mu\nu}\,\partial_\mu\varphi^i\,\partial_\nu\varphi^j - V(\varphi)\right]\,,$$

$g_{\mu\nu}(x)$ - spacetime metric ,

Ansatz: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$, $\varphi^i = \varphi^i(t)$,

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)}$$ - Hubble parameter ,

$G_{ij}(\varphi)$ - target space metric: $i,j = 1,2$

Gaussian curvature of G_{ij} - constant and negative

Two-field α-attractors:

Kallosh, Linde et al. (arXiv:1311.0472 [hep-th], arXiv:1405.3646 [hep-th], arXiv:1503.06785 [hep-th], arXiv:1504.05557 [hep-th])

Two-dim. manifold $\mathcal{M}$ with metric $ds_G^2 = G_{ij} d\varphi^i d\varphi^j$ and Gaussian curvature $K_G = const < 0$: hyperbolic surface

$\rightarrow$ simplest example: Poincaré disk

Initial studies: radial trajectories on the Poincaré disk

Generalization to any hyperbolic surface:

Lazaroiu and Shahbazi (arXiv:1702.06484 [hep-th])

Two-field α-attractors:

Note:

In single-field models potential $V(\varphi)$ plays key role:

Always: field redefinition $\rightarrow$ canonical kinetic term

(Can transfer complexity to the potential)

In multi-field models:

Cannot redefine away the curvature of G_{ij} !

$\Rightarrow$ kinetic term becomes important

In particular: Can have genuine two (or multi-) field trajectories even when $\partial_{\varphi^i} V = 0$!

Action:

Substituting ansatz $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$, $\varphi^i = \varphi^i(t)$:

$$L = -3a\dot{a}^2 + a^3 \left[\frac{1}{2} G_{ij} \dot{\varphi}^i \dot{\varphi}^j - V(\varphi) \right]$$

$\rightarrow$ classical mechanical action for $\{a, \varphi^i\}$ ds.o.f.

Euler-L. eqs of $L \equiv$ original EoMs, when imposing constraint:

$$E_L \equiv \dot{a} \frac{\partial L}{\partial \dot{a}} + \dot{\varphi}^i \frac{\partial L}{\partial \dot{\varphi}^i} - L = 0$$

Note: $E_L = const$ on solutions of EL eqs., so Hamiltonian constraint $\rightarrow$ relation between integration constants

Noether Symmetry

Will impose condition that L has Noether symmetry

Motivation:

- can restrict:
  - form of potential V (expected)
  - value of Gaussian curvature K_G (unexpected!)

  (hence: may help for embedding in fundamental theory)

- can facilitate finding exact solutions of EoMs

  (as opposed to numerical ones)

- conserved quantity may play important role

Noether symmetry:

Recall: $L = -3a\dot{a}^2 + a^3\left[\frac{1}{2}G_{ij}\dot{\varphi}^i\dot{\varphi}^j - V(\varphi)\right]$

Denote $q^I \equiv \{a, \varphi^i\}$ - generalized coordinates on $\widetilde{\mathcal{M}}$

Consider transformation $q^I \rightarrow Q^I(q)$:

– generated by: $X = X^a(a,\varphi)\partial_a + X^i(a,\varphi)\partial_{\varphi^i}$

– induces transf. on tangent bundle $T\widetilde{\mathcal{M}}$, generated by :

(with coord. $\{q^I, \dot{q}^I\}$)

$$\hat{X} = X + \dot{X}^a(a,\varphi,\dot{a},\dot{\varphi})\partial_{\dot{a}} + \dot{X}^i(a,\varphi,\dot{a},\dot{\varphi})\partial_{\dot{\varphi}^i}$$

Symmetry condition: $\mathcal{L}_{\hat{X}}(L) = 0$

Noether symmetry:

(arXiv:1905.01611 [hep-th])

$\mathcal{L}_{\hat{X}}(L) = 0 \quad \Rightarrow$ set of equations equivalent with:

$$X^a = \frac{\Lambda(\varphi)}{\sqrt{a}} \quad , \quad X^i = Y^i(\varphi) - \frac{4}{a^{3/2}} G^{ij} \partial_j \Lambda \quad ,$$

where Λ and Y^i satisfy:

- $\nabla_i Y_j + \nabla_j Y_i = 0 \quad , \quad Y^i \partial_i V = 0$

  $\rightarrow$ Y^i - Killing vector on $\mathcal{M}$, preserving $V(\varphi)$

- $\nabla_i \nabla_j \Lambda = \frac{3}{8} G_{ij} \Lambda \quad , \quad G^{ij} \partial_i V \partial_j \Lambda = \frac{3}{4} V \Lambda$

  $\rightarrow$ Λ - Hessian symmetry (hidden symmetry)

Rotationally-invariant G_{ij}: (recall: $i, j = 1, 2$)

Consider G_{ij} : $ds^2_G = dr^2 + f(r)d\theta^2$

– Showed that Hessian equation $\nabla_i \nabla_j \Lambda = \frac{3}{8} G_{ij} \Lambda$ implies:

$$K_G = -\frac{3}{8}$$

$\rightarrow$ Λ-symmetry requires hyperbolic $\mathcal{M}$!

– Found general Λ-solution for any rotationally-invariant hyperbolic surface

With known Λ: $G^{ij} \partial_i V \partial_j \Lambda = \frac{3}{4} V \Lambda$ - equation for V

– Found general form of V compatible with Λ-symmetry

Summary

Found so far:

- Most general hidden symmetries of cosmological two-field α-attractor models with rot.-invariant scalar manifold metric

  [In particular: Gaussian curvature - fixed !]

- Form of scalar potential compatible with hidden symmetry
- Exact solutions in special case [separation-of-variables Ansatz]

Open issues:

- Exact solutions in general case ?...
- Embedding in string theory (points of enhanced symmetry) ?...
- Perturbations, cosmological observables ?...

Thank you!