$O(D, D)$ completion of the Friedmann equations

Stephen Angus
Ewha Woman's University, Seoul

based on

1905.03620 (SA, K. Cho, G. Franzmann, S. Mukohyama, J.H. Park)

String Phenomenology 2019
CERN, Geneva

Introduction

General Relativity is a successful theory of gravity.

- Geometry $\Leftrightarrow$ Matter; expressed via Einstein's equations

$$G_{\mu\nu} = 8\pi G_{\mathrm{N}} T_{\mu\nu} \ .$$

- GR accurately describes astrophysical/cosmological phenomena: perihelion precession, gravitational lensing, Friedmann eqns...
- However, some results cannot be explained by GR + visible matter e.g. rotation curves, accelerating expansion, horizon problem, ...

Broadly, two types of solutions to such problems:

1. GR is correct, but there is dark matter, dark energy, inflation, ...
2. Theory of gravity should be modified...

Introduction

- String theory predicts its own gravity, the **O**(D, D)-completion of General Relativity.
- In this Stringy Gravity, there are $(D^2 + 1)$ off-shell degrees of freedom, $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$, which couple independently to matter, yielding a richer spectrum of gravitational solutions than GR.
- Traditionally, we study GR-like solutions of string theory on a $D = 4$ background, fixing $B_{\mu\nu} = 0$ and $\phi \sim$ constant (usually via compactification and moduli stabilization, e.g. using R–R fluxes).
- However, in the most general case, all gravitational components may be dynamical.

Cosmology in Stringy Gravity

Goal of my talk:

To introduce a new framework for cosmology, based on the $\mathbf{O}(D, D)$ *invariant formulation of Stringy Gravity (Double Field Theory).*

A brief introduction to Double Field Theory

In Double Field Theory (Hull, Zwiebach; 2009) we describe D-dim. physics using $D+D$ coordinates, $x^A = (\tilde{x}_\mu, x^\nu)$, $A = 1, \ldots, D+D$.

- $\exists$ a manifest $\mathbf{O}(D,D)$ T-duality gauge symmetry (& doubled diff.s).
- Section condition: $\partial_A \partial^A = 2\,\partial_\mu \tilde{\partial}^\mu = 0$ (e.g. choose $\tilde{\partial}^\nu = 0$).
- Basic idea: D coordinates $\{\tilde{x}_\mu\}$ are gauged; gauge orbits $\simeq$ points in D-dimensional spacetime spanned by $\{x^\nu\}$ (Park; 2013).
- Gravity sector: $\{d, \mathcal{H}_{AB}\}$, DFT dilaton and $\mathbf{O}(D,D)$-symm. metric.
- $\mathcal{H}_{AB}$ can be decomposed into a pair of vielbeins, $\{V_{Ap}, \bar{V}_{A\bar{p}}\}$ (left- and right-moving string modes have independent local frames).
- On Riemannian backgrounds, $\tilde{\partial}^\mu = 0$: $\{d, \mathcal{H}_{AB}\} \to \{g_{\mu\nu}, B_{\mu\nu}, \phi\}$.

Aside: DFT can also be defined on 'non-Riemannian' backgrounds
$\Rightarrow$ moduli-free compactification (Cho, Morand, Park; 2018).

Einstein Double Field Equations

Local geometry $\Rightarrow$ DFT Ricci tensor $S_{p\bar{q}}$ and scalar $S_{(0)} = R + \ldots$

Couple to matter fields $\{\Upsilon_a\}$: **O**(D, D)-covariant action,

$$\int_{\Sigma} e^{-2d}\left[\tfrac{1}{16\pi G}S_{(0)} + L_{\rm matter}(\Upsilon_a)\right] \qquad (\Sigma\text{: } D\text{-dim. section})\,.$$

Gravitational equations of motion: Einstein Double Field Equations,

$$S_{p\bar{q}} = 8\pi G K_{p\bar{q}}\,, \qquad S_{(0)} = 8\pi G T_{(0)} \qquad \Rightarrow \quad G_{AB} = 8\pi G T_{AB}\,,$$

where the stringy energy-momentum tensor has (D^2+1) components,

$$K_{p\bar{q}} := \frac{1}{2}\left(V_{Ap}\frac{\delta L_{\rm matter}}{\delta \bar{V}_A{}^{\bar{q}}} - \bar{V}_{A\bar{q}}\frac{\delta L_{\rm matter}}{\delta V_A{}^{p}}\right)\,, \quad T_{(0)} := e^{2d}\times\frac{\delta\left(e^{-2d}L_{\rm matter}\right)}{\delta d}\,.$$

Note: $T_{(0)}$ depends on the Lagrangian density, $\mathcal{L}_{\rm matter} := e^{-2d}L_{\rm matter}$.

On-shell conservation: $\mathcal{D}_A T^{AB} \equiv 0$; $\;T_{AB} := 4V_{[A}{}^{p}\bar{V}_{B]}{}^{\bar{q}}K_{p\bar{q}} - \tfrac{1}{2}\mathcal{J}_{AB}T_{(0)}$.

Homogeneous and isotropic backgrounds

- Consider $D = 4$ solutions which are homogeneous and isotropic.
- Solving the DFT-Killing equations $\Rightarrow$ cosmological ansatz

$$\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + a(t)^2\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right] ,$$

$$B_{(2)} = \frac{hr^2}{\sqrt{1-kr^2}}\cos\vartheta\,\mathrm{d}r\wedge\mathrm{d}\varphi\,, \quad \phi = \phi(t)\,.$$

  Note: can choose e.g. cosmic gauge where the function $N(t) = 1$; solutions characterized by $a(t)$, $\phi(t)$, and parameters h and k.

- Note that the corresponding H-flux is homogeneous and isotropic:

$$H_{(3)} \equiv \mathrm{d}B_{(2)} = \frac{hr^2}{\sqrt{1-kr^2}}\sin(\vartheta)\mathrm{d}r\wedge\mathrm{d}\vartheta\wedge\mathrm{d}\varphi\,.$$

- Homogeneous and isotropic stringy energy-momentum tensor:

$$K^{\mu}{}_{\nu} = \mathrm{diag}(K^t{}_t(t), K^r{}_r(t), \ldots, K^r{}_r(t))\,, \quad T_{(0)} = T_{(0)}(t)\,.$$

Energy density and pressure

- Define energy density and pressure as

$$\rho := \left(-K^t{}_t + \frac{1}{2}T_{(0)}\right)e^{-2\phi}, \qquad p := \left(K^r{}_r - \frac{1}{2}T_{(0)}\right)e^{-2\phi}\ .$$

- Why? First of all, we should demand $\rho \equiv \mathcal{H}$ (Hamiltonian). Next, recall matter action $\int e^{-2d}L_{\mathrm{matter}}$, where $e^{-2d} = e^{-2\phi}\sqrt{-g}$; $-K^t{}_t = \pi^a\partial_0\Upsilon_a$, $T_0 = -2L_{\mathrm{matter}}$ (if L_{matter} is dilaton-independent).
- Stringy e-m tensor conserved $\Rightarrow$ one non-trivial conservation law:

$$\dot\rho + 3H(\rho + p) + \dot\phi T_{(0)}e^{-2\phi} = 0\ ,$$

where $H \equiv \frac{\dot a}{a}$ (in cosmic gauge), and $\dot{\{\}} = \frac{\mathrm{d}\{\}}{\mathrm{d}t}$.

O(*D*, *D*)-complete Friedmann Equations

In the homogeneous and isotropic case, EDFEs reduce to ($N(t) = 1$)

$$\frac{8\pi G}{3}\rho e^{2\phi} + \frac{h^2}{12a^6} = H^2 - 2\dot{\phi}H + \frac{2}{3}\dot{\phi}^2 + \frac{k}{a^2}\,,$$
$$\frac{4\pi G}{3}(\rho + 3p)e^{2\phi} + \frac{h^2}{6a^6} = -H^2 - \dot{H} + \dot{\phi}H - \frac{2}{3}\dot{\phi}^2 + \ddot{\phi}\,,$$
$$\frac{4\pi G}{3}\left(2\rho e^{2\phi} - T_{(0)}\right) = -H^2 - \dot{H} + \frac{2}{3}\ddot{\phi}$$

→ "**O**(D, D)-complete Friedmann Equations" (OFEs).

- Note: 3 OFEs + 1 conservation law ⇒ 3 independent equations.
- If $\dot{\phi} = \ddot{\phi} = 0$, $h = 0$ ⇒ standard GR cosmology; $T_{(0)} \equiv \rho - 3p$.
- For $h = k = 0$, covariance under **O**(3, 3) spatial T-duality:

Before	a	H	ϕ	ρ	p	$T_{(0)}$
After	a^{-1}	$-H$	$\phi - 3\ln a$	$a^6\rho$	$-a^6\left(p + T_{(0)}e^{-2\phi}\right)$	$T_{(0)}$

Generalized perfect fluid

- It is useful to define two equation-of-state parameters,

$$w := \frac{p}{\rho}\,; \qquad \lambda := \frac{T_{(0)}}{\rho e^{2\phi}}\,.$$

- For constant w and λ ("generalized perfect fluid"),

$$\rho = \rho_0 \frac{e^{-\lambda\phi}}{a^{3(1+w)}}\,.$$

- Power-law ansatz: $a \propto t^n$, $e^\phi \propto t^{-s}$ (set $h = k = 0$). Solution:

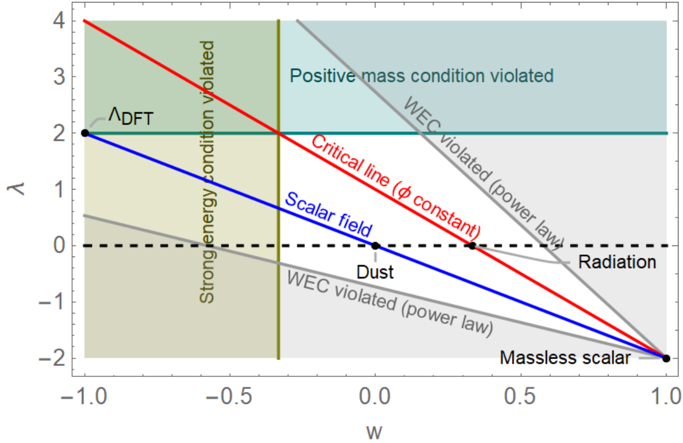
$$n = \frac{2(2w+\lambda)}{2+6w^2+6w\lambda+\lambda^2}\,, \qquad s = \frac{2(1-3w-\lambda)}{2+6w^2+6w\lambda+\lambda^2}\,,$$

Special cases:

- Constant dilaton, $s = 0$ on the 'critical line', $\lambda = 1 - 3w$
  $\Rightarrow$ recover standard GR cosmology on this line, with $T_{(0)} \equiv \rho - 3p$.
- Static universe, $n = 0$ on the line $\lambda = -2w$. Scalar fields also lie on this line (but can have varying w and λ).

Cosmological solutions

Identify various regions and types of matter in the (w, λ)-plane.



Also, pure DFT vacuum: $\rho = 0$ (Copeland, Lahiri, Wands; 1994).

Note: Usual supergravity case is $\lambda = 0 \Rightarrow$ radiation critical, dust is not.

Example: radiation with H-flux and freezing dilaton

Beyond power law, other analytic solutions. E.g. $\exists$ an analytic solution for radiation ($w = 1/3$, $\lambda = 0$), in the presence of non-vanishing spatial curvature and H-flux, in which the dilaton is frozen at late times.

- Conformal scale factor $b := ae^{-\phi}$ (in terms of conformal time η):

$$b^2 = \frac{\tau(C_1 + \Omega_{\rm rad} H_0^2 \tau)}{1 + k\tau^2} \; ; \quad \tau = \begin{cases} \tan(\eta - \eta_0) & \text{for} \quad k = 1 \, , \\ \eta - \eta_0 & \text{for} \quad k = 0 \, , \\ \tanh(\eta - \eta_0) & \text{for} \quad k = -1 \, . \end{cases}$$

- The dilaton profile is (c.f. $h = 0$: Copeland, Lahiri, Wands; 1994)

$$e^{2\phi} = \left(\frac{C_1 \tau}{\tau_* \left(C_1 + \Omega_{\rm rad} H_0^2 \tau \right)} \right)^{\pm\sqrt{3}} + \frac{1}{12} \frac{h^2}{C_1^2} \left(\frac{C_1 \tau}{\tau_* \left(C_1 + \Omega_{\rm rad} H_0^2 \tau \right)} \right)^{\mp\sqrt{3}} ,$$

which converges to a constant as $\eta \to \infty$ (for $k \in \{0, -1\}$).

de Sitter solutions?

Can we realize de Sitter in $\mathbf{O}(D,D)$ cosmology? We considered exponential scale factors in both string and Einstein frames.

For simplicity, we focused on $\lambda = -2w$ (e.g. Λ_{DFT}, canonical scalar).

String frame: $a = e^{Ht}$, $N = 1$, $\lambda = -2w$, $k = 0$:

$$8\pi G e^{2\phi}\rho + \frac{h^2}{4}e^{-6Ht} = -\frac{3}{2}H^2 + \frac{h^4}{8H^2}e^{-12Ht} < 0 \quad \text{as} \quad t \to \infty\,.$$

Einstein frame: $b = e^{Ht}$ (recall $a = e^{\phi}b$), $N = e^{\phi}$, $\lambda = -2w$, $k = 0$:

$$4\pi G\,(\rho_{\mathrm{E}} + p_{\mathrm{E}}) := 4\pi G e^{4\phi}\,(\rho + p) = -\dot{\phi}^2 - \frac{h^2}{4}e^{-6H_{\mathrm{E}}t-4\phi} \leq 0\,.$$

In both cases, weak energy condition violated; scalars tachyonic.

More generally: $(w,\lambda) = (-1,4) \Rightarrow$ GR cosmological constant!? Hence de Sitter solves the OFEs... but this case does not correspond to any known $\mathbf{O}(D,D)$-covariant Lagrangian. Would need $\rho = K_t{}^t e^{-2\phi}$, but this is minus the kinetic energy, so expect $\rho < 0 \Rightarrow$ WEC violated.

Summary

- Stringy Gravity (Double Field Theory) coupled to matter satisfies the Einstein Double Field Equations, $G_{AB} = 8\pi G T_{AB}$.
- From the Einstein Double Field Equations on homogeneous and isotropic Riemannian backgrounds, we derived the **O**(D, D)-complete Friedmann equations.
- We found various solutions, including a radiation solution with non-vanishing H-flux and frozen dilaton at late times.
- de Sitter solution for a Λ_{DFT}/scalar violates the weak energy condition in both string and Einstein frames. Another GR-like dS solution exists but no DFT origin: is de Sitter an artefact of GR?
- New **O**(D, D)-complete framework for cosmology.

Future directions

We have only scratched the surface of $\mathbf{O}(D, D)$-complete cosmology. Many issues yet to be addressed, such as:

- inflation. . . or, in general, generating (almost) scale-invariant curvature perturbations which match observations of the CMB;
- frame dependence: is string frame or Einstein frame correct? (In DFT, point particles follow geodesics in string frame...) Our universe is 13.8 billion years old... but in which frame?
- Maxwell fields couple to the dilaton... consequences?
- consistency with local measurements of G_{N}... slowly-varying dilaton at late times? (quintessence?)
- and many more...