$U(1)_R$ inspired inflation model in no-scale Supergravity

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Introduction

- **Planck satellite's four years data of the cosmic microwave background radiation and the large structure in the universe support the predictions of cosmological inflation.**

- **The recent data confirmed that spectral index (scalar density fluctuations) is given by**

$$n_s = 0.96 \pm 0.007$$

  and the upper bound on the tensor-to-scalar ratio is

$$r < 0.08$$

- **These results imposed severe challenges on several inflationary models. The simple chaotic and hybrid inflationary models are now ruled out.**

- **On the other hand, some other models of inflation with compatible cosmological fluctuation predictions receive a growing interest.**

- **One of these models is the Starobinsky inflation, which is based on modified gravity:**

$$\mathbf{R + \frac{1}{6m^2} R^2 \quad \Rightarrow \quad V(\phi) = \frac{3}{4} m^2 M_P^2 \left(1 - e^{-\sqrt{2/3}\phi/M_P}\right)^2 .}$$

Inflation in no-scale supergravity

- In supergravity, effective scalar potential is proportional to e^K. Thus, scalars typically pick up masses proportional to $H^2 \sim V$, where H is the Hubble parameter.

- This is the basis for the infamous η-problem of SUGRA inflation, which has to do with the fact that the slow-roll parameter η was pushed to order unity by SUGRA corrections, where

$$\eta = m_P^2 \frac{V''}{V} \simeq \frac{1}{3}\left(\frac{m}{H}\right) = \mathcal{O}(1),$$

  with $m \sim H$ being the mass of the scalar field. Hence, slow-roll is destabilized.

- An attractive way to avoid the η problem is provided by no-scale supergravity, due to the involvement of logarithmic form in the Kähler potential.

$$K = -3 \ln\left(T + \bar{T} - \frac{|\phi|^2}{3}\right),$$

  where T is the compactification volume modulus and and ϕ is another field in the hidden sector which will be responsible for inflation.

Starobinsky-like inflation in no-scale supergravity

- **It was shown that no-scale SUGRA can behave like a Starobinsky inflationary model: Ellis et al (2013) & (2016). The following superpotential was considered**

$$W = \frac{\mu}{2}\phi^2 - \frac{\lambda}{3}\phi^3.$$

- **In this case, the kinetic terms for the scalar fields T and ϕ become**

$$\mathcal{L}_{KE} = (\partial_\mu \phi^*, \partial_\mu T^*) \left(\frac{3}{(T + T^* - |\phi|^2/3)^2} \right) \begin{pmatrix} (T+T^*)/3 & -\phi/3 \\ -\phi^*/3 & 1 \end{pmatrix} \begin{pmatrix} \partial^\mu \phi \\ \partial^\mu T \end{pmatrix}$$

and the effective potential becomes

$$V \;=\; \frac{\hat{V}}{(T + T^* - |\phi|^2/3)^2} : \hat{V} \;\equiv\; \left| \frac{\partial W}{\partial \phi} \right|^2$$

- **Assuming that the field T is stabilized, by some dynamics, at $T = c/2$, one finds**

$$\mathcal{L}_{eff} = \frac{c}{\left(c - \frac{|\phi|^2}{3}\right)} |\partial_\mu \phi|^2 - \frac{\hat{V}}{\left(c - \frac{|\phi|^2}{3}\right)}$$

- **By considering the following field redefinition $\phi = \sqrt{3c}\tanh\left(\frac{\chi}{\sqrt{3}}\right)$, the Lagrangian becomes**

$$\mathcal{L}_{eff} = \mathrm{sech}^2((\chi - \chi^*)/\sqrt{3}) \left[|\partial_\mu \chi|^2 - (\frac{3}{c}) \left| \sinh(\chi/\sqrt{3}) \left(\hat{\mu}\cosh(\chi/\sqrt{3}) - \sqrt{3c}\lambda \sinh(\chi/\sqrt{3}) \right) \right|^2 \right]$$

- **Writing $\hat{\mu} = \mu\sqrt{c/3}$, the potential becomes**

$$V = \mu^2 \left| \sinh(\chi/\sqrt{3}) \left(\cosh(\chi/\sqrt{3}) - \frac{3\lambda}{\mu} \sinh(\chi/\sqrt{3}) \right) \right|^2$$

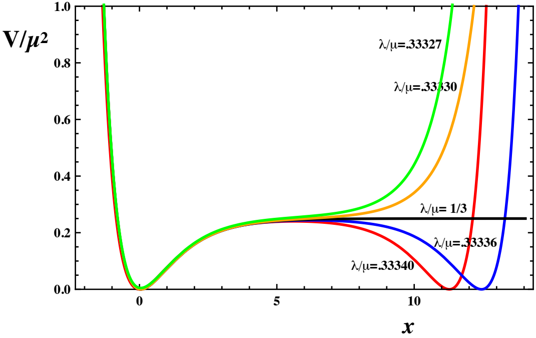
- **With $\chi = (x + iy)/\sqrt{2}$ and $\lambda = \mu/3$, we have**

$$\mathcal{L}_{eff} = \frac{1}{2}\sec^2(\sqrt{2/3}y)\left((\partial_\mu x)^2 + (\partial_\mu y)^2\right) - \mu^2 \frac{e^{-\sqrt{2/3}x}}{2}\sec^2(\sqrt{2/3}y)\left(\cosh\sqrt{2/3}x) - \cos\sqrt{2/3}y\right)$$

- **For real parameters, this potential is always minimized by $y = 0$ for any value of x in the range of interest for inflation.**

- **Setting $y = 0$ and $\lambda/\mu = 1/3$, we then identify the inflaton as x, with the inflationary (Starobinsky-like) potential**

$$V = \mu^2 e^{-\sqrt{2/3}x}\sinh^2(x/\sqrt{6}) = \frac{\mu^2}{4}\left(1 - e^{-\sqrt{2/3}x}\right)^2$$



The potential V in the NSWZ model for choices of $\lambda \sim \mu/3$ in Planck units. *Ellis, et al, PRL 111 (2013) 111301*

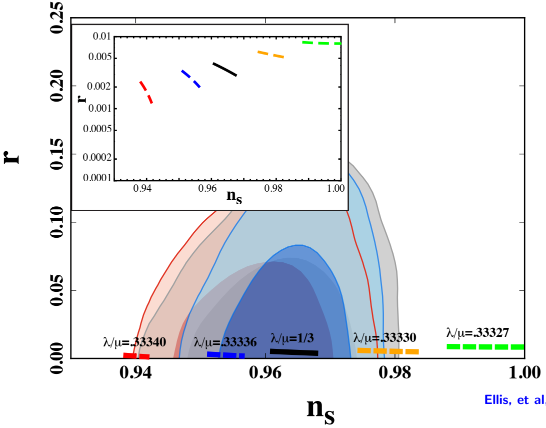
- **For the special case $\lambda = \mu/3$, the scalar amplitude is given by**

$$A_s = \frac{V}{24\pi^2\epsilon} = \frac{\mu^2}{8\pi^2}\sinh^4(x/\sqrt{6}),$$

implying a value $\mu = 2.2 \times 10^{-5}$ in Planck units for $N = 55$.

- **In the following plot, we display the predictions for (n_s, r) of the NSWZ model for five choices of the coupling λ that yield $n_s \in [0.93, 1.00]$ and $N \in [50, 60]$.**

- **As one can see, the values of λ are constrained to be close to the critical value $\mu/3$.**



Ellis, et al, PRL 111 (2013) 111301

$U(1)_R$ inspired inflation model in no-scale Supergravity

- **In the above NSWZ model, no $U(1)_R$ symmetry is prevailing in this construction.**

- **R symmetry plays important roles in many supersymmetric constrctions. R symmetry is a necessary condition in order to realize supersymmetry breaking. However an exact R symmetry forbids gauginos and Higgsinos to have mass. Hence it must be broken.**

- **We assume**

$$W = \frac{1}{2}\mu S^2 - \frac{1}{4}\lambda \frac{S^4}{M_*},$$

where the inflaton superfield S has an R charge unity. The tree level superpotential does not lead to a slowly rolling scalar potential.

- **We propose a deformation of the superpotential by including an explicit R symmetry breaking term. T his term is suppressed by the cut-off scale M_*. A natural choice $M_* \sim M_P$.**

- **In this case, we get**

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\mathbf{K.E}} - V_F = \frac{c/M_P}{\left(\frac{c}{M_P} - \frac{|S|^2}{3M_P^2}\right)^2}(\partial_\mu S^*)(\partial^\mu S) - \frac{1}{\left(\frac{c}{M_P} - \frac{|S|^2}{3M_P^2}\right)^2}\left|\mu S - \lambda \frac{S^3}{M_*}\right|^2.$$

- **We redefine S field in terms of χ to have canonically normalized kinetic term:**

$$S = \sqrt{3cM_P}\tanh\left(\frac{\chi}{M_P\sqrt{3}}\right).$$

- **Considering** $\chi = (\chi_1 + i\chi_2)/\sqrt{2}$, $\mathcal{L}_{\rm K.E.}$ **becomes** $\mathcal{L}_{\rm K.E.} = \sec^2\left(\frac{2\chi_2}{\sqrt{3}M_P}\right)(\partial_\mu \chi^*)(\partial^\mu \chi)$,
- **The F-term scalar potential responsible for inflation will have the form**

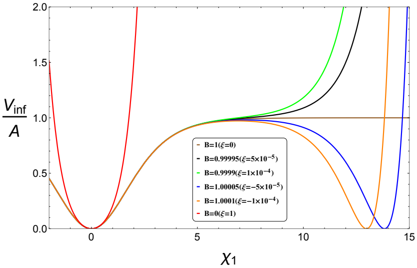
$$V_F = 3M_P^4\left(\frac{\mu^2}{M_P^2}\right)\left(\frac{M_P}{c}\right)\left[1 - \left|\tanh\left(\frac{\chi_1 + i\chi_2}{\sqrt{6}M_P}\right)\right|^2\right]^{-2}\left|\tanh\left(\frac{\chi_1 + i\chi_2}{\sqrt{6}M_P}\right) - 3\frac{c\lambda M_P}{\mu M_*}\tanh^3\left(\frac{\chi_1 + i\chi_2}{\sqrt{6}M_P}\right)\right|^2.$$

- **Imaginary part** χ_2 **will be stabilized at zero during the inflation. Hence**

$$V_{\bf Inf} = A\cosh^4\left(\frac{\chi_1}{\sqrt{6}}\right)\tanh^2\left(\frac{\chi_1}{\sqrt{6}}\right)\left[1 - B\tanh^2\left(\frac{\chi_1}{\sqrt{6}}\right)\right]^2 \simeq A\tanh\left(\frac{\chi_1}{\sqrt{6}}\right)^2\left\{1 + 2\xi\sinh\left(\frac{\chi_1}{\sqrt{6}}\right)^2\right\}.$$

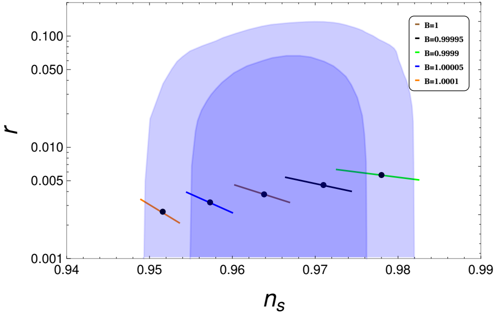
$A = 3\frac{\mu^2}{cM_P}$ **and** $B = 1 - \xi = \lambda\frac{3cM_P}{\mu M_*}$.

- **For** $B = 1$, $V_{\bf Inf}^{(B=1)} = A\tanh^2\left(\frac{\chi_1}{\sqrt{6}}\right)$, **not exactly Starobinsky potential,** $\sim \left(1 - e^{-2x/\sqrt{6}}\right)^4$.



- **To show the importance of the** R **symmetry breaking term** λ, **we include the case of** $\lambda = 0$ **(***i.e* $B = 0$**)**

- **The parameter A will be fixed by observed value of scalar perturbation spectrum $P_s = 2.2 \times 10^{-9}$.**
- **We show the Logarithmic plot of the spectral index n_s versus the tensor to scalar ratio r, as predicted by our model.**



Here, a single colored line segment represents the variation of the number of e-foldings N_e from 50 to 60, where the prediction for $N_e = 55$ is denoted by a black dot over the respective line.

Conclusions

- **we proposed a global R symmetry motivated inflation model within no-scale SUGRA. We found that the minimal $U(1)_R$ symmetric superpotential (quadratic in inflaton superfield) is unable to provide a successful inflation as the associated scalar potential turns out to be extremely steep.**
- **We the introduced an explicit R symmetry breaking term in the superpotential at a non-renormalizable level which provides the required flatness for inflation.**
- **The effective inflation potential resulted from our proposed set-up carries similarity with Staborinsky like inflation models in the limit, one combination of parameters of the superpotential and no-scale Kahler potential as $B = 1$. Varying B from unity by tiny amount leads to the predictions for the spectral index and tensor-to-scalar ratio.**
- **Such a construction involving explicit R symmetry breaking term may also have some interesting consequences while supersymmetry breaking will also be involved. Since any dynamical supersymmetry breaking model requires that R symmetry should spontaneously be broken leading to the presence of R axion, such an explicit breaking term, connected with inflation, in our set-up can be helpful in providing the mass of it.**