Holography, relaxions and the self-tuning of the cosmological constant

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Fine tuning problems

- Cosmological constant
  natural value is $(\text{SUSY breaking})^4 \gg (\text{meV})^4$

- Higgs mass
  natural value is $(\text{SUSY breaking})^2 \gg (125\text{GeV})^2$

Our project: Try to solve these problems in the context of the brane world inspired by holography.
Fine tuning problems

- Cosmological constant  \( (\text{Holographic) self tuning} \)
  natural value is \( (\text{SUSY breaking})^4 \gg (\text{meV})^4 \)

- Higgs mass \( \text{relaxion} \)
  natural value is \( (\text{SUSY breaking})^2 \gg (125\text{GeV})^2 \)

Our project: Try to solve these problems in the context of the \textit{brane world} inspired by \textit{holography}. 
Self-tuning of $cc$

- In the brane world setup, the solution of eom with flat brane exists for generic parameter.
  
  [Arkani Hamed-Dimopoulos-Kaloper-Sundrum ’00]
  [Kachru-Schulz-Silverstein ’00]
  [Csaki-Erlich-Grojean ’00]

- Problems: For generic choice of integration constant of eom, the (bad) bulk singularity exists, whose interpretation is not clear.
Holographic Self Tuning

- If fundamental theory is 4d QFT, dual to brane world setup, we should choose integration constant in such a way that there is no singularity, or there is “good” singularity. [Gubser, ’00]

- Good Singularity may be resolved in string theory. e.g. by adding KK states.

[Charmousis-Kiritsis-Nitti, ’17]
Holographic Self Tuning

- 5d Einstein-Dilaton theory with brane.

\[
S_{\text{bulk}} = M^{d-1} \int d^{d+1}x \sqrt{-g} \left[ R - \frac{1}{2} g^{ab} \partial_a \phi \partial_b \phi - V(\phi) \right] + S_{\text{GH}}
\]

\[
S_{\text{brane}} = M^{d-1} \int d^d x \sqrt{-\gamma} \left[ -W_B(\phi) - \frac{1}{2} Z_B(\phi) \gamma^{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi + U_B(\phi) R(\gamma) \right] + \ldots
\]

Ansatz: flat metric on the brane.

\[
ds^2 = du^2 + e^{2A(u)} \eta_{\mu \nu} dx^\mu dx^\nu, \quad \varphi = \varphi(u)
\]

- For generic choice of the potential, flat brane solution exists.  
  [Charmousis-Kiritsis-Nitti, '17]
Setup

IR: AdS or good singularity
Relaxion

- The smallness of Higgs mass is explained by
  - Many vacua, one of which realized the small negative Higgs mass.
  - Mechanism to relax into desired vacuum through cosmological evolution.
The relaxion goes to the vacuum where Higgs mass is small.
Our setup

- We use the idea of relaxion in the holographic self-tuning setup.
- We work in Einstein-Axion-Dilaton theory with brane where SM lives.

Bulk

\[ S = M_p^{d-1} \int d^{d+1}x \sqrt{-g} \left[ R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - \frac{1}{2} Y(\varphi) g^{ab} \partial_a a \partial_b a - V(\varphi) \right] + S_{GHY} \]

Brane

\[ S_{brane} = M_p^{d-1} \int d^d x \sqrt{-\gamma} \left[ - W_B(\varphi, a) - X_H(\varphi, a) |H|^2 - S_H(\varphi, a) |H|^4 + \cdots \right] \]
Setup

UV AdS

Brane

IR: good singularity

[YH-Kiritsis-Nitti-Witkowski, work in progress]
Axion & $\theta$ angle

- $a_{UV}$: source of axion (field value in the UV).
- $\theta_{UV}$: $\theta$ angle in dual QFT.

$$\theta_{UV} + 2\pi k = N_c a_{UV}, \quad k \in \mathbb{Z}, \quad 0 \leq \theta_{UV} < 2\pi.$$  

- For large $N_c$,
  
  One $\theta_{UV}$  \quad many $a_{UV}$ \quad $k = 0, \pm 1, \pm 2, \ldots$

$$a_{UV} = \frac{\theta_{UV} + 2\pi k}{N_c}$$
Axion & $\theta$ angle

- There are many saddle point (vacua) parametrized by integer $k$.

$$a_{UV} = \frac{\theta_{UV} + 2\pi k}{N_c} \quad k = 0, \pm 1, \pm 2, \cdots$$

- The small Higgs mass would be realized in one of the vacua.
Numerics

- We **numerically** check the existence of vacuum where Higgs mass is small.

- Solve the equation of motion w/ ansatz
  \[ ds^2 = du^2 + e^{2A(u)} \eta_{\mu\nu} dx^\mu dx^\nu, \quad \varphi = \varphi(u), \quad a = a(u) \]

- Israel **junction condition** at brane position.

- Regularity condition at IR.

- Only free integration constant is axion source
  (axion field value at UV)
Functions

- Bulk&Brane functions are chosen as

\[ S = M_p^{d-1} \int d^{d+1}x \sqrt{-g} \left[ R - \frac{1}{2} g^{ab} \partial_a \varphi \partial_b \varphi - \frac{1}{2} Y(\varphi) g^{ab} \partial_a a \partial_b a - V(\varphi) \right] + S_{GHY} \]

\[ S_{brane} = M_p^{d-1} \int d^d x \sqrt{-\gamma} \left[ - W_B(\varphi, a) - X_H(\varphi, a) |H|^2 - S_H(\varphi, a) |H|^4 + \cdots \right] \]

\[ W_B = \Lambda^4 (-1 - \varphi + e^{\varphi/s}), \quad X_H = \Lambda^2 - \Lambda_a^2 a, \quad S_H = 1 \]

\[ V = -\frac{1}{\ell^2} \left[ d(d-1) + \left( \frac{1}{2} (d - \Delta_-) \Delta_- - b^2 V_\infty \right) \varphi^2 + 4V_\infty \sinh^2 \left( \frac{b\varphi}{2} \right) \right], \quad Y = Y_\infty e^{\gamma\varphi} \]

\[ \Delta_- = 1.2, \quad d = 4, \quad b = 1.3, \quad \gamma = 1.5, \quad V_\infty = 1, \quad Y_\infty = 1, \quad s = 5, \quad \Lambda = 1, \quad \Lambda_a = 1.7 \]
NUMERICS

\[ a_{UV} = \frac{\theta_{UV} + 2\pi k}{N_c} \]

\((\text{Higgs mass})^2\)

\[
\chi_H^2
\]

Axion source
If we take $\theta_{UV}=0.1$, $N_c=100$.
If we take $\theta_{UV} = 0.1$, $N_c = 100$

$$a_{UV} = \frac{\theta_{UV} + 2\pi k}{N_c}$$
Numerics

If we take $\theta_{UV}=0.1$, $N_c=100$

For large $N_c$, small Higgs mass is realized in one vacuum.
Possibilities

- If small Higgs mass saddle is lowest free energy, it is stable vacuum.

- If not, small Higgs mass is realized in metastable vacuum.
  - Cosmological evolution like relaxion case
  - Anthropic principle
Summary

- We investigate fine tuning problems in the brane world setup motivated by holography.
- The solution where the brane is flat exists.
- For fixed $\theta$ angle in QFT, there are many saddle points in bulk, one of which realizes the negative small Higgs mass.