Geometrical destabilisation of light fields in string inflation

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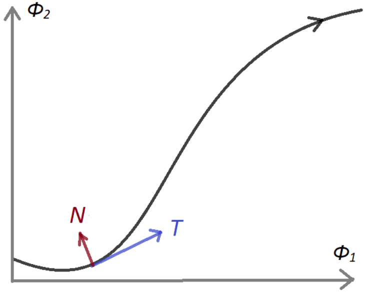
[1807.03818], [1903.01497] in collaboration with: M. Cicoli, F. G. Pedro, G. P. Vacca

General setup

Multifield inflation in curved field space:

$$\mathcal{L} = \frac{1}{2} G_{ij}(\phi^i) \partial_\mu \phi^i \partial^\mu \phi^j - V(\phi)$$

Split the background trajectory in tangent and normal directions



$$T^i = \frac{\dot\phi^i}{\dot\phi_0}$$

$$(N_a)_i T^i = 0 \qquad a = 1 \dots d-1$$

$$(N_a)_i (N_a)^i = T_i T^i = 1$$

$$\dot\phi_0 = \sqrt{G_{ij} \dot\phi^i \dot\phi^j}$$

$$\ddot\phi^i + 3H\dot\phi^i + \Gamma^i_{jk} \dot\phi^j \dot\phi^k + G^{ij} V_j = 0$$

What do we look at?

Superhorizon behaviour of isocurvature modes *[Sasaki et al. '96] [Di Marco et al. '03] [Achucarro et al. '11] [Petel et al. '15]*

$$\delta Q^a = \delta\phi^a + \psi\frac{\dot{\phi}^a}{H} \qquad \rightarrow \qquad \mathcal{R} = \frac{H}{\dot{\phi}_0}Q^T \qquad \mathcal{S} = \frac{H}{\dot{\phi}_0}Q^N$$

$$m^2_{\mathrm{NN\,eff}} \equiv N^i N^j \left(V_{ij} - \Gamma^k_{ij}V_k\right) + \left(\epsilon\mathbb{R} + 3\eta_\perp^2\right) H^2$$

where

$$\eta_\perp = \frac{N^i V_i}{(H\dot{\phi}_0)}$$

Tachyonic effective mass $\Longrightarrow$ Exponential growth

Negative contributions can come from:

- **Negative field space curvature**
- **Metric connection**

Simplified setup

Let us consider

$$G_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & f^2(\phi_1) \end{pmatrix} \qquad V = V(\phi_1) + V(\phi_2)$$

Conjugate momenta

$$\pi_1 = a^3 \dot{\phi}_1 \qquad \pi_2 = a^3 f^2 \dot{\phi}_2$$

EOMs

$$\dot{\pi}_1 = a^3 \left(f f_1 \dot{\phi}_2^2 - V_1 \right) \qquad \dot{\pi}_2 = -a^3 \, V_2$$

Curvature

$$\mathbb{R} = -2 \frac{f_{11}}{f}$$

Nearly massless normal mode and canonical inflaton: (ϕ_1 inflaton)

- Model dependent sign of $\eta_\perp$ and m^2_{eff}
- General in string inflation and SUGRA

Building inflation in String Theory:

- Complete UV theory
- Suitable for inflation: symmetries
- After compactification plethora of massless fields called moduli $\rightarrow$ moduli stabilisation
- Many light spectator fields (perturbative axion shift symmetry)

Concrete model building:

Kähler moduli: $T_i = \tau_i + i\theta_i$

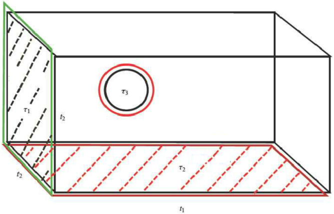
Kähler potential: $K = -2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right) + K_{g_s}$

Superpotential: $W = W_0 + W_{\rm np}$

Scalar potential: $V_F = e^K\left(K^{i\bar{j}} D_i W D_{\bar{j}}\bar{W} - 3|W|^2\right)$

Lagrangian: $\mathcal{L}_{\rm kin} = \frac{\partial^2 K}{\partial T_i \partial \bar{T}_j}\,\partial_\mu T_i \partial^\mu \bar{T}_j = \frac{\gamma_{ij}(\tau)}{2}\left(\partial_\mu \tau_i \partial^\mu \tau_j + \partial_\mu \theta_i \partial^\mu \theta_j\right)$

Fibre Inflation: *[Cicoli, Quevedo, Burgess '09]*



Fibred CY volume: $\mathcal{V} = \alpha\left(\sqrt{\tau_1}\tau_2 - \lambda_3\tau_3^{3/2}\right)$

Superpotential: $W = W_0 + A_3\, e^{-a_3 T_3}$

Large Volume Scenario $\rightarrow$ expansion of V_F in inverse powers of $\mathcal{V}$

$$V_{\rm LVS} = \frac{8a_3^2A_3^2\sqrt{\tau_3}}{3\alpha\lambda_3\mathcal{V}}e^{-2a_3\tau_3} + \frac{4a_3A_3\tau_3W_0\cos(a_3\theta_3)}{\mathcal{V}^2}e^{-a_3\tau_3} + \frac{3\xi W_0^2}{4g_s^{3/2}\mathcal{V}^3}$$

$$V_{\rm inf} \propto \frac{W_0^2}{\mathcal{V}^{10/3}} \qquad \Longrightarrow \qquad V_{\rm LVS} \gg V_{\rm inf}$$

Stabilisation of heavy fields:

$$a_3\langle\theta_3\rangle = \pi\,, \qquad a_3\langle\tau_3\rangle \sim \frac{1}{g_s}\,, \qquad \langle\mathcal{V}\rangle \sim W_0\,\sqrt{\langle\tau_3\rangle}\,e^{a_3\langle\tau_3\rangle}$$

$$m_{\theta_3}^2 \simeq m_{\tau_3}^2 \simeq \frac{W_0^2}{\mathcal{V}^2} \gg m_{\tau_2}^2 \simeq \frac{W_0^2}{\mathcal{V}^3} \gg H^2 \simeq \frac{W_0^2}{\mathcal{V}^{10/3}}$$

Light fields: $\underbrace{\tau_1}_{\text{inflaton}} \quad \underbrace{\theta_1\,,\ \theta_2}_{\text{massless axions}}$

Two field subspaces - Heavy fields

Main feature:

- Constant curvature $\mathbb{R}$
- Canonically normalised inflaton ϕ_1
- Field space metric

$$\gamma_{ij}(\tau)(\partial_\mu \tau_i \partial^\mu \tau_j + \partial_\mu \theta_i \partial^\mu \theta_j) \underbrace{\rightarrow}_{\text{local diag}} \partial_\mu \phi_i \partial^\mu \phi_j + \gamma_{ij}(\phi) \partial_\mu \theta_i \partial^\mu \theta_j$$

Heavy fields:

$$\begin{cases} \gamma_{ij}(\phi) \sim \text{diag} \\ \mathbb{R} = \text{const} \\ V_{\text{LVS}} \gg V_{\text{inf}} \end{cases} \implies$$

NO Geomerical Destabilisation!

Two field subspaces - Light fields

Massless axions:

$$\begin{cases} \gamma_{ij}(\phi) \sim \begin{pmatrix} 1 & 0 \\ 0 & f^2(\phi_1) \end{pmatrix} \\ \mathbb{R} = -2\frac{f_{11}}{f} \leq 0 \quad \text{const} \end{cases} \implies \begin{array}{l} f(\phi_1) \in \{A_+ e^{\lambda\phi_1}\,;\, A_- e^{-\lambda\phi_1}\} \\ \lambda = \sqrt{|\mathbb{R}|/2} \end{array}$$

$$m^2_{\perp,\,\mathrm{eff}} = -\lambda^2 \dot{\phi}_0^2 \pm \lambda \alpha_1^2 V_1 + \alpha_2^2 \left(\frac{3V_1^2}{\dot{\phi}_0^2} + V_{11} \right) \geq 0$$

Equation of motion and effective mass

$$\left(f\theta_i'\right)(N) = \left(f\theta_i'\right)(0)\, e^{-g(N)} \qquad \text{where} \qquad g(N) \simeq 3N \pm \lambda \left(\hat{\phi}_1(N) - \hat{\phi}_1(0) \right)$$

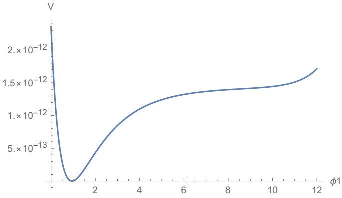
If the inflationary trajectory is not able to overcome Hubble friction $\alpha_2 \sim 0$

$$m^2_{\perp,\,\mathrm{eff}} \simeq \lambda \left(\pm V_1 - \lambda \dot{\phi}_0^2 \right) > 0 \qquad \frac{\lambda \dot{\phi}_0^2}{|V_1|} \sim \frac{\lambda\sqrt{2\epsilon}}{3}$$

Axions with different exponential couplings $\implies$ **Potential geometrical destabilisation!**

Results for massless axions

	θ_1	θ_2
γ_{ij}	$\begin{pmatrix} 1 & 0 \\ 0 & A_-^2\, e^{-\frac{4}{\sqrt{3}}\phi_1} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & A_+^2\, e^{+\frac{2}{\sqrt{3}}\phi_1} \end{pmatrix}$
$\mathbb{R}$	$-8/3$	$-2/3$



Inflationary potential from g_s-loop corrections:

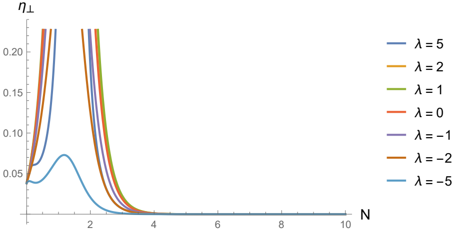
$$V = V_0 \left(3 - 4\, e^{-\frac{\hat\phi_1}{\sqrt{3}}} + e^{-\frac{4\hat\phi_1}{\sqrt{3}}}\right) \qquad \text{where} \qquad V_0 \sim \frac{W_0^2}{\mathcal{V}^{10/3}}$$

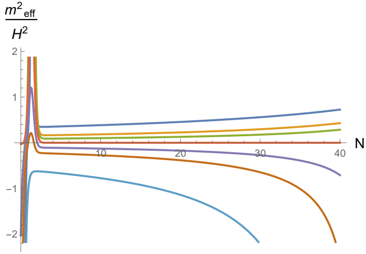
$$\hat\phi_1(N) - \hat\phi_1(0) \simeq \frac{1}{k_2} \ln\left(1 - \frac{4N}{9}\, e^{-k_2 \hat\phi_1(0)}\right) \qquad k_2 = 1/\sqrt{3}$$

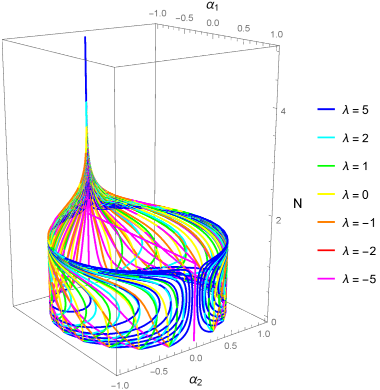
$$g(N) = 3N \pm \frac{\lambda}{k_2} \ln\left(1 - \frac{4N}{9}\, e^{-k_2 \hat\phi_1(0)}\right) > 0 \qquad \forall\, \lambda_i\,; \quad i = 1, 2 \qquad \alpha_2 \to 0!$$

Since $V_1 > 0$ we have $m^2_{\theta_1,\,\mathrm{eff}} < 0$, $m^2_{\theta_2,\,\mathrm{eff}} > 0$ $\implies$ **GD of θ_1!**

Numerics:







$$\alpha_1 = \frac{\phi_1'}{\phi_0'} \qquad \alpha_2 = \frac{f\phi_2'}{\phi_0'}$$

$$\alpha_1^2 + \alpha_2^2 = 1$$

$$\epsilon_i = \{0, 0.5, 1, 2, 3\}$$

Giving a mass to axions

Axions can receive a mass through

$$W = W_0 + A_3\, e^{-a_3 T_3} + A_i\, e^{-a_i T_i}\,, \qquad i = 1, 2$$

New terms in the potential:

$$V(\theta_i, \hat{\phi}_1) = \Lambda_i^{(0)}\, e^{-g_i(\hat{\phi}_1)} \cos(a_i \theta_i)\,, \qquad \text{with} \quad \Lambda_i^{(0)} = \frac{4 a_i A_i W_0 \langle \tau_i \rangle}{\mathcal{V}^2} \quad \forall i = 1, 2$$

$$g_1(\hat{\phi}_1) = -2 k_2 \hat{\phi}_1 + a_1 \langle \tau_1 \rangle\, e^{2 k_2 \hat{\phi}_1}$$

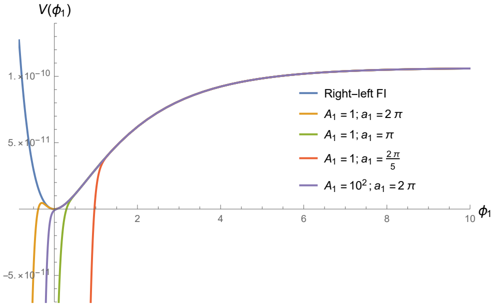
Effective mass:

$$m^2_{\theta_i,\,\mathrm{eff}} = \left(\frac{V_{\theta_i \theta_i}}{f^2} + \frac{f_{\hat{\phi}_1}}{f} V_{\hat{\phi}_1} + 3 \frac{V^2_{\theta_i}}{\dot{\phi}_0^2 f^2} \right) - \dot{\phi}_0^2 \frac{f_{\hat{\phi}_1 \hat{\phi}_1}}{f}$$

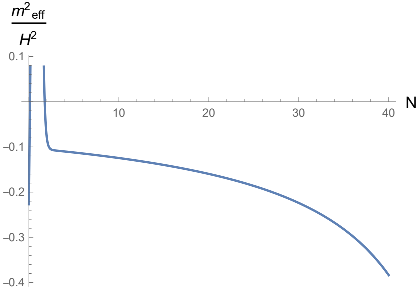
Corrections proportional to:

$$\delta_1 \equiv \frac{V(\theta_i, \hat{\phi}_1)}{V(\hat{\phi}_1)} \propto \exp\left\{ -a_1 \langle \tau_1 \rangle\, e^{2 k_2 \hat{\phi}_1} \right\}$$

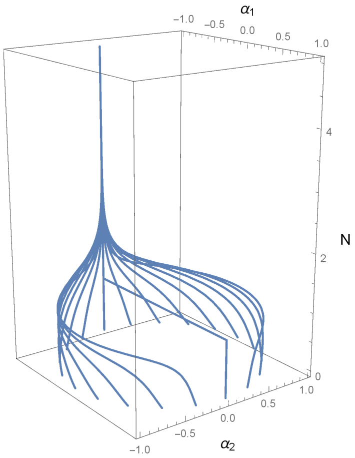
Numerics:



$V(\phi_1)$
$1.\times10^{-10}$
$5.\times10^{-11}$
$-5.\times10^{-11}$
2
4
6
8
10
ϕ_1
Right–left FI
$A_1 = 1; a_1 = 2\pi$
$A_1 = 1; a_1 = \pi$
$A_1 = 1; a_1 = \frac{2\pi}{5}$
$A_1 = 10^2; a_1 = 2\pi$



$\frac{m^2_{\text{eff}}}{H^2}$
0.1
−0.1
−0.2
−0.3
−0.4
10
20
30
40
N



α_1
−1.0
−0.5
0.0
0.5
1.0
N
0
2
4
α_2
−1.0
−0.5
0.0
0.5
1.0

Conclusions:

- Presence of **many light fields** is common in string setup (Axiverse)
- **Massless axions** can lead to geometrical destabilisation of inflation
- Giving a **mass** to axions does not ensure absence of problems

What to do:

- Check the presence of the instability without projecting 1 orthogonal dimension at time
- Beyond perturbation theory: Numerical GR or stochastic inflation
- Perturbation theory valid until the end of inflation? (nearly massless field, no DM)
- End of inflation? Backreaction of perturbations?
- New phenomenological features: non-gaussianities, gravity waves, PBH?

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Merci pour votre attention!

A quintessence like model:

We choose $V = V(\phi_1) = V_0\, e^{-k_2 \phi_1}$ $\qquad f = f_0\, e^{-k_1 \phi_1}$

EOMs
$$\begin{cases} \phi_1'' + (3-\epsilon)\,(\phi_1' - k_2) + k_1 (f\phi_2')^2 = 0 \\ (f\phi_2')' + (3 - \epsilon - k_1 \phi_1')\,(f\phi_2') = 0 \end{cases}$$

Asymptotic solution $\qquad \phi_1'' = (f\phi_2')' = 0$

- $(f\phi_2') \neq 0$ $\implies$
$$\begin{cases} k_2 \geq \sqrt{k_1^2 + 6} - k_1 \\ \frac{m^2_{\perp,\,\mathrm{eff}}}{H^2} = \frac{6k_1(k_2^2 + 2k_1k_2 - 6)}{2k_1 + k_2} \geq 0 \\ \eta_\perp^2 = \frac{k_1}{2k_1 + k_2} \frac{m^2_{\perp,\,\mathrm{eff}}}{H^2} \neq 0 \end{cases}$$

- $(f\phi_2') = 0$ $\implies$
$$\begin{cases} k_2 \leq \sqrt{k_1^2 + 6} - k_1 \\ \frac{m^2_{\perp,\,\mathrm{eff}}}{H^2} = \frac{k_1 k_2}{2} \underbrace{\left(6 - 2k_1k_2 - k_2^2\right)}_{\geq 0} \\ \eta_\perp^2 = 0 \end{cases}$$

GD if $k_2 < 0$

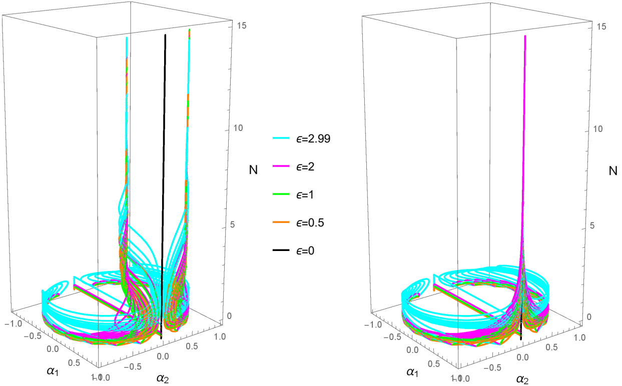
Analytics vs numerics:

Fraction of kinetic energy: $\alpha_1 = \frac{\phi_1'}{\phi_0'}$ $\alpha_2 = \frac{f\phi_2'}{\phi_0'}$ $\rightarrow$ $\alpha_1^2 + \alpha_2^2 = 1$

Initial conditions:

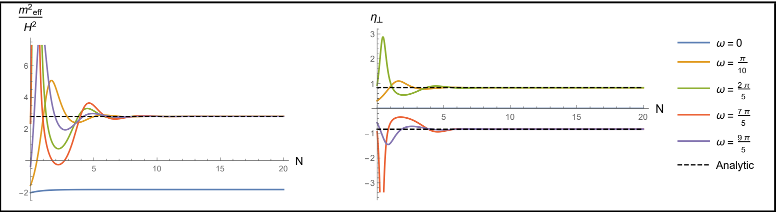
$$\phi_1'(0)|_{ik} = \sqrt{2\epsilon_i(0)}\cos(\omega_k) \qquad \epsilon_i = \{0, 0.5, 1, 2, 3\}$$

$$(f\phi_2)'(0)|_{ik} = \sqrt{2\epsilon_i(0)}\sin(\omega_k) \qquad \omega_k = \frac{k\pi}{10} \qquad k = 0, \ldots, 19$$



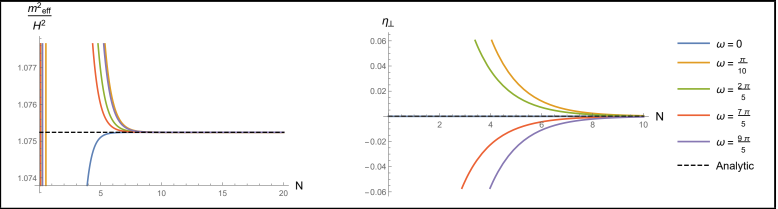
Analytics vs numerics:

$$(f\phi_2') \neq 0$$



$$(f\phi_2') = 0$$

$$k_2 \geq 0$$



$$(f\phi_2') = 0$$

$$k_2 < 0$$