A Tower WGC from IR consistency

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based on SA, D. Junghans, T. Noumi, G. Shiu 1802.04287

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Outline

Introduction and review

- Landscape and Swampland
- WGC and CHC

Infrared Consistency and WGC

- Positivity Bounds (causality & analiticity)
- **A stronger CHC**

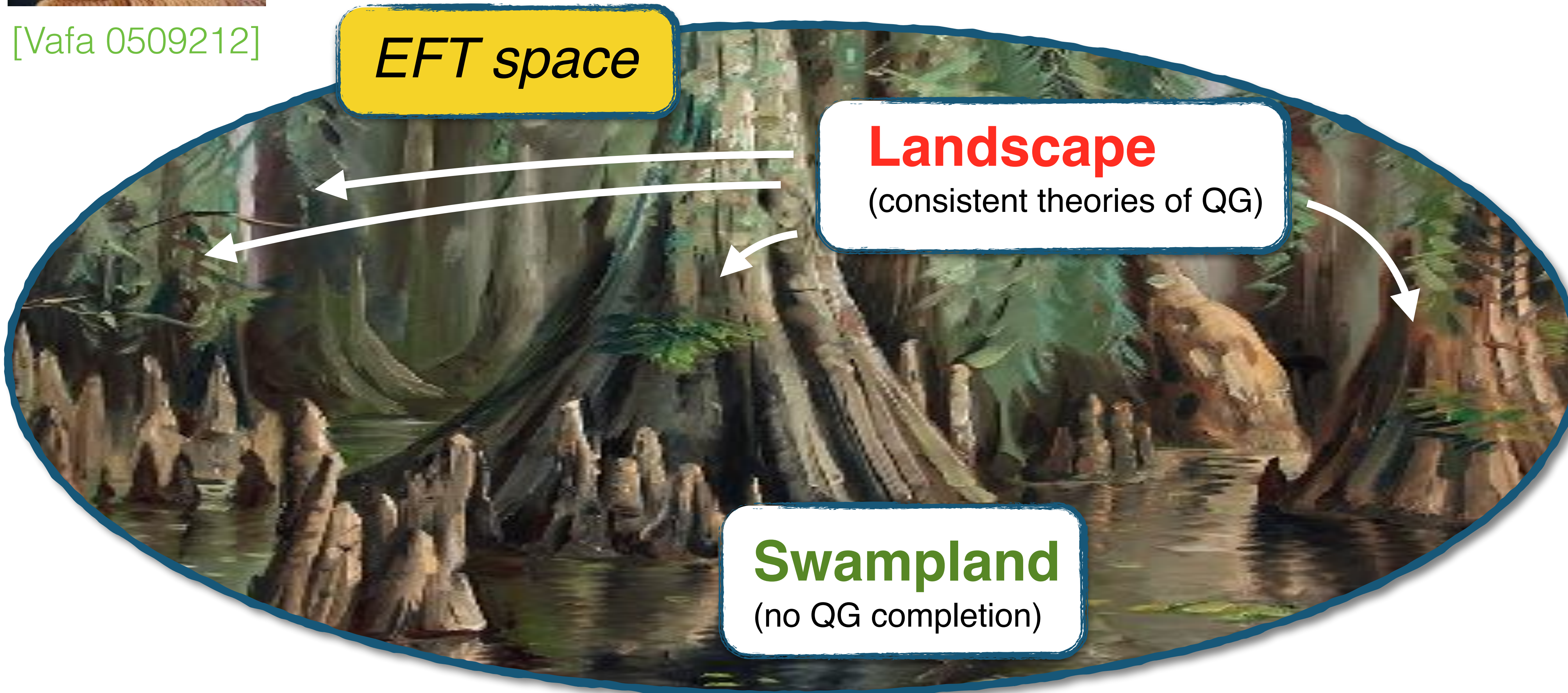
Consistency under dimensional reduction

- (sub-)Lattice WGC
- **Tower WGC**

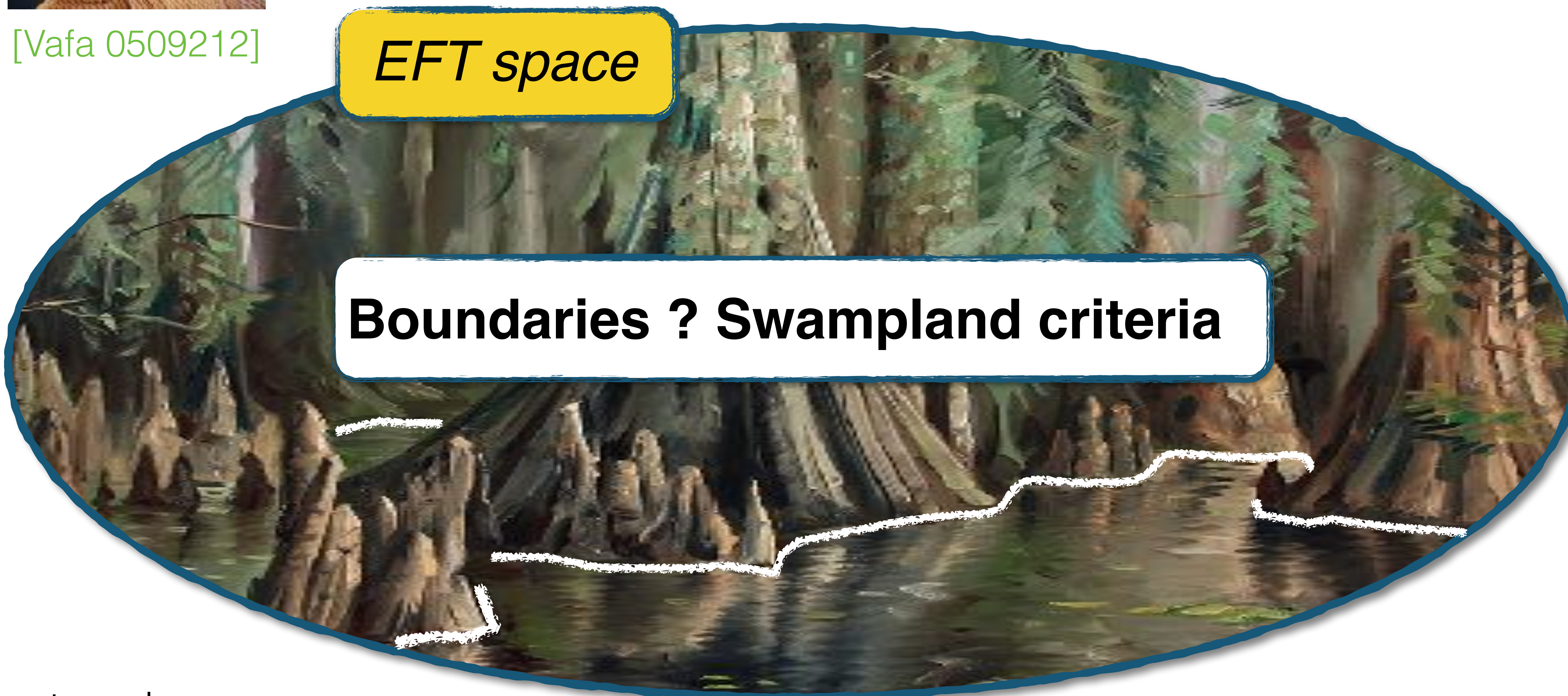
Conclusions

[Vafa 0509212]

SWAMPLAND!



[Vafa 0509212]



Recent work

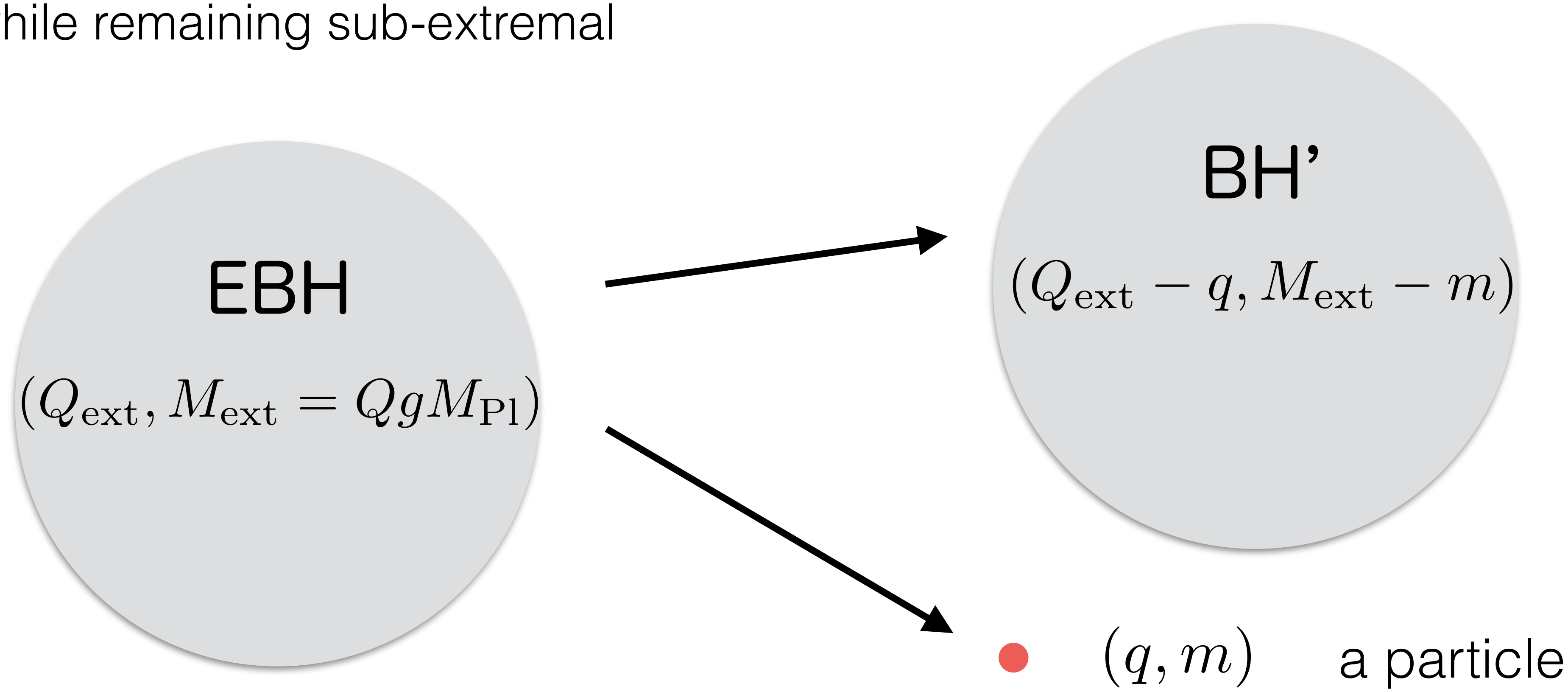
- **Distance conjecture** (see Palti's, Grimm's and Marchesano's talks)
- **dS** (see McAllister's and Grana's talks)
- **new criteria** (see Vafa's and Shiu's talks)
- **WGC** (see also Montero's and Heidenreich's talks)

WGC & CHC

In any consistent EFT of gauge U(1) coupled to gravity
There must exist a particle (m,q) with charge-mass ratio

$$z \equiv \frac{gqM_P}{m} \geq 1$$

To avoid troubles with remnants: EBH's must decay into charged object while remaining sub-extremal



$$g(Q_{\rm ext} - q) \leq (M_{\rm ext} - m)/M_{\rm Pl} \leftrightarrows gq \geq m/M_{\rm Pl}$$

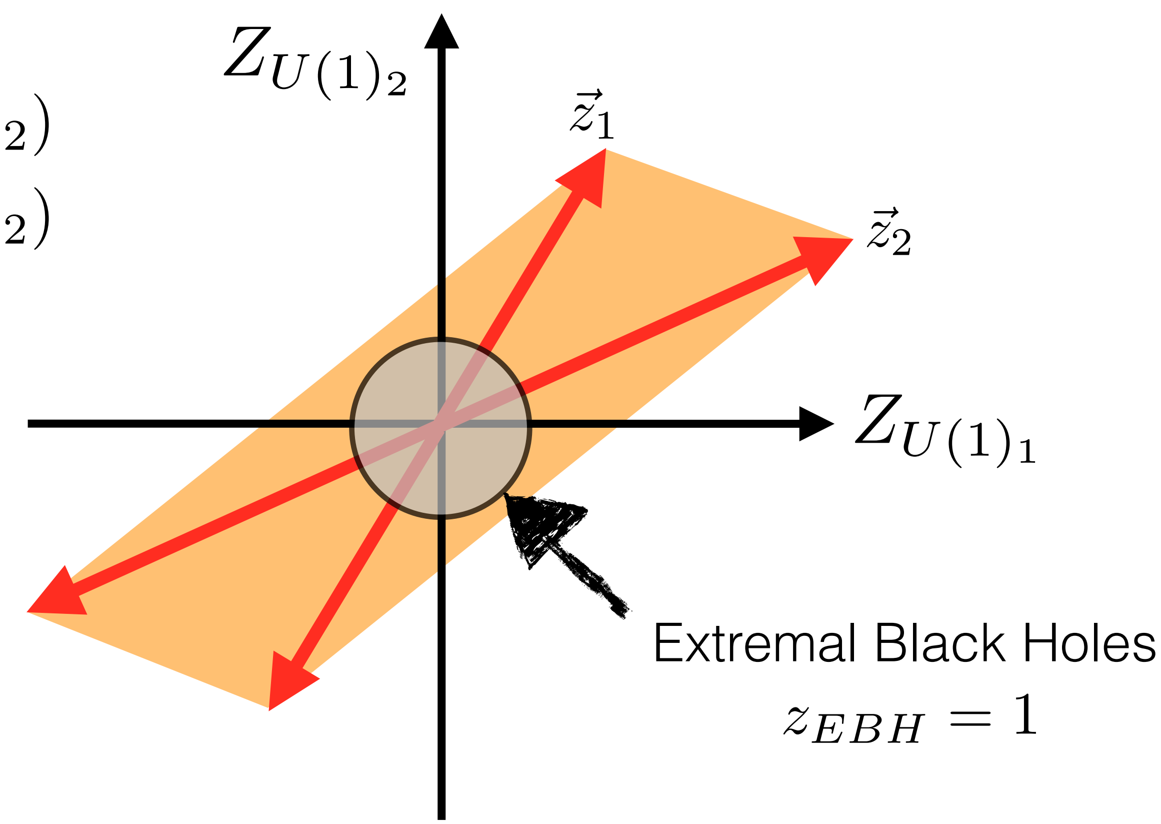
WGC & CHC

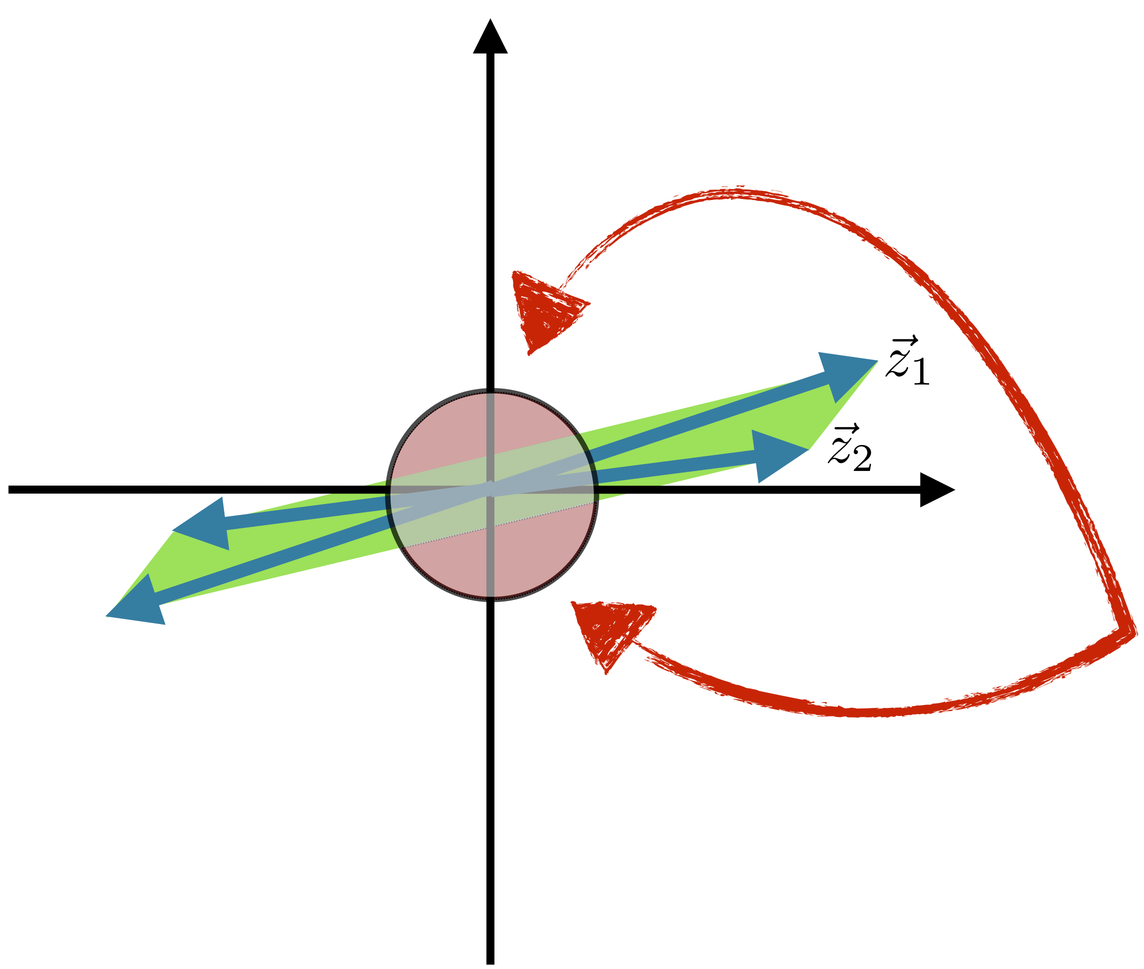
Multiple U(1)’s:

(sub-)Extremal Black holes can decay if they lie inside the Convex Hull!

Ex.: $U(1)^2$, 2 particles

$$\vec{z}_1 = (z_{11}, z_{12})$$
$$\vec{z}_2 = (z_{21}, z_{22})$$





There are no states which can discharge Black Holes in the red regions!

Positivity bounds (IR)

An EFT with HO operators = higher derivatives corrections

Eg. $$\mathcal{L}_{1-loop} = \frac{M_P}{2} R - \frac{F^2}{4e^2} + C F^4$$

is "IR consistent" i.e. respects: **causality** (subluminal fluctuations)

analyticity of S-matrix $\mathcal{M}(s)$

(non-anal in s where particles are produced)

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(non-anal in s where particles are produced)

if

$$C > 0$$

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi 0602178]

Causality (eg 3D)

Dualisation: $$\mathcal{L}_1 = \frac{M_3}{2}R - \frac{(\partial\phi)^2}{2} + 4C(\partial\phi)^4 \qquad F \sim *d\phi$$

Require the speed of propagation of fluctuations $\phi = \bar{\phi} + \varphi$ is sub-luminal (in any locally flat frame η_{ab})

EOMs —Fourier space→ dispersion relation

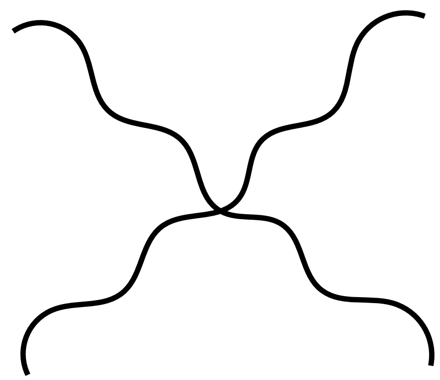
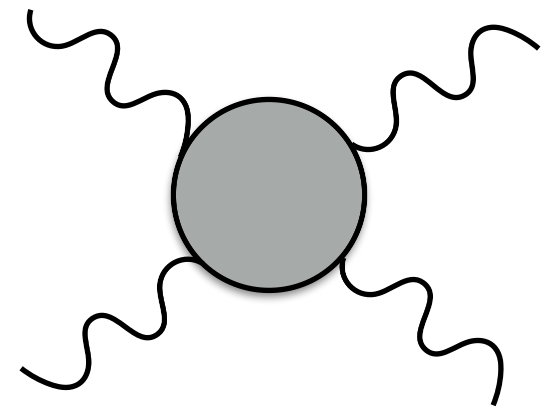
$$(\eta_{ab} + 16C\ \overline{\partial_a\phi\partial_b\phi})\ k^a k^b = 0$$ True for any bg choice

Simplest bg: constant EM field $\overline{\partial_a\phi} = w_a = (w_0, \vec{w}) = const$

$$v = \frac{k_0}{|\vec{k}|} = 1 - 8C(w_0 - \vec{w}\cdot\hat{k})^2$$ → $$\boxed{C > 0}$$

Analyticity - 3D/4D

Consider 4-pts photon scatterings

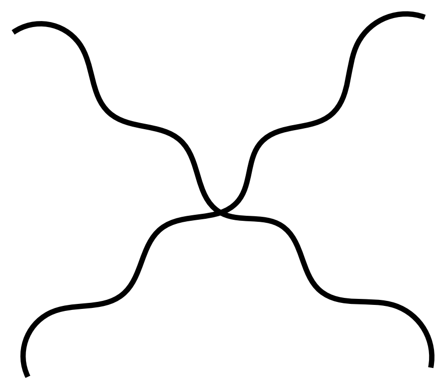
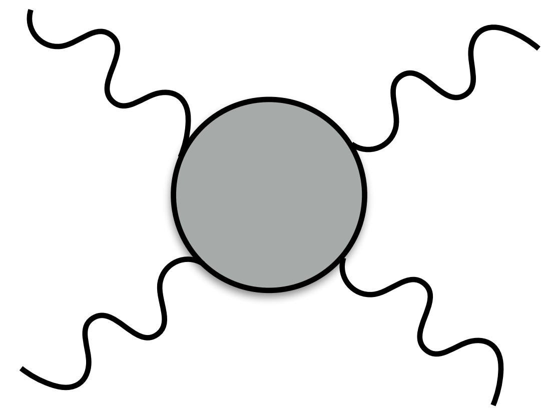
IR  from 

In the forward limit $$\mathcal{M}(s,t) = 8C(s^2+t^2+u^2) \quad \underset{t\to 0}{\longrightarrow} \quad \mathcal{M}(s) = 16Cs^2$$

We can extract C from

Analyticity - 3D/4D

Consider 4-pts photon scatterings

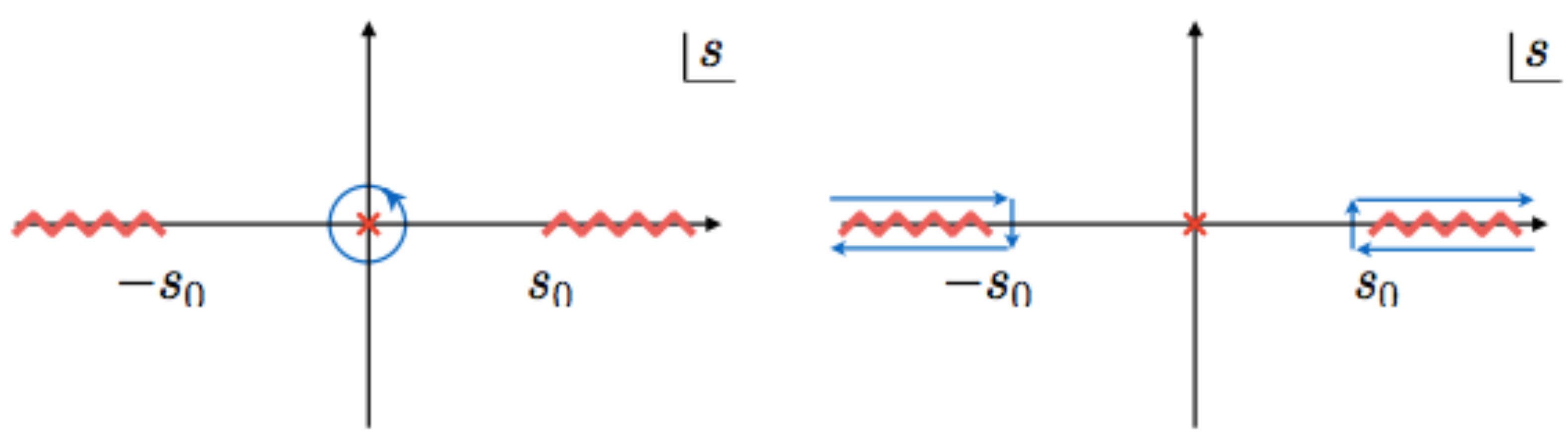
IR  from 

$$16C = \oint_\gamma \frac{ds}{2\pi i}\frac{\mathcal{M}(s)}{s^3} = \left(\int_{-\infty}^{-s_0} + \int_{s_0}^{\infty}\right)\frac{ds}{2\pi i}\frac{\mathrm{Disc}[\mathcal{M}(s)]}{s^3} = \frac{2}{\pi}\int_{s_0}^{\infty} ds\,\frac{\mathrm{Im}\mathcal{M}(s)}{s^3} > 0$$

contour def
+ **analyticity**
+ Froissart bound

analyticity
+ Schwarz reflection
+ crossing symm

Opt Th
$\mathrm{Im}\mathcal{M}(s) = s\sigma(s)$



$$C > 0$$

The S-matrix is **analytical** along the real axis $|s| < s_0$, up to the lowest energy where on-shell intermediate states are created (= red discontinuities)

Positivity bounds $\longleftrightarrow$ WGC ?

$$C > 0 \quad \longleftrightarrow \quad f(z) > 0$$

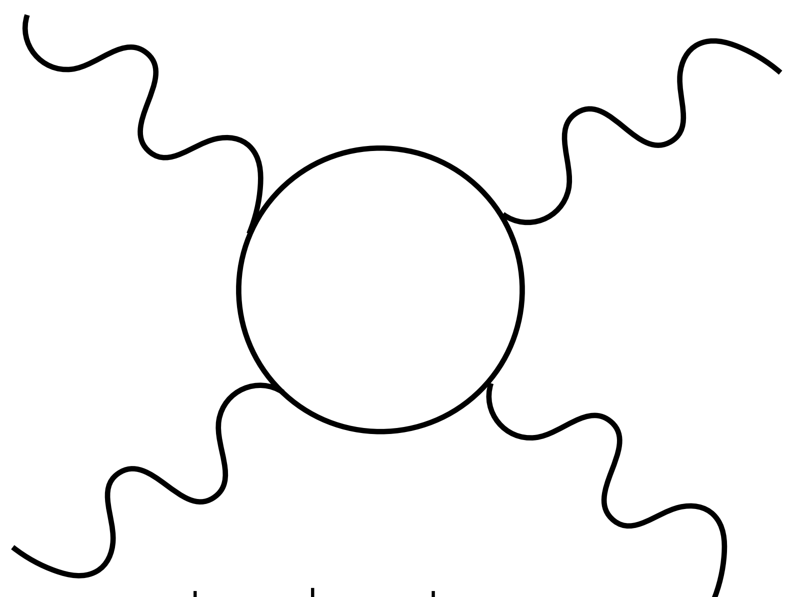
bound for z ?

[Cheung-Remmen '14]

YES! *

*** caveats…**

Suppose the 1-loop EFT is obtained by **integrating out charged matter**



at each vertex = qg

$$\longrightarrow \quad C(z)$$

$$C > 0 \quad \Rightarrow \quad \text{Conditions on } z$$

Setup (3d)

EFT multiple scalar/fermions charged under multiple U(1)'s

$$\Gamma = \int d^3x\sqrt{-g}\left[\frac{M_3}{2}R - \frac{1}{4}\sum_i F_i^2\right] + \Gamma_{s/f} + H.O.$$

UV

$$\Gamma_{\text{s}} = \int d^3x\sqrt{-g}\sum_a\left(-|D_\mu\phi_a|^2 - m_a^2|\phi_a|^2\right)$$

$$\Gamma_{\text{f}} = \int d^3x\sqrt{-g}\sum_a\bar{\psi}_a(-\Gamma^\mu D_\mu - m_a)\psi_a$$

$$H.O. = \sum_{ijkl} c_{ijkl}(F_i\cdot F_j)(F_k\cdot F_l)$$

UV-Physics dof
(kept **generic/unknown**)

Setup (3d)

Λ

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UV-Physics dof
(kept **generic/unknown**)

"Elephant in the room"…

Nonetheless, there is a regime where we can extract interesting results!

Setup (3d)

Integrating out matter, we obtain

$\min(m_a)$

$$\Gamma_1 = \int d^3x \sqrt{-g} \left[\frac{M_3}{2} R - \frac{1}{4} \sum_{i,j} \delta_{ij} F_i \cdot F_j + \sum_{i,j,k,l} C_{ijkl} (F_i \cdot F_j)(F_k \cdot F_l) \right]$$

Setup (3d)

$\min(m_a)$

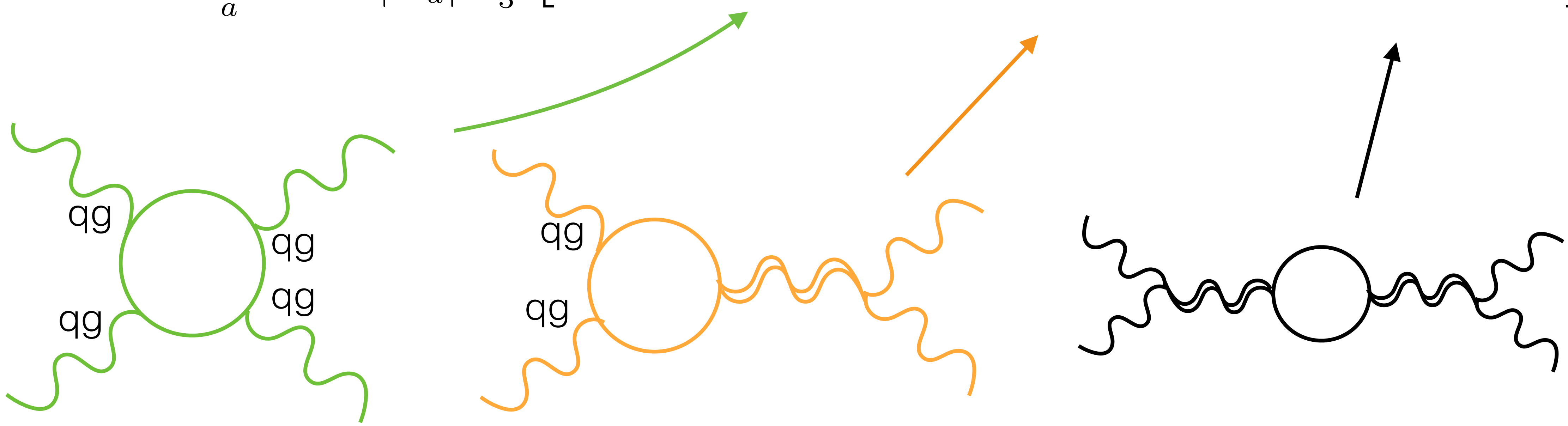
Integrating out matter, we obtain

$$\Gamma_1 = \int d^3x\sqrt{-g}\left[\frac{M_3}{2}R - \frac{1}{4}\sum_{i,j}\delta_{ij}F_i\cdot F_j + \sum_{i,j,k,l}C_{ijkl}(F_i\cdot F_j)(F_k\cdot F_l)\right]$$

$$C^s_{ijkl} = c_{ijkl} + \sum_a \frac{1}{1920\pi|m_a|M_3^2}\left[\frac{7}{8}z_{ai}z_{aj}z_{ak}z_{al} + \frac{3}{2}z_{ai}z_{aj}\delta_{kl} - z_{ai}z_{ak}\delta_{jl} + \frac{1}{2}\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl}\right]$$

UV

$$C^f_{ijkl} = c_{ijkl} + \sum_a \frac{1}{1920\pi|m_a|M_3^2}\left[z_{ai}z_{aj}z_{ak}z_{al} + z_{ai}z_{aj}\delta_{kl} - \frac{3}{2}z_{ai}z_{ak}\delta_{jl} - \frac{1}{2}\delta_{ij}\delta_{kl} + \frac{3}{2}\delta_{ik}\delta_{jl}\right]$$



Implications:

- **single particle**, charged under **single U(1)**

$$C > 0 \begin{cases} \text{scalar:} \quad z^2\left(z^2 + \frac{4}{7}\right) + \mathcal{O}_s(z^0) > 0 \\ \\ \text{fermion:} \quad z^2\left(z^2 - \frac{1}{2}\right) + \mathcal{O}_f(z^0) > 0 \end{cases}$$

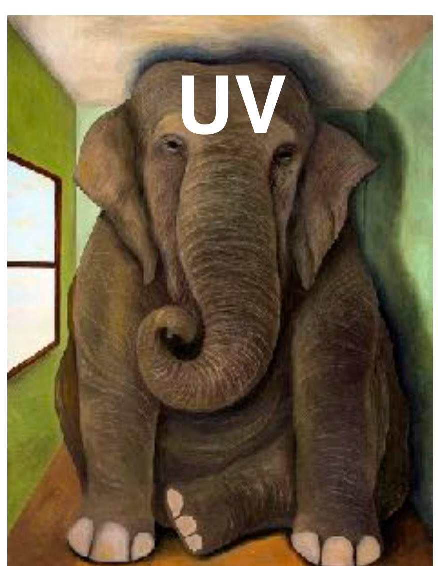
Implications:

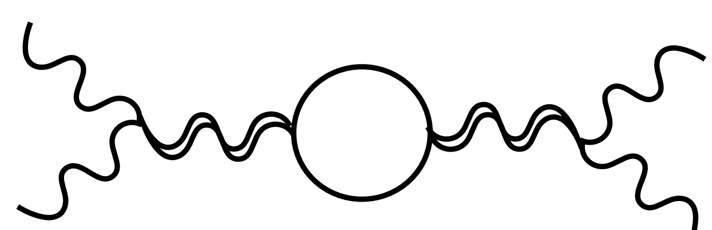
- **single particle**, charged under **single U(1)**

$$C > 0 \begin{cases} \text{scalar:} \quad z^2\left(z^2 + \frac{4}{7}\right) + \mathcal{O}_s(z^0) > 0 \quad \longrightarrow \quad \text{trivial} \\ \text{fermion:} \quad z^2\left(z^2 - \frac{1}{2}\right) + \mathcal{O}_f(z^0) > 0 \quad \longrightarrow \quad z > \frac{1}{\sqrt{2}} \end{cases}$$

WGC!

In the regime where $\mathcal{O}(z^0)$ is negligible

$\mathcal{O}(z^0) \sim$ 

$+$  $\longrightarrow 0$

Implications:

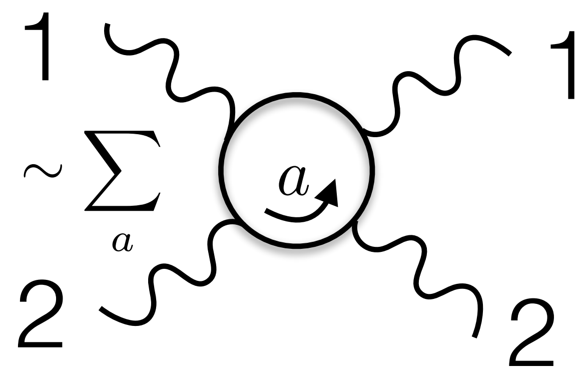
- **more particles**, **multiple U(1)'s**: ➡ Several positivity conditions on

$$C_{ijkl}$$

Strongest positivity conditions are given by **mixed** scatterings

e.g.: **U(1)xU(1)**

$$C_{1212} \sim \sum_a$$



a=matter field ϕ_a, ψ_a

$m_a, \vec{q}_a = (q_{1a}, q_{2a})$

$$C_{(12)(12)} > 0$$

$$\mathcal{O}(z^0) \to 0$$

scalar: $\displaystyle\sum_a \frac{1}{3840\pi|m_a|}\left[z_{1a}^2 z_{2a}^2 - \frac{2}{7}z_{1a}^2 - \frac{2}{7}z_{2a}^2\right] > 0$

fermion: $\displaystyle\sum_a \frac{1}{480\pi|m_a|}\left[z_{1a}^2 z_{2a}^2 - \frac{3}{8}z_{1a}^2 - \frac{3}{8}z_{2a}^2\right] > 0$

Presence of **bifundamentals** is crucial for IR consistency

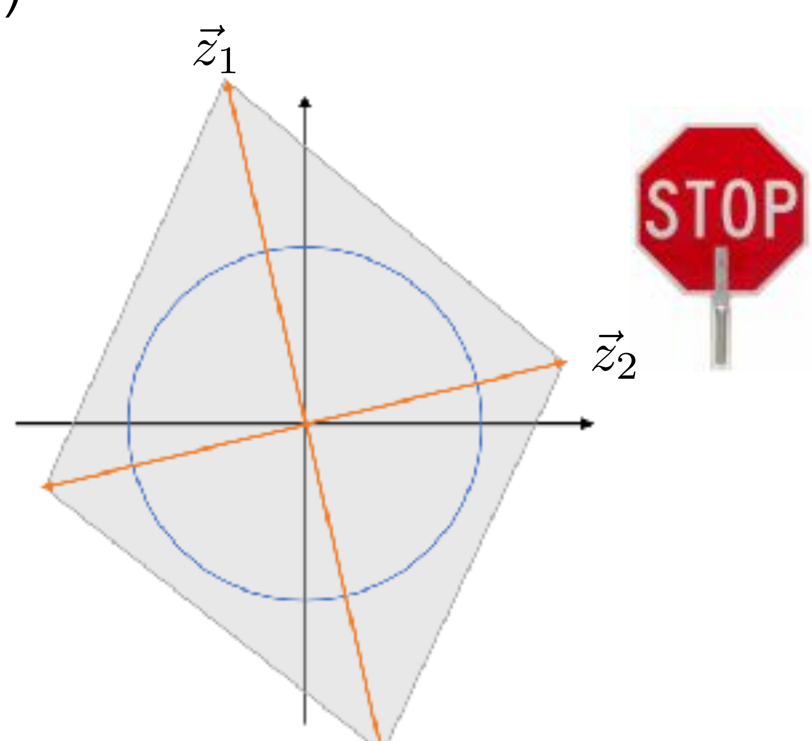
A stronger CHC (3d/4d)

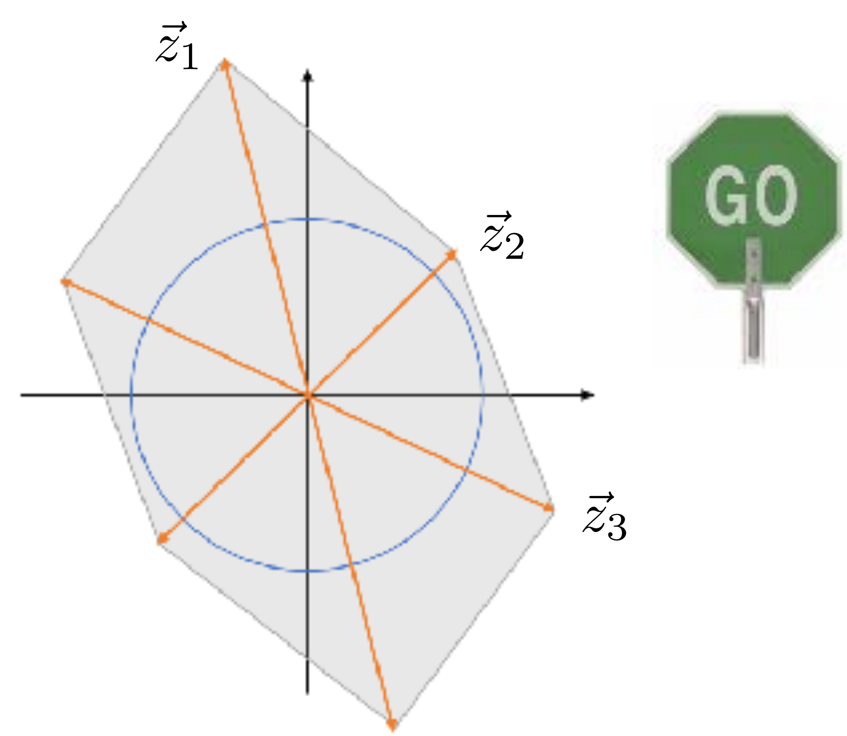
Positivity conditions ⟺

1) CHC is satisfied

2) There must exist (at least) a bifundamental particle for any (orthogonal) basis choice for the U(1)'s gauge fields

E.g., U(1)xU(1)





Dimensional reduction

Is the WGC **consistent** under dimensional reduction ?

[B. Heidenreich, M. Reece, T. Rudelius 1509.06374]

D dim: 1 particle, single U(1) $z_0 \geq 1$ WGC ✔

$S^1_{(r)} \downarrow$

D-1 dim: KK tower, U(1)xU(1)$_{KK}$ WGC (CHC) ? **NOT** always!

(problem $r \to 0$ limit)

Unless…

A super-extremal particle $z > 1$ should exist for every charge in the charge lattice

→ Using **positivity bounds** ?

Same problem in D-1 for $r \to 0$

$$\vec{Z}_{(n)} = (z_F, z_{KK}) = \left(\frac{q}{\sqrt{m^2 + n^2/r^2}}, \frac{n}{\sqrt{m^2 r^2 + n^2}} \right)$$

- lowest mode (n=0): $(z_F, z_{KK}) = (q/m, 0)$
- KK modes (n≠0): $(z_F, z_{KK}) \sim (0, 1)$

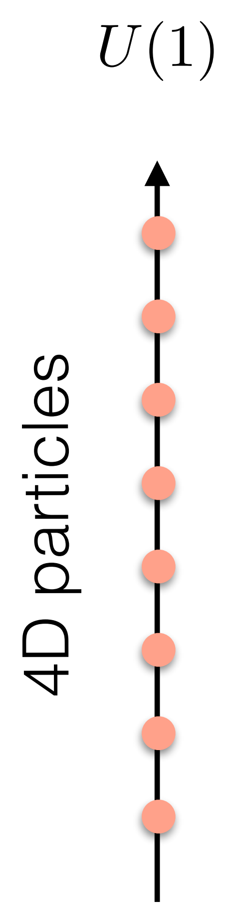
Absence of bifundamentals ➡ **IR inconsistent**

Solution…

Tower-WGC*

(*in absence of fermions)

Replace the 4D field with a **tower** of 4D fields Φ_l charged under U(1) with masses and charges (m_l, q_l) s.t. **bifundamental** contributions (at any r) saves IR cons.



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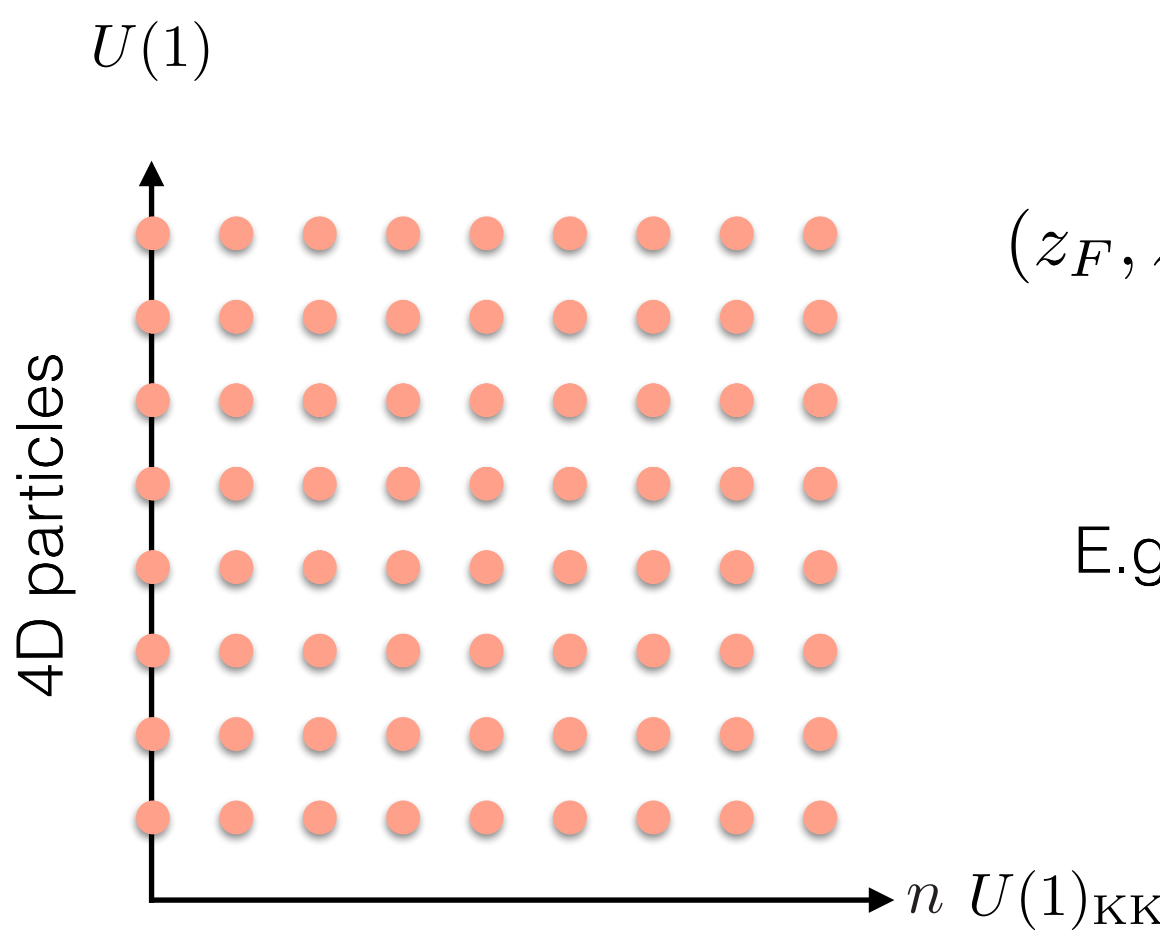


$$(z_F, z_{KK}) = \left(\frac{q_l}{\sqrt{m_l^2 + n^2/r^2}}, \frac{n}{\sqrt{m_l^2 r^2 + n^2}} \right)$$

E.g.: $m_l = \sqrt{m^2 + l^2 \mu^2}$ $\quad q_l = (l+1)q$

(depending on μ)

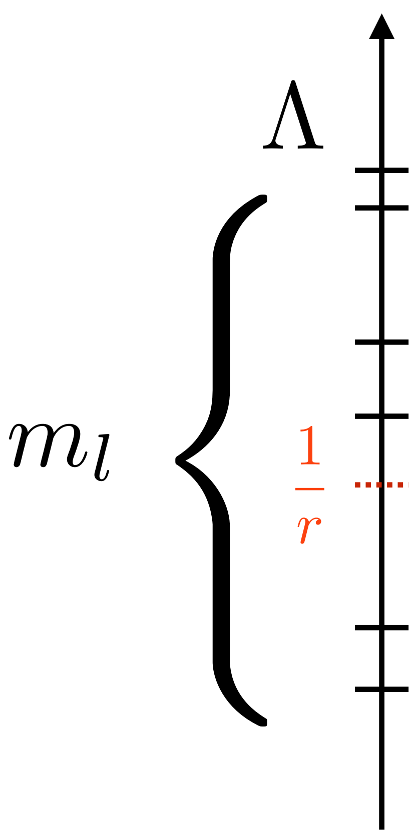
there may be bifundamentals at small radii: IR-OK

Many other possibilities…

Tower-WGC

Replace the 4D field with a **tower** of 4D fields Φ_l charged under U(1) with masses and charges (m_l, q_l) s.t. bifundamental contributions (at any r) saves IR cons.

more precisely:



- There must exist particles with mass near the cut-off $m_l \lesssim \Lambda$
- Such particles must have $z_l \gtrsim \mathcal{O}(1)$
- In case the lightest particle has mass $m \ll r^{-1}, \Lambda$
  then the number of particles in the tower is *at least* of order $(mr)^{-1}$

TWGC is *weaker* than the LWGC

no counterexample
+ agreement with swampland distance conjecture

Conclusions

Relation IR consistency - WGC ?

★ Clear connection when **UV contribution** to the EFT in a certain regime. Possibly, compute UV contributions and check…

★ Existence of **bifundamentals** is crucial for multiple U(1)'s:

stronger CHC

under KK reduction: necessity of a tower of particles in the parent D-dimensional theory **(TWGC)**

TWGC weaker than (sub-)LWGC and agrees with literature


Thank You
Spasibo
Juspaxar
Kiitos
Nirringrazzjak
Multumesc
Merci
Obrigado
Kiitos
Raibh Maith Agat
Chokrane
Mochchakkeram
Obrigado
Grazie
Arigato
Grazie
Asante
Dank Je
Mochchakkeram
Chokrane
Dankon
Spasibo
Maake
Matur Nuwun
Kiitos
Cam on ban
Merci
Raibh Maith Agat
Matondo
Maake
Welatin
Salamat
Kia Ora
Chokrane
Multumesc
Ua Tsaug Rau Koj
Obrigado
Multumesc
Vinaka
Mochchakkeram
Terma Kasih
Ua Tsaug Rau Koj
Obrigado
Raibh Maith Agat
Merci
Mochchakkeram
Mamana
Obrigado
Matondo
Salamat
Nirringrazzjak