Swampland Distance Conjecture for One-Parameter Calabi-Yau Manifolds

A Test

Abhinav Joshi

BCTP, University of Bonn

June 25, 2019

Based on arXiv:1903.00596 with Albrecht Klemm.

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   - Swampland Distance Conjecture
2. The Setup
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Swampland Conjectures

These are statements about the properties of a consistent quantum gravity effective field theory.

A consistent theory is a theory with a (string theoretic) UV completion.

[Vafa, 2005]

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Swampland Distance Conjecture

Consider a point p_0 on the moduli space defining an effective theory, $\mathcal{L}_0$. For another point p, the effective theory $\mathcal{L}$ compared to $\mathcal{L}_0$ develops a tower of exponentially light states with mass of the order

$$e^{-\alpha d(p_0,p)} \ ,$$

where $d(p_0, p)$ is the geodesic distance. As this distance diverges, one finds infinitely many massless states leading to breakdown of the effective theory.

The parameter α is arbitrary and conjectured to be $\mathcal{O}(m_{Pl})$ in the refined version.

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The fundamental object of interest is the period vector Π.

$$\Pi = \begin{pmatrix} F_I(z) \\ X^I(z) \end{pmatrix} = \begin{pmatrix} \int_{B_I} \Omega(z) \\ \int_{A^I} \Omega(z) \end{pmatrix} .$$

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This allows us to find the Kähler potential and the metric on M_{cs} via

$$K(z, \bar{z}) = -\log(i\Pi^\dagger \Sigma \Pi) ,$$
$$G_{i\bar{j}} = \partial_i \bar{\partial}_{\bar{j}} K(z, \bar{z}) ,$$

where

$$\Sigma = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix} .$$

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where $\vec{S} = \left(B_I, A^I\right)^{\mathsf{T}}$ and $\mathbf{q}$ is the RR charge vector.

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The mass of the BPS 3-brane **[Ceresole, D'Auria, Ferrara, Van Proeyen, 1995]**

$$m_{\mathbf{q}} = |Z_{\mathbf{q}}| \ ,$$

where

$$Z_{\mathbf{q}} = e^{K/2} \int_S \Omega = e^{K/2} \mathbf{q}^{\mathsf{T}} \cdot \Sigma \cdot \Pi$$

is called the central charge.

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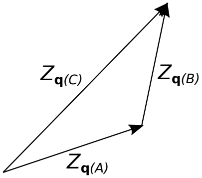
2. Are they stable against decay?
   One uses following argument to test the stability.
   Consider three 3-branes, A, B and C

$$Z_{\mathbf{q}(C)} = Z_{\mathbf{q}(A)} + Z_{\mathbf{q}(B)} \ ,$$
$$m_{\mathbf{q}(C)} \leq m_{\mathbf{q}(A)} + m_{\mathbf{q}(B)} \ .$$



Hence, if $\arg(Z_{\mathbf{q}(A)}) = \arg(Z_{\mathbf{q}(B)})$, C can decay to A and B or vice-versa. Otherwise, C will be stable.

One-Parameter Calabi-Yau Threefolds

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The most interesting case is that of two cubics in $\mathbb{P}^5$, let's call it Y. It's mirror X is given by zero locus of **[Libgober, Teitelbaum, 1993]**

$$P_1 = x_1^3 + x_2^3 + x_3^3 - z^{-\frac{1}{6}} x_4 x_5 x_6 \text{ and}$$
$$P_2 = x_4^3 + x_5^3 + x_6^3 - z^{-\frac{1}{6}} x_1 x_2 x_3 \ ,$$

where z parameterises M_{cs}.

Period Calculation

The period vector can be calculated using the Picard-Fuchs equation

$$\left(\theta^4 - 3^6 z\,(\theta + 1/3)^2\,(\theta + 2/3)^2\right) f_i(z) = 0\ ,$$

where $\theta = z\frac{d}{dz}$.

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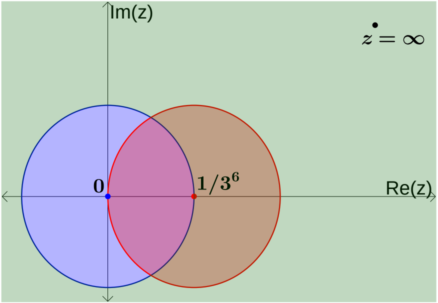
$$\tilde{\Pi} = \begin{pmatrix} f_1(z) \\ f_2(z) \\ f_3(z) \\ f_4(z) \end{pmatrix} \xrightarrow{\text{base change}} \Pi = T\tilde{\Pi} = \begin{pmatrix} F_0(z) \\ F_1(z) \\ X^0(z) \\ X^1(z) \end{pmatrix} \ .$$

Period Calculation

Three singular points

$$z = \left\{0,\ \frac{1}{3^6},\ \infty\right\}.$$



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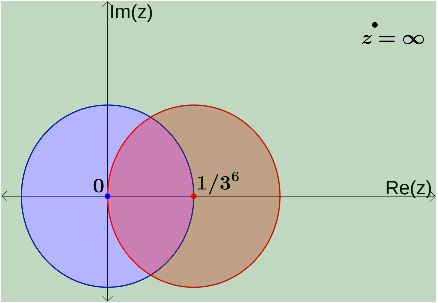

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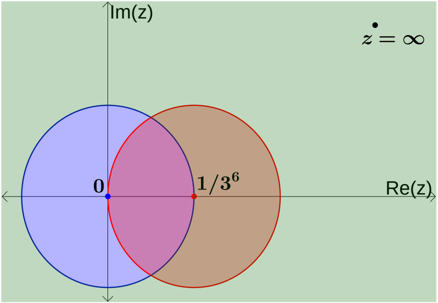

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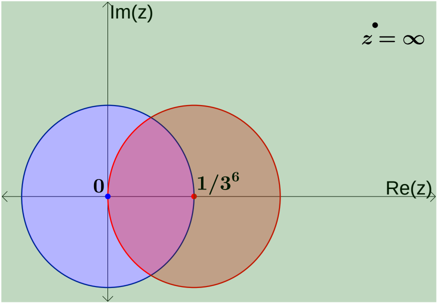

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Using analytic continuation, we match them to get symplectic basis

$$\tilde{\Pi}(z) \to \Pi(z) \ .$$

Results: Metric

Define: $\psi = \frac{1}{3z^{1/6}}$. Singular points: $\overbrace{\left\{0, \frac{1}{3^6}, \infty\right\}}^{z} \to \overbrace{\{\infty, 1, 0\}}^{\psi}$.

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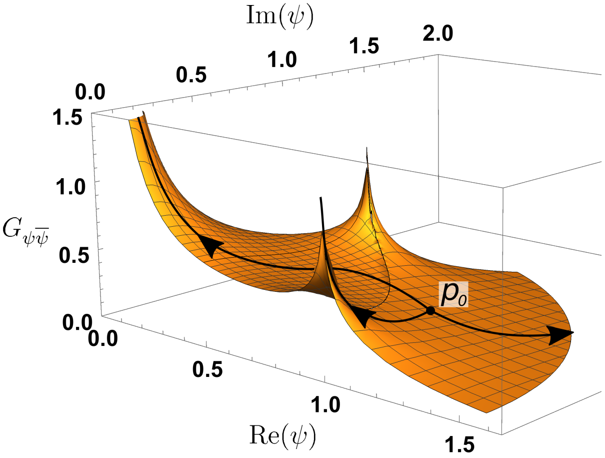

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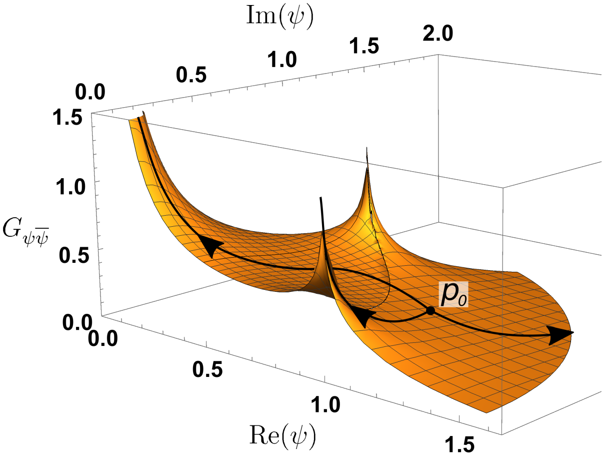
ψ	$G_{\psi\overline{\psi}}$	Distance
0	∞	∞
1	∞	Finite
∞	0	∞



Results: Metric

Two infinite distance points, two infinite tower of states according to Swampland Distance Conjecture.

$$\boxed{\psi \to 0} \quad \text{and} \quad \boxed{\psi \to \infty}$$

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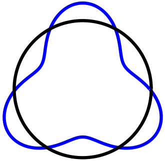

$$m \sim \frac{1}{\lambda} \sim \frac{1}{R}$$

These are called Kaluza-Klein states.

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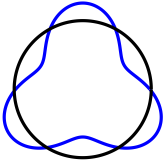

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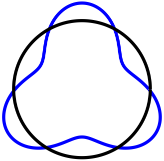

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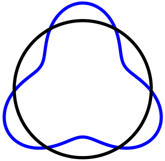

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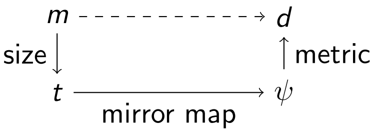

$$\boxed{m \sim m_0 e^{-\alpha d}}, \quad \alpha = \frac{1}{\sqrt{3}}$$

[Blumenhagen, Klaewer, Schlechter, Wolf, 2018]

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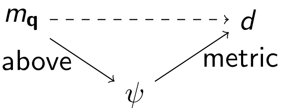

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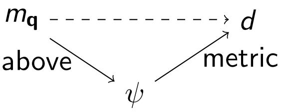

$$\boxed{m \sim m_0 e^{-d}}$$

Results: Tower of States (Stability)

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For stability, consider the set of all possible $\mathbf{q}$ which leads to all possible $Z_{\mathbf{q}}$.

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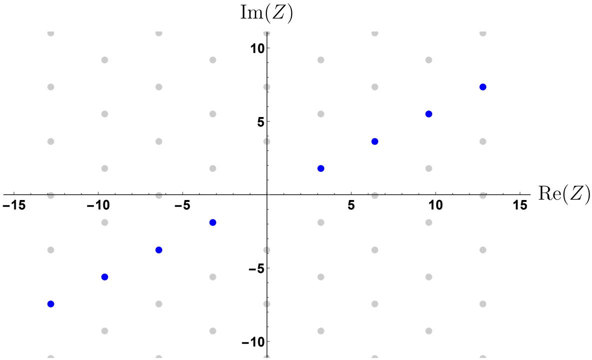

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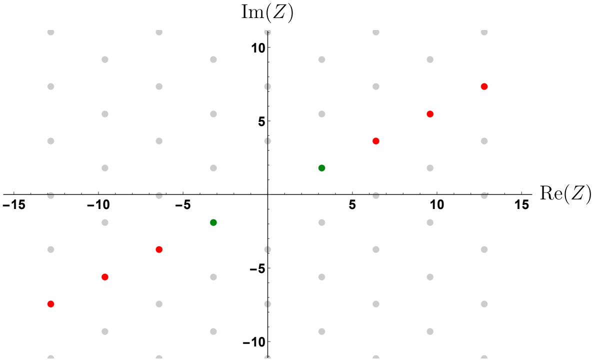

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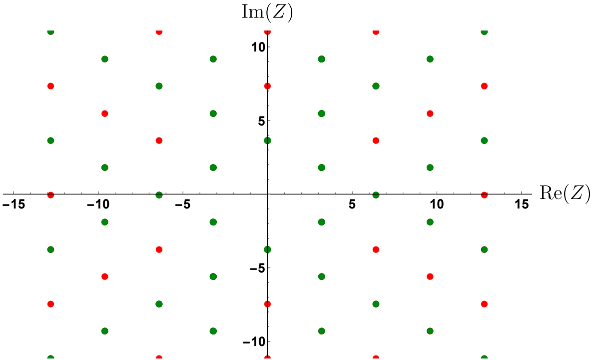
- Green: infinite set of stable states.



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We studied all 14 cases and the one presented here is one of the interesting ones since it has both type of infinite distance singularities. K at $\psi = 0$ and M at $\psi = \infty$.

Conclusions

We find a charge sub-lattice of infinite light BPS-states at these infinite distance points (In addition to KK states at large volume point.) showing the **validity of Swampland Distance Conjecture**.

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