Carving out the Swampland of 6D SCFTs

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EXCELENCIA SEVERO OCHOA

6D SCFT and Geometry

$6D\,,\ \mathcal{N}=(\geq 1,0)$ SCFTs occupy a very special place in modern QFT [Nahm, '78]
From a Stringy point of view, SCFTs usually studied through their tensor branch.

- $B_{\mu\nu} \quad \leftrightarrow \quad$ String

  Smoking gun for SCFTs : Tensionless strings.

- F-theory : 6D strings from D3-branes wrapping $\Sigma_i \subset \mathcal{B}$.

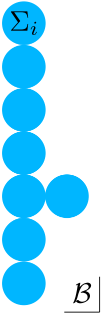
- CFT point reached when contracting all curves to a point

$$T = \text{Vol}(\Sigma) \to 0 \qquad \Sigma_i \cdot \Sigma_j \preceq 0$$

- All possible bases constructed gluing NHCs.

  $\longrightarrow$ **Generalised quivers**

$$\mathcal{B} = \mathbb{C}^2/\Gamma\,, \qquad \Gamma \subset U(2)$$



[Heckman,Morrison,Tomasiello,Rudelius,Vafa,'13-15]

What really happens at the SCFT point ?

Spectrum along the tensor branch reorganises into **superconformal multiplets**.

$$\begin{array}{ccc} SO(5,1)\times Sp(\mathcal{N}) & \longrightarrow & SO(6,2)\times Sp(\mathcal{N}) \\ \Phi\,,A_\mu\,,T_{\mu\nu} & \longrightarrow & ??? \end{array}$$

CFT operators are **gauge neutral**, e.g. $\mathcal{O}_k = \mathrm{Tr}_G(\Phi^k)$.

- Goal : Use **unitarity** to obtain constraints **directly** at CFT point.

  Possibly exclude theories not captured by the classification.

  Dream : Exclude all spectra but countable set.

- Eliminate theories that do not even satisfy unitarity constraints (swampland of the swampland), e.g.

$$C_T \geq 0\,, \qquad \Delta \geq \Delta_{\text{bound}}(\ell) = \begin{cases} 2\,, & \ell = 0 \\ \ell+4\,, & \ell > 0 \end{cases}$$

- String Theory : unitarity very powerful (anomaly cancellation, $\Sigma_i \cdot \Sigma_j \preceq 0\ldots$)

The Conformal Bootstrap : New twist on an old Idea

- OPE : reduces 4-point functions as a sum of **conformal blocks**

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{(x_{12}\,x_{34})^{\Delta_\phi}} \sum_{\mathcal{O}\in\phi\times\phi} \lambda^2_{\phi\phi\mathcal{O}}\; g_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(u,v)$$

Casimir Equation [Dolan,Osborn,'04+'11]

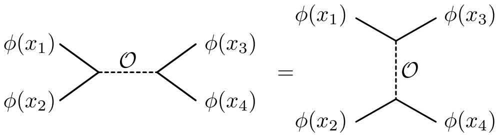
$$\mathcal{D}\, g_{\Delta,\ell} = c\, g_{\Delta,\ell}\,, \qquad c = c_{\Delta,\ell} \in \mathbb{R}^+$$

In even dimension, $g_{\Delta,\ell}$ has known closed form.

The Conformal Bootstrap : New twist on an old Idea

- OPE should be **associative**

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle =$$



Sum Rule [Rattazzi,Rychkov,Tonni,Vichi,'08]

$$\sum_{\mathcal{O}\in\phi\times\phi} \lambda_{\mathcal{O}}^2 \underbrace{\left(v^{\Delta_\phi}\, g_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(u,v) - u^{\Delta_\phi}\, g_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(v,u)\right)}_{F_{\mathcal{O}}(u,v)} = 0$$

- By unitarity $\lambda_{\mathcal{O}} \in \mathbb{R}$.

  Consider would-be spectrum $\{\Delta_{\mathcal{O}}, \lambda_{\mathcal{O}}\}$, if we can find a functional such that

$$\alpha(F_{\mathcal{O}}) > 0\,, \quad \forall\,\ell\,,\ \Delta \geq \Delta_{\text{bound}} \qquad \Rightarrow \qquad \sum_{\mathcal{O}} \underbrace{\text{const}}_{>0} = 0$$

CONTRADICTION !

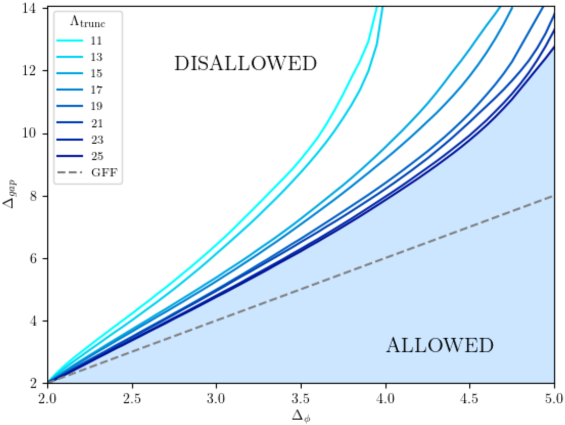
$\longrightarrow$ **semi-definite program** (linear optimisation). $\exists$ standard tools (SDPB).
[Poland,Simmons-Duffin,Vichi,'12 ; Simmons-Duffin,'15]

In practice : $\alpha_{\text{trunc}} = \sum_{m+n\leq\Lambda_{\text{trunc}}} \alpha_{m,n}\partial_u^m\partial_v^n F_{\mathcal{O}}|_{u=v}$

- As $\Lambda \to \infty$, more and more spectra can be excluded.
- Consider gapped spectrum

  $\Delta \geq \Delta_{\text{gap}} + 2\,, \qquad \forall\,\text{scalars}$

- Only deviations from GFF for non-SUSY w/ $\Delta < 4$.



Generalised free theories :

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \langle\phi(x_1)\phi(x_2)\rangle\,\langle\phi(x_3)\phi(x_4)\rangle + (\text{perms.})\,, \qquad \langle\phi(x)\phi(y)\rangle = \frac{1}{|x-y|^{\Delta}}$$

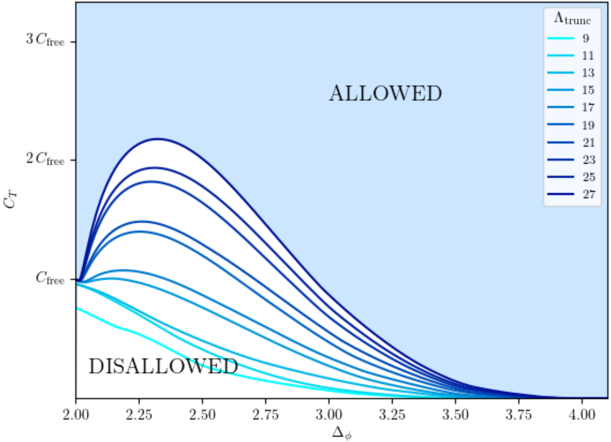
Also used to bound OPE coefficients. Singling out energy-momentum tensor $T_{\mu\nu}$:

$$\lambda^2_{T_{\mu\nu}} \underbrace{\alpha(F_{T_{\mu\nu}})}_{=1} = -\alpha(F_{\mathbb{1}}) - \sum_{\{\mathcal{O}\}\setminus\{\mathbb{1},T_{\mu\nu}\}} \lambda^2_{\mathcal{O}}\alpha(F_{\mathcal{O}}) \leq -\alpha(F_{\mathbb{1}})$$

- Maximise $\alpha(F_{\mathbb{1}})$
- Get lower bound :

$$\lambda^2_{T_{\mu\nu}} = \frac{5}{6}\frac{\Delta^2_\phi}{C_T} \leq -\alpha(F_{\mathbb{1}})$$

- Weakest where supermultiplets appear.



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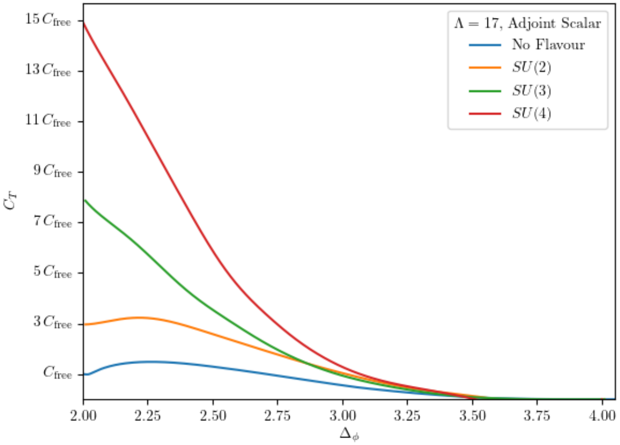

Bootstrapping $\mathcal{D}$-multiplets

While the bootstrap program has grown into a field of physics of its own, the literature on 6D SCFT is sparse, but has yielded interesting results

- $\mathcal{N} = (2,0)$ with $\mathcal{D}[2,0] \ni T_{\mu\nu}$ [Beem,Lemos,Rastelli,van Rees,'16]

$$C_T^{\text{min}} \to 25 \;,\; \Lambda_{\text{trunc}} \to \infty \qquad (A_1 \text{ theory !})$$

- $\mathcal{N} = (1,0)$ with $\mathcal{D}[1] \ni J_\mu$ [Chang,Lin,'17]

$$C_J^{\text{min}} \to 150 \,,\; \Lambda_{\text{trunc}} \to \infty \qquad (\text{E-String !})$$

Superblocks satisfy a **super Casimir equation**

$$(\mathcal{D}_{\text{bos}} + \mathcal{D}_{\text{SUSY}})\, G_\chi = (c_{\text{bos}} + c_{\text{SUSY}})\, G_\chi \,, \qquad G_\chi = \sum_{m,n} f_{m,n}\, g_{\Delta+m,\ell+n}$$

Can use bosonic Casimir equation to find prefactors.

Bootstrapping $\mathcal{D}$-multiplets

$\mathcal{D}$-multiplets correlators most constrained (shortness conditions)

$\Rightarrow$ Casimir equation simpler ($Q^n \mathcal{O} = 0$ for some operators inside multiplet)

- Generalises known results for $\mathcal{D}[1]$ to arbitrary $\mathcal{D}[J_r]$ ($\forall\ 2 < D \leq 6$ w/ $8Q$)
  [Chang,Lin,'17 ; Bobev,Lauria,Mazáč,'17]

- Also for 4-pt functions involving different $\mathcal{D}[J_R]$

$\longrightarrow$ Enables bootstrap for **mixed** correlators.

- Will put strong constraints on theories without known F-theory realisation

$$G_F = SU(3) \qquad \text{with} \qquad C_T = \frac{19488}{5}\,, \qquad C_J = \frac{195}{5}$$

(stay tuned !)

[Shimizu,Tachikawa,Zafrir'17]

Conclusion

- The bootstrap is a very powerful tool to study SCFTs
- Bootstrap "knows" about supersymmetry and String theory !
- We found the blocks for 4-pt function of any $\mathcal{D}$-multiplets

A lot of exciting direction to explore :

- Relate the bounds to geometric quantities ?
- How much freedom is there for non F-theoretic SCFTs ?
- Are SCFTs embeddable into compact geometries special (swampland) ?
- ...

Thank you for your attention !