F-theory on Quotient Elliptic Threefolds

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Based on
- arXiv:1801.08658 with: L. Anderson, J. Gray, A. Grassi
- arXiv:1906.XXXXX with: L. Anderson, J. Gray

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Connect field theories by geometric quotients

Relate SUGRA theories from two quotient geometries

$$\begin{array}{cccc} SUGRA: & A & \longleftarrow ? \longrightarrow & B \\ & \uparrow & & \uparrow \\ Geometry: & X & \leftarrow Quotient \rightarrow & X/\Gamma_n \end{array}$$

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Example: 4d Heteortic String and model building with $\mathbb{Z}_2$ quotient [Ovrut, Donagi; Braun, Brodie, Lukas; Andreas, Curio, Klemm; Blaszczyk, Groot-Nibbelink, Ratz, Ruehle, Trapletti, Vaudrevange]

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Gauge Group	$SU(5)$	$SU(3)SU(2)U(1)$
# Families	6	3

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F-theory in 6 dimensions

smooth, compact, torus $\mathcal{E}$-fibered 3-fold

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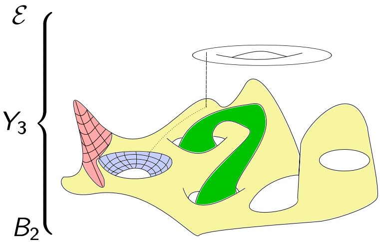

- $\tau(\mathcal{E})$ treated as space dependent **axio-dilaton of IIB** (forget the $\mathrm{Vol}(\mathcal{E})$)
  - B_2: **physical** compactification space
  - D7-(p,q) brane **monodromies** of τ traced geometrically
  - 7-Brane stacks **localized** in B_2 where $\mathcal{E}$ becomes **singular**

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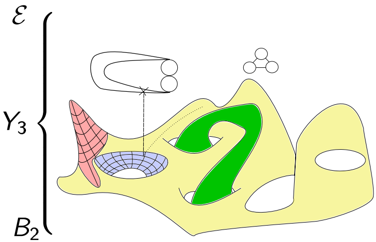

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- Y_3 **smooth**: Resolve the singularity $\rightarrow$ additional divisors [Kodaira,Tate]
- Curves in intersect like affine **ADE Dynkin diagram** $\rightarrow$ **Gauge algebra**
- 6d SUGRA **anomaly cancellation automatic** [Sodov; Grassi,Morrison]

Quotient Calabi-Yau Geometries

Torus fibered threefold Y_3 **quotient** by $\mathbb{Z}_n$ automorphism $\widehat{Y}_3 = Y_3/\Gamma_n$ [Braun, Candelas, Davis, He, Donagi, Ovrut, Pantev, Waldram'99, Andreas,Curio,Klemm; Braun, Brodie, Lukas'17]

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3. Γ_n **respects fibration**/projection π:

$$\begin{array}{ccc} T/\Gamma_{n,f} & \rightarrow & \hat{Y} = Y/\Gamma_n \\ & & \downarrow \pi \\ & & \hat{B} = B/\Gamma_{n,b} \end{array}$$

$\rightarrow \Gamma_n$ **decomposable** into fiber & base action $\Gamma_n = \Gamma_{F,n} \oplus \Gamma_{b,n}$

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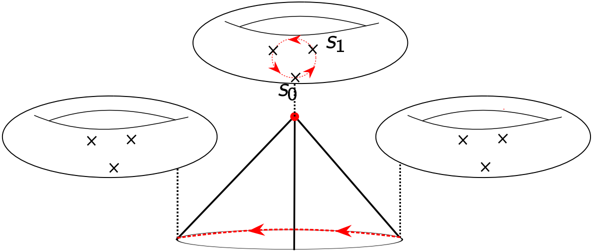
Quotient Fiber Action



Locally: $X_{loc} = T^2 \times \mathbb{C}^2$ with coordinates (ω, z_0, z_1)

- Pick **automorphism** that acts as

$$\Gamma_n : (\omega, z_0, z_1) \quad \rightarrow (\omega + \tfrac{1}{n}\tau s_n, z_0 e^{2\pi i/n}, z_1 e^{-2\pi i/n})$$

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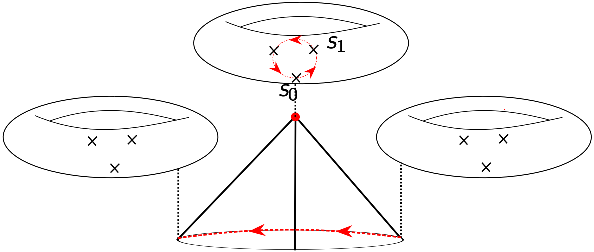

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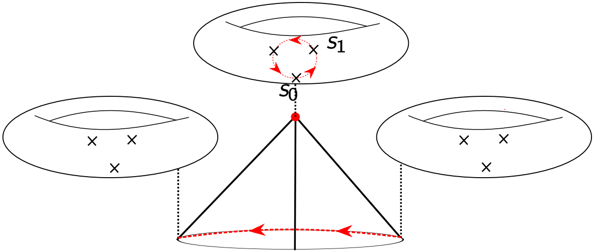

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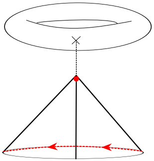
- $\Gamma_{n,f}$ **shifts** points on the torus $s_i \to s_{i+1} \sim s_{i+1+n}$
- **Smooth Quotient** $Y_{loc} = (T^2 \times \mathbb{C}^2)/\Gamma_n$ **identifies** all these points $s_0 \sim s_i$ over base orbifold
- No well defined $n-$section and a **multiple fiber** [Gross; de Boer, Dijkgraaf, Hori,Keurentjes, Morgan, Morrison, Sethi; Bhardwaj, Del Zotto, Heckman, Morrison, Rudelius, Vafa]

Tensor branch physics



Tensor branch in 6d

Scalar component of (2,0) tensor: $\langle s \rangle \sim 1/g \sim vol(\mathbb{P}^1)$
Blow-up: finite vev $\rightarrow$ finite coupling [Seibterg, Witten'96]

ungauged: $A_{n-1}^{(2,0)} \rightarrow (n-1)(T_{(1,0)} \oplus H_{\mathbf{1_0}})$

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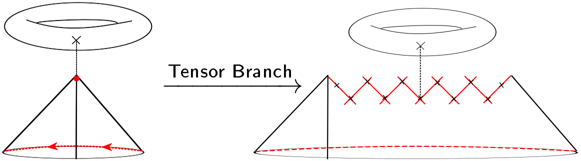

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- Ungauged A_{n-1} (2,0) case: **only tensors, no new vectors or hypers**

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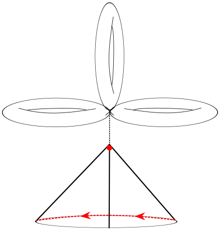

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Multi-Fiber case: total space smooth!

- **Local equation**: $x_1 y_2 - y_1 x_2 = \epsilon$ subject to $(x_m, y_m) \sim (xe^{2\pi i m/n}, ye^{-2\pi i m/n})$
- **Tensor branch**: $\epsilon \to 0$ **and** resolve: **hyperconifold** [Davis'13]

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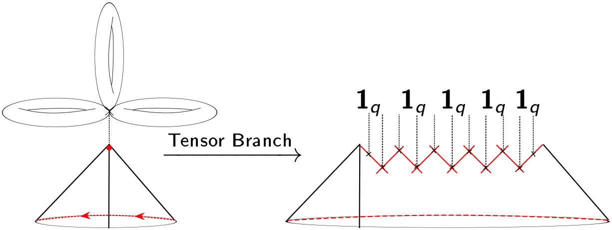

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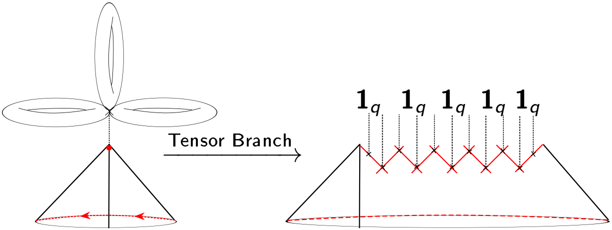

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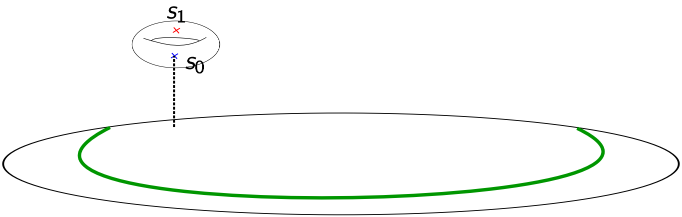
A $\mathbb{Z}_2$ Example



Extend to a global geometry

- The base B_2 must have an $\mathbb{Z}_n$ **automorphism**
- Fibration needs **additional sections** with global shift symmetry
  $\rightarrow$ Finite order **Mordell-Weil** group $MW(Y_3) \ni \mathbb{Z}_n$
- **Example:** $\mathbb{Z}_2$ torsion Weierstrass model: [Aspinwall, Morrison]

$$y^2 = x(x^2 + a_2 x + a_4), \quad \Delta = a_4^2(4a_4 - a_2^2)$$

$\rightarrow$ **Second Torsion section** $s_1 : y = x = 0$

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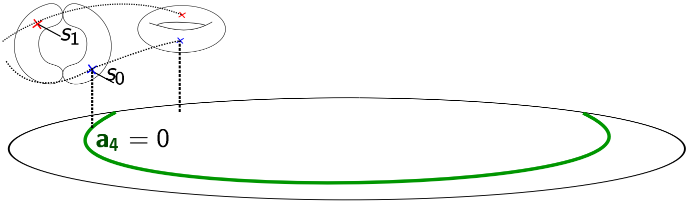

The Global Covering Geometry

- $\mathbb{Z}_2$ torsion WSF: $y^2 = x(x^2 + a_2 x + a_4)$, $\quad \Delta = \mathbf{a_4}^2(4a_4 - a_2^2)$
- **Torsion forces** the presence of $SU(2)/\mathbb{Z}_2 \sim SO(3)$ over $\mathbf{a_4} = 0$

  [Aspinwall,Morrison/Till,Mayrhover, Weigand]
- **Torsion relation** induced by intersection of SU(2) res. divisor by s_1
- **Charged hypermultiplet spectrum: 3** $\times\, g_{\mathbf{a_4}}$ the only matter

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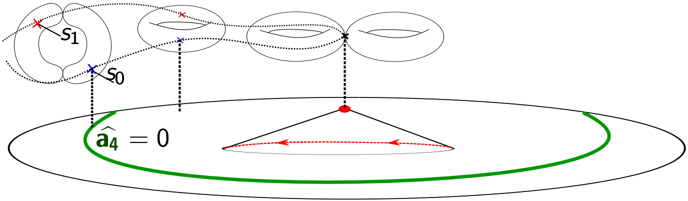

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- **Genus-one geometry:** sections combine: $s_0 \sim s_1 \to s^{(2)}$
- Identification **extends** to SO(3): fibral curves have **same** volume!

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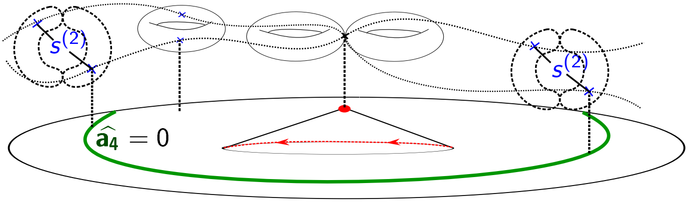

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  - **Gauge group:** $SO(3) \to \mathbb{Z}_2$ no **independently shrinkable curve** !
  - **Singlets** from M2's: $\mathbf{3} \times g_{a_4} \to (\mathbf{1}_1 \oplus \mathbf{1}_1)(g_4 - 1)/2$ [Witten; Braun,Morrison]
- **Add** discrete gauged **superconformal matter** at fixed points!
- **All anomalies canceled!**

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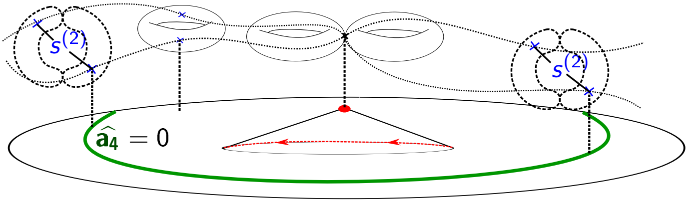

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Geometry: Turned a smooth elliptic fibration Y_3 with an order n freely acting automorphism into a non-simply connected, genus-one fibered threefold $\widehat{Y_3}$ with multiple fibers.
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State of the Art: $\mathbb{Z}_6$ quotient of the Schoen manifold [Bouchard,Donagi'06]

	Cover	Quotient
$h^{2/1,1}$:	19	$\rightarrow$ 3
V :	$(SU(2)xSU(3)xSU(6))/\mathbb{Z}_6$	$\rightarrow$ $\mathbb{Z}_6$
H :	$20 \times \mathbf{1} + \mathbf{3} + \mathbf{8} + \mathbf{35}$	$\rightarrow$ $4 \times \mathbf{1}$
T :	$9T_{(1,0)}$	$\rightarrow$ $1T_{(1,0)} + \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_5$

- **Anomalies canceled**: $H - V + 29T_{(1,0)} + 30\mathcal{A}_n - 273 = 0$ ✓