[Barbosa, Cvetic, Heckman, Lawrie, Torres, GZ: 1906.02212]

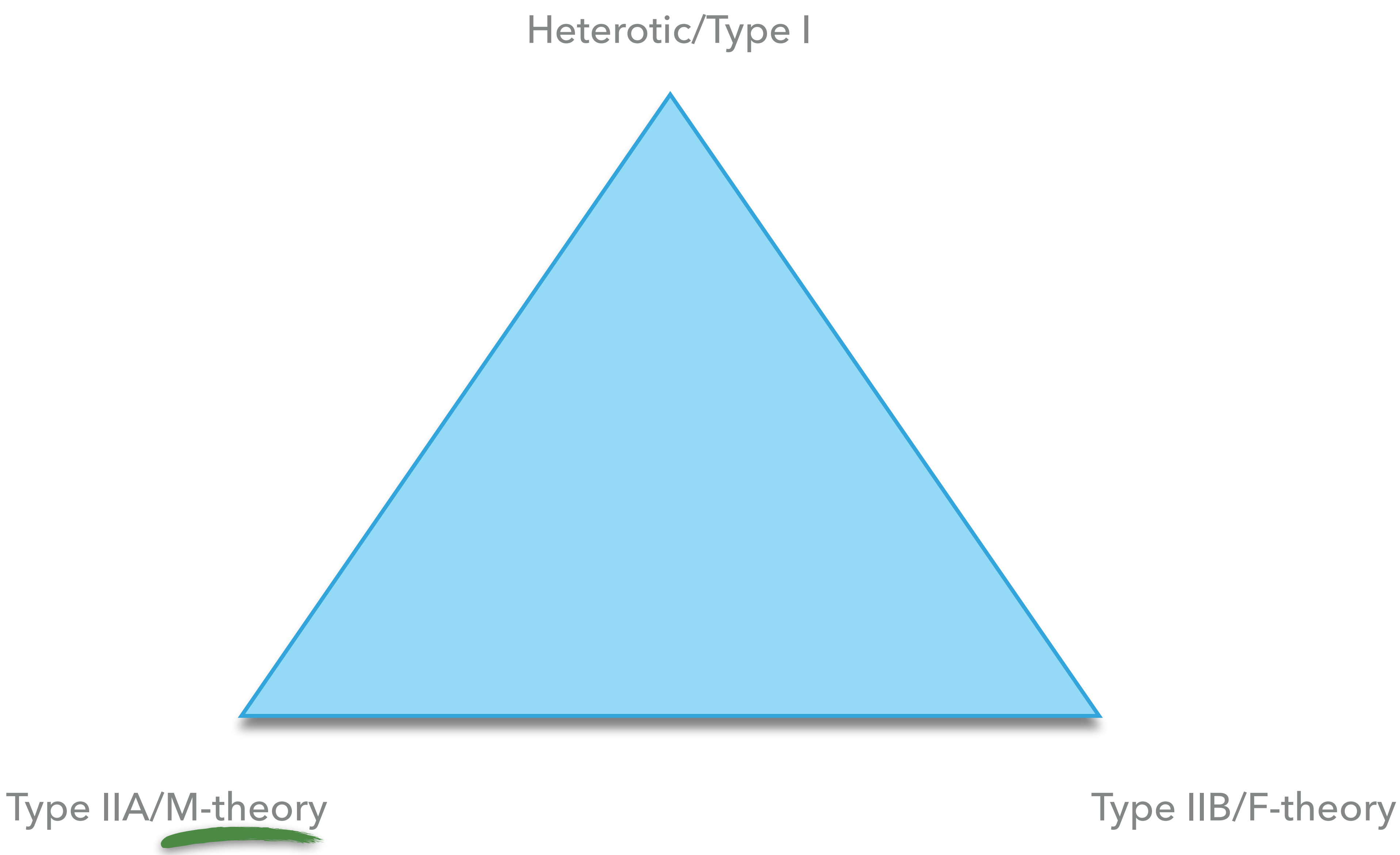
GIANLUCA ZOCCARATO

Penn
University of Pennsylvania

T-BRANES IN G2 BACKGROUNDS

June 25th, 2019

String Phenomenology 2019


Heterotic/Type I
Type IIA/M-theory
Type IIB/F-theory

M-THEORY ON G2-MANIFOLDS

See Schäfer-Nameki's talk

Geometry of G2 manifolds is captured by a harmonic 3-form

$$d\Phi = 0 \qquad d \star \Phi = 0$$

Gauge sector is on codimension 4 ADE singularities

Brane configuration is reflected in the geometry

New techniques are available to build G2 manifolds

[Kovalev '03]
[Corti, Haskins, Nordström, Pacini '03 - '05]
[Halverson, Morrison '14 - '15]
[Andriolo, Shiu, Triendl, Van Riet, Venken, GZ '18]

However it is not known how to get codimension 7 singularities

Local models

TYPE IIB/F-THEORY MODELS

7-branes wrapping a complex surface [Beasley, Heckman, Vafa '08]

Matter content: gauge field and (2,0) form (A, Φ)

BPS equations

$$\bar{\partial}_A \Phi = 0$$

$$F^{(0,2)} = 0$$

$$\omega \wedge F + \frac{1}{2}[\Phi, \Phi^\dagger] = 0$$

Matter fields are localised on curves, Yukawa interactions at points

Chirality is generated via fluxes

See Lin's talk

T-BRANES

[Donagi, Katz, Sharpe '03]
[Cecotti, Cordova, Heckman, Vafa '10]

Local configuration of 7-branes is captured by spectral equation

$$\det(s\,\mathbb{I} - \Phi) = 0$$

Some configurations can not be captured by spectral equation

$$\Phi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

In particular we have that $[\Phi, \Phi^\dagger] \neq 0$

See Savelli's talk

T-BRANES

[Donagi, Katz, Sharpe '03]
[Cecotti, Cordova, Heckman, Vafa '10]

Local configuration of 7-branes is captured by spectral equation

$$\det(s\,\mathbb{I} - \Phi) = 0$$

Some configurations can not be captured by spectral equation

$$\Phi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

In particular we have that $[\Phi, \Phi^\dagger] \neq 0$

See Savelli's talk

[Anderson, Apruzzi, Ashfaque, Bena, Blåbäck, Carta, Cecotti, Chiou, Cicoli, Collinucci, Cordova, Cvetic, Del Zotto, Donagi, Esole, Faraggi, Font, Fredrickson, García-Etxebarria, Giacomelli, Hassler, Hayashi, Heckman, Katz, Kawano, Lin, Marchesano, Mayrhofer, Mekareeya, Minasian, Morrison, Quevedo, Regalado, Rochais, Rudelius, Savelli, Schaposnik, Schwieger, Sharpe, Shukla, Tatar, Tizzano, Tomasiello, Tsuchiya, Vafa, Valandro, Walters, Watari, Wijnholt, GZ]

LOCAL MODELS IN TYPE IIA/M-THEORY

[Pantev, Wijnholt '09]
[Braun, Cizel, Hübner, Schäfer-Nameki '18]

6-branes wrapping a 3-manifold M

Matter content: gauge field and 1-form (A, ϕ)

BPS equations

$$F = \phi \wedge \phi$$
$$D_A \phi = 0$$
$$D_A \star \phi = 0$$

Truncation of Hermitian-Yang-Mills on $T^* M$

Matter fields are localised at points

We would like to consider non-abelian solutions $\phi \wedge \phi \neq 0$

PANTEV-WIJNHOLT SYSTEM

Introduce a complexified connection $\mathcal{A} = A + i\phi$

and the field strengths

$$\mathcal{F} = [D_{\mathcal{A}}, D_{\mathcal{A}}]$$

$$\mathcal{D} = [D_{\mathcal{A}}, D_{\overline{\mathcal{A}}}]$$

BPS equations can be written as

$$\mathcal{F} = 0$$

$$g^{ij}\mathcal{D}_{ij} = 0$$

and moduli space of solutions is $\{\mathcal{F} = 0,\ g^{ij}\mathcal{D}_{ij} = 0\}/G^{\text{gauge}}$

or equivalently $\{\mathcal{F} = 0\}/G^{\text{gauge}}_{\mathbb{C}}$

subject to a stability condition

[Donaldson '87]
[Corlette '88]

NON-ABELIAN PW FLOWS

Look for solutions on a three manifold $I \times \Sigma$

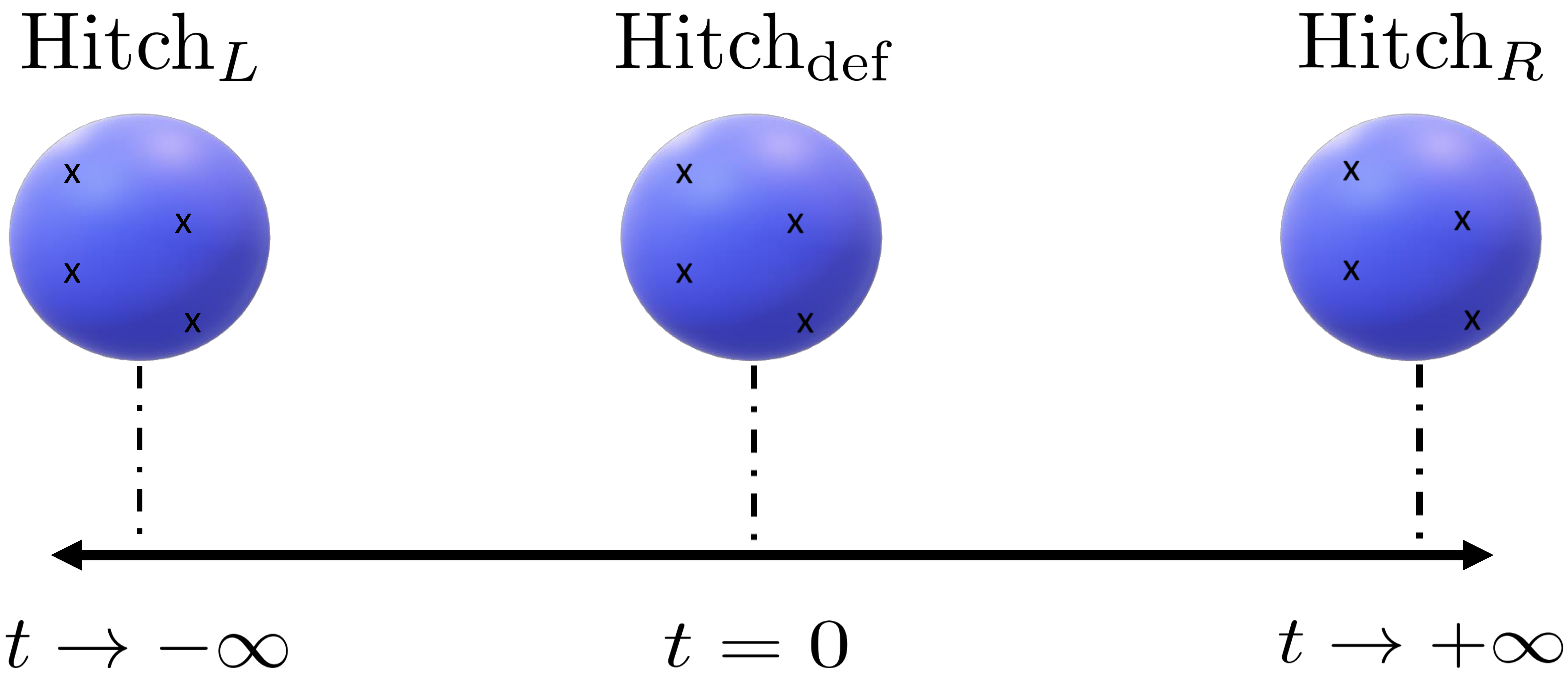
Restricting on the Riemann surface the PW system becomes

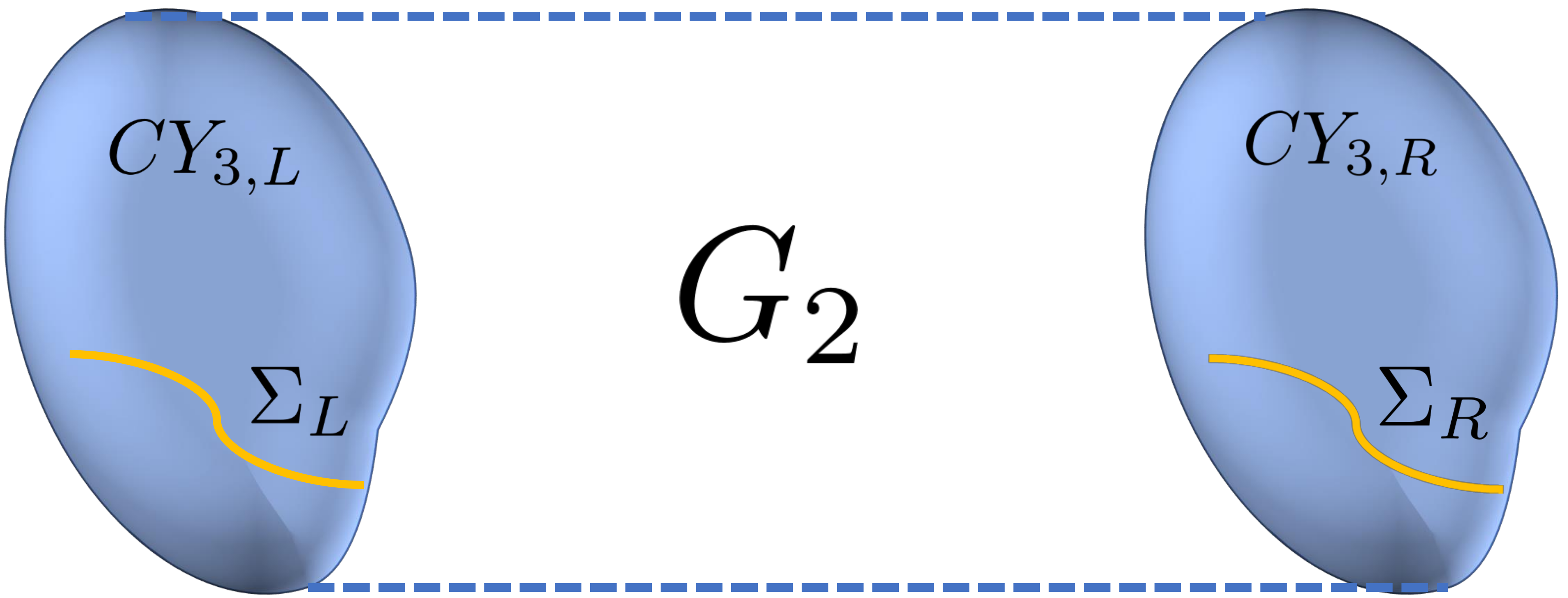
$$\begin{aligned} F_\Sigma &= \phi_\Sigma \wedge \phi_\Sigma \\ D_\Sigma \phi_\Sigma &= 0 \\ \star_\Sigma D_\Sigma \star_\Sigma \phi_\Sigma &= -g^{tt} D_t \phi_t \end{aligned}$$

which is a deformed Hitchin system [Hitchin '87]

Solution takes the form of a flow along the interval

Asymptotically solutions approach two Hitchin systems at boundaries


Hitch_L
$\mathrm{Hitch}_{\mathrm{def}}$
Hitch_R
$t \to -\infty$
$t = 0$
$t \to +\infty$


$CY_{3,L}$
$CY_{3,R}$
G_2
Σ_L
Σ_R

SOLUTIONS I: ABELIAN

Simplest examples are abelian solutions. For SU(2) take

$$\phi = -i \begin{bmatrix} df & 0 \\ 0 & -df \end{bmatrix}$$

The Laplacian of f has to be zero

Matter is localised at the critical points of f (brane intersection)

Structure of hessian of f gives different possible geometries

i. One zero eigenvalue: codimension 6 singularity (non-chiral matter)
ii. All eigenvalues non-zero: codimension 7 singularity (chiral matter)

SOLUTIONS II: NON-ABELIAN

Take an SU(3) model. Within an SU(2) sub-algebra put a Hitchin system (on the Riemann surface) with Higgs field

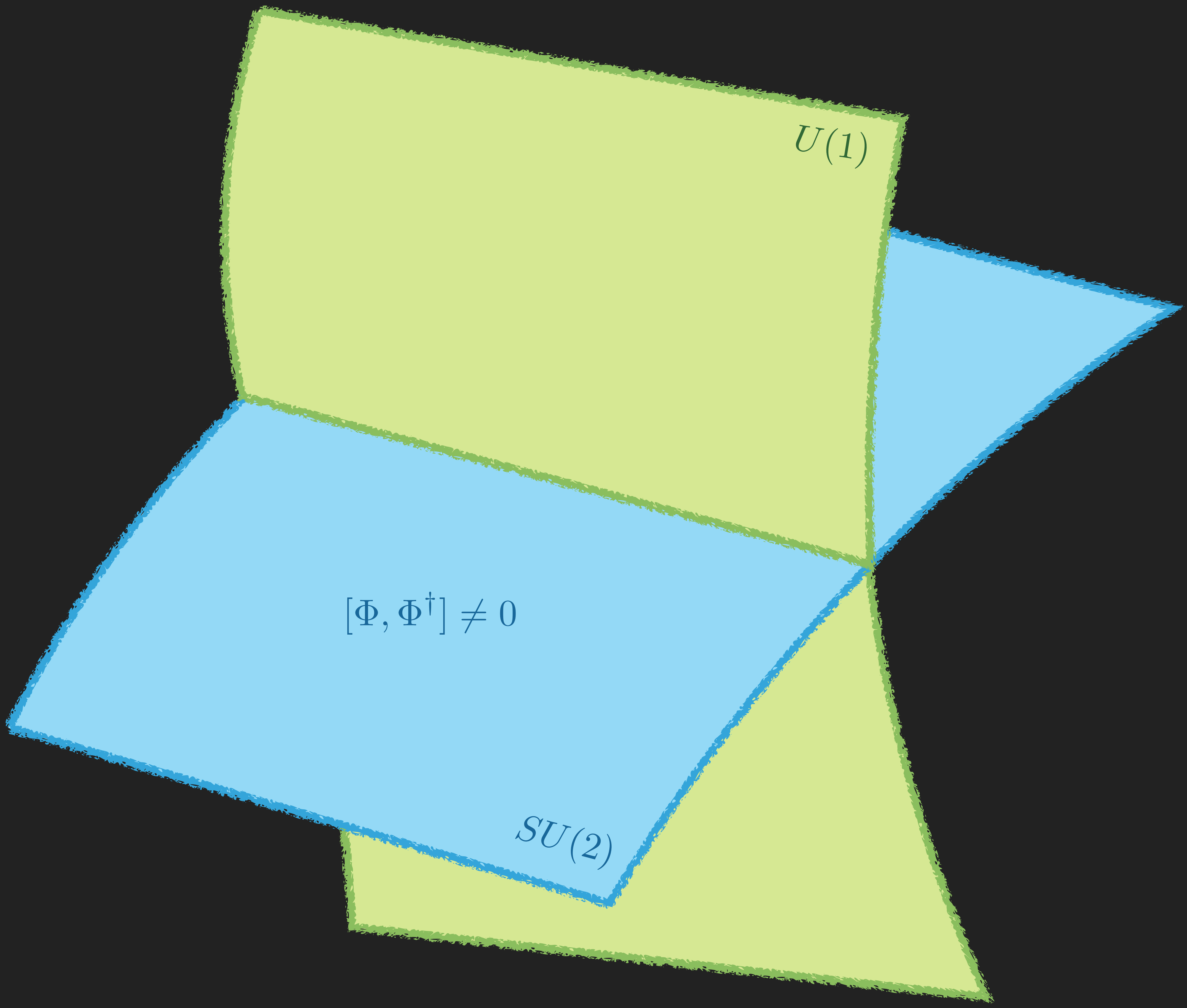
[Hitchin '87]

$$\Phi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

This gives a 5d hypermultiplet localised at the origin

On top of this add an abelian system giving a codimension 6 singularity along the Cartan of SU(3) commuting with SU(2)

Combining both systems gives a solution with a chiral 4d mode



$U(1)$
$[\Phi, \Phi^\dagger] \neq 0$
$SU(2)$

ZERO MODES

Zero modes are linear fluctuations around a given background

$$\mathcal{A} = \langle \mathcal{A} \rangle + \psi$$

The zero modes equations are

$$D_{\langle \mathcal{A} \rangle} \psi = 0$$
$$D^{\dagger}_{\langle \mathcal{A} \rangle} \psi = 0$$

Counting of zero modes can be turned into a cohomology problem

$$D_{\langle \mathcal{A} \rangle} : \Omega^{\bullet}(M, E_{\mathcal{R}}) \to \Omega^{\bullet+1}(M, E_{\mathcal{R}})$$

and zero modes are 1-forms that are closed and co-closed

ZERO MODES – SPECTRAL METHODS

To assess whether localised modes exist one can use spectral equations

$$\det(s_a \, \mathbb{I} - i\phi_a) = 0$$

which give the brane configuration in T^*M

Modes localise at loci where the various sheets intersect each other (intersection of 6-branes)

This however does not work for the non-abelian case

Unlike the 7-brane case there is no holomorphicity

ZERO MODES - LOCAL MATTER RING

Zero modes ~ linearised solutions of F-terms mod complex gauge

$$D_{\langle \mathcal{A} \rangle} \psi = 0 \,, \qquad \psi \simeq \psi + D_{\langle \mathcal{A} \rangle} \chi$$

In a local patch all modes are gauge equivalent to zero

However gauge parameter blows off at infinity

We need to restrict only to gauge transformations that do not change the boundary conditions (that do not destabilise the background)

Following this it is possible to show that the space of zero modes is

$$\frac{\mathcal{O} \otimes \mathfrak{g}_{\mathbb{C}}}{\langle \ker \, \mathrm{ad}_{\Phi_z} , \ker \, \mathrm{ad}_{\Phi_t} \rangle}$$

CONCLUSIONS

M-theory on G2 manifolds produces N=1 theories in 4d

However it is difficult to build compact geometries with chiral matter

We revisited local models for 6-branes on 3-manifolds

We built solutions with non-abelian configurations (and fluxes)

Some non-abelian effects can be hidden in the geometry

Using non-abelian solutions it is possible to get chiral matter

Algebraic methods can be used to check existence of localised modes

CONCLUSIONS

M-theory on G2 manifolds produces N=1 theories in 4d

However it is difficult to build compact geometries with chiral matter

We revisited local models for 6-branes on 3-manifolds

We built solutions with non-abelian configurations (and fluxes)

Some non-abelian effects can be hidden in the geometry

Using non-abelian solutions it is possible to get chiral matter

Algebraic methods can be used to check existence of localised modes

Thank you!