NO-GO THEOREMS FOR THE INFLATION AND EKPYROSIS FROM STRING THEORY

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(JHEP 06 (2018) 041)

[1] Introduction

- The strong no-go theorems which exclude tree-level de Sitter compactifications have been much explored.

  (S. Kachru et al., hep-th/0301240)
  (O. DeWolfe et al., hep-th/0505160)
  (T. Wrase & M. Zagermann, arXiv:1003.0029 [hep-th])

- However, the no-go theorems for ekpyrotic scenario which is alternative to inflation model in string theory is much less extensive.

- One motivation for the present work is to improve this situation.

- The Ekpyrosis inspired by string theory and brane world model suggests alternative solutions to the early universe puzzles such as inflation and dark energy.

  (P. Horava, E. Witten, hep-th/9510209)
  (A. Lukas et al., hep-th/9710208)
  (J. Khoury et at., hep-th/0103239)

- Since the big bang is described as a collision of branes, there is also a new ekpyrotic phase or a cyclic universe due to another brane collision with the creation of new matter.

  (M. Bastero-Gil, E. J. Copeland, J. Gray, A. Lukas, M. Plumacher, hep-th/0201040)

- We consider inflation whenever the potential energy dominates. This will be possible provided the potential is flat enough, as the scalar field would then be expected to roll slowly.

- The standard strategy for solving field equations is the slow-roll approximation

$$\varepsilon_{\rm s} \ll 1\,, \qquad |\eta_{\rm s}| \ll 1$$

where we define slow-roll parameter:

$$\varepsilon_{\rm s} = \frac{1}{2}\left(\frac{V'}{V}\right)^2\,, \qquad \eta_{\rm s} = \frac{V''}{V}$$

- The potential during ekpyrosis is negative and steeply falling; it can be modeled by the exponential form $V(\phi)=-V_0 \exp(-c\phi)$ $(c\gg 1)$.
  (J. Khoury et at., hep-th/0103239)

$$a(t) = (-t)^p\,, \quad p = \frac{2}{c^2}$$

- There was plenty of time before the big bang for the universe to be in causal contact over large regions.

- The scalar potential obeys fast-roll condition.
  (S. Gratton, et al., astro-ph/0301395)

$$\varepsilon_{\mathrm{f}} = \frac{V^2}{\sum_i \left(\partial_{\phi_i} V\right)^2}\,, \quad \eta_{\mathrm{f}} = 1 - \frac{V \sum_i \partial^2_{\phi_i} V}{\sum_i \left(\partial_{\phi_i} V\right)^2}$$

❑ Our work:

❀ We investigate whether the inflation and Ekpyrosis can be embedded into 10D string theory (no go theorem).

☞ We use that the scalar potential obtained from compactifications of type II string with sources has a universal scaling with respect to the dilaton and the volume mode.

[2] Compactifications of the type II theory

Compactifications of the type II theory to 4-diemnsional spacetime on compact manifold

☆10-dimensional action

$$S = \frac{1}{2\bar{\kappa}^2} \int d^{10}x \sqrt{-g} \left[\mathrm{e}^{-2\phi} \left(R + 4g^{MN} \partial_M \phi \partial_N \phi - \frac{1}{2} |H|^2 \right) - \frac{1}{2} \sum_p |F_p|^2 \right]$$

$$- \sum_p (T_{\mathrm{D}p} + T_{\mathrm{O}p}) \int d^{p+1}x \sqrt{-g_{p+1}}\, \mathrm{e}^{-\phi} \,,$$

F_p : R-R p-form field strengths

T_{Dp} , (T_{Op}) : Dp-brane (Op-plane) tension

★To compactify the theory to 4 dimensions, we consider the metric ansatz of the form:

6-dimensional internal space

$$ds^2 = g_{MN} dx^M dx^N = q_{\mu\nu} dx^\mu dx^\nu + \rho\, u_{ij}(\mathrm{Y}) dy^i dy^j$$

4-dimensional universe

$q_{\mu\nu}$: 4-dimensional metric

ρ : volume modulus of the compact space

4-dimensional effective action (Einstein frame)

(M. P. Hertzberg, arXiv:0711.2512)

$$S_{\mathrm{E}} = \int d^4x \sqrt{-\bar{q}} \left[\frac{1}{2\kappa^2}\bar{R} - \frac{1}{2}\bar{q}^{\mu\nu}\partial_\mu\bar{\rho}\partial_\nu\bar{\rho} - \frac{1}{2}\bar{q}^{\mu\nu}\partial_\mu\bar{\tau}\partial_\nu\bar{\tau} - V(\bar{\tau}\,,\bar{\rho}) \right]$$

$\bar{R}$: Ricci scalar constructed from $\bar{q}_{\mu\nu}$

$$q_{\mu\nu} = \left(\frac{\bar{\kappa}}{\tau\kappa}\right)^2 \bar{q}_{\mu\nu}$$

κ^2 : 4-dimensional gravitational constant

τ : dilaton modulus

$$\bar{\rho} = \sqrt{\frac{3}{2}}\kappa^{-1}\ln\rho\,, \quad \bar{\tau} = \sqrt{2}\kappa^{-1}\ln\tau\,, \quad \tau = \mathrm{e}^{-\phi}\rho^{3/2}$$

☆ moduli potential

$$V(\bar{\tau}\,,\bar{\rho}) = V_{\rm Y} + V_{\rm H} + V_p + V_{\rm DO}$$

$$V_{\rm Y}(\bar{\tau}\,,\bar{\rho}) = -A_{\rm Y}\,(\phi_i)\exp\left[-\kappa\left(\sqrt{2}\bar{\tau} + \frac{\sqrt{6}}{3}\bar{\rho}\right)\right] R({\rm Y})\,,$$

$$V_{\rm H}(\bar{\tau}\,,\bar{\rho}) = A_{\rm H}\,(\phi_i)\exp\left[-\kappa\left(\sqrt{2}\bar{\tau} + \sqrt{6}\bar{\rho}\right)\right],$$

$$V_p(\bar{\tau}\,,\bar{\rho}) = \sum_p A_p\,(\phi_i)\exp\left[-\kappa\left\{2\sqrt{2}\bar{\tau} + \frac{\sqrt{6}}{3}(p-3)\bar{\rho}\right\}\right],$$

$$V_{\rm DO}(\bar{\tau}\,,\bar{\rho}) = \sum_p \left[A_{{\rm D}p}\,(\phi_i) - A_{{\rm O}p}\,(\phi_i)\right]\exp\left[-\kappa\left\{\frac{3\sqrt{2}}{2}\bar{\tau} + \frac{\sqrt{6}}{6}(6-p)\bar{\rho}\right\}\right] \times \int d^{p-3}x\sqrt{g_{p-3}}$$

positive

A_Y, A_H, A_p, A_{Dp}, A_{Op} : coefficients

[3] The scenario with vanishing flux

☆ Statement :

Inflation and ekpyrosis are prohibited in string theory with D-branes, O-planes source and zero fluxes.

☆ moduli potential with vanishing flux

$$
\begin{aligned}
V(\bar{\tau}\,,\bar{\rho}) &= V_{\rm Y} + V_{{\rm D}p} + V_{{\rm O}p} \\
&= -A_{\rm Y}\left(\phi_i\right)\exp\left[-\kappa\left(\sqrt{2}\bar{\tau} + \frac{\sqrt{6}}{3}\bar{\rho}\right)\right]R({\rm Y}) \\
&\quad + \sum_p \left[A_{{\rm D}p}\left(\phi_i\right) - A_{{\rm O}p}\left(\phi_i\right)\right]\exp\left[-\kappa\left\{\frac{3\sqrt{2}}{2}\bar{\tau} + \frac{\sqrt{6}}{6}\left(6-p\right)\bar{\rho}\right\}\right]\int d^{p-3}x\sqrt{g_{p-3}}
\end{aligned}
$$

★Inflation:

If we set $R(Y)=0$, $A_Y=1$, $A_H=A_p=A_{0p}=0$, and $A_{Dp} \int d^{p-3}x (g_{p-3})^{1/2}=1$, ($p=4, 6, 8$ for IIA and $p=3, 5, 7, 9$ for IIB), in the moduli potential, the slow-roll parameters ε_s obeys

$$\varepsilon_{\rm s} = \frac{1}{2\kappa^2}\frac{(\partial_{\bar{\tau}}V)^2+(\partial_{\bar{\rho}}V)^2}{V^2} > \frac{9}{4}, \quad \text{For IIA}$$

$$\varepsilon_{\rm s} = \frac{1}{2\kappa^2}\frac{(\partial_{\bar{\tau}}V)^2+(\partial_{\bar{\rho}}V)^2}{V^2} > \frac{9}{4}, \quad \text{For IIB}$$

Compactifications tend to provide potentials that are not sufficiently flat.

★Ekpyrosis:

(E. Meeus & T. Riet, (2016), K. Uzawa, JHEP06 (2018) 041)

For the case of $R(Y)=0$, $A_Y=1$, $A_H=A_p=A_{Dp}=0$, and $A_{0p}\int d^{p-3}x(g_{p-3})^{1/2}=1$, ($p$=4, 6, 8 for IIA and p=3, 5, 7, 9 for IIB), in the moduli potential, the fast roll parameters ε_f satisfy

$$\varepsilon_{\rm f} = \kappa^2 \frac{V^2}{(\partial_{\bar\tau}V)^2 + (\partial_{\bar\rho}V)^2} > \frac{6}{31}\,, \qquad \text{For IIA}$$

$$\varepsilon_{\rm f} = \kappa^2 \frac{V^2}{(\partial_{\bar\tau}V)^2 + (\partial_{\bar\rho}V)^2} > \frac{1}{6}\,, \qquad \text{For IIB}$$

The result gives the contradiction with the fast-roll condition for ekpyrosis.

❑ Our results:

❀ We find strong constraints ruling out ekpyrosis from analyzing the fast-roll conditions.

☞ We conclude that a compactification in type II string theory tend to provide potentials that are not too steep and negative (ekpyrosis).

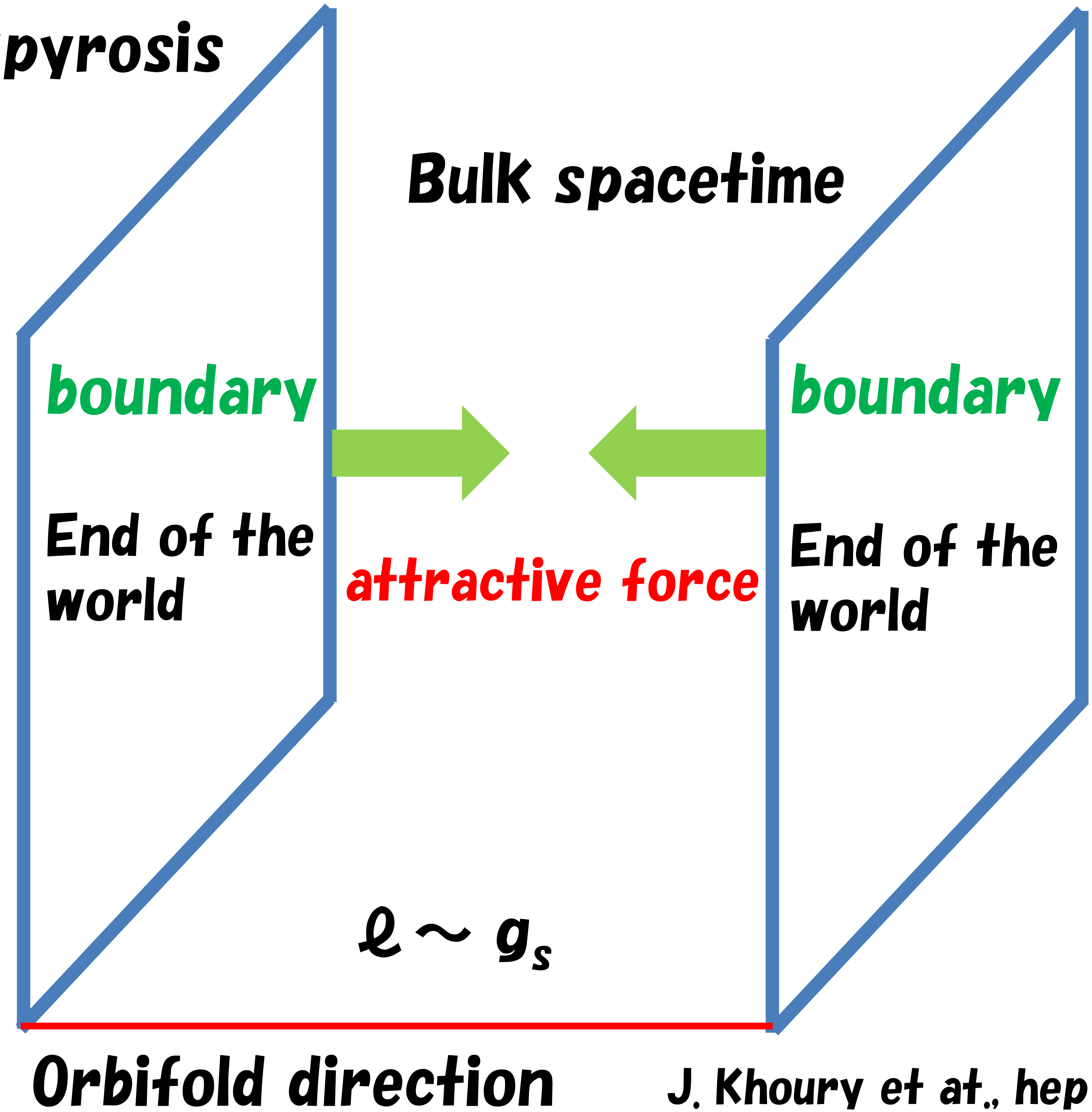
[4] Summary and comments

(1) We studied the No-Go theorem of the inflation and ekpyrosis for string theory with vanishing flux.

(2) The 4-dimensional effective potential of two scalar fields can be constructed by postulating suitable emergent gravity, orientifold planes in terms of the compactification with smooth manifold.

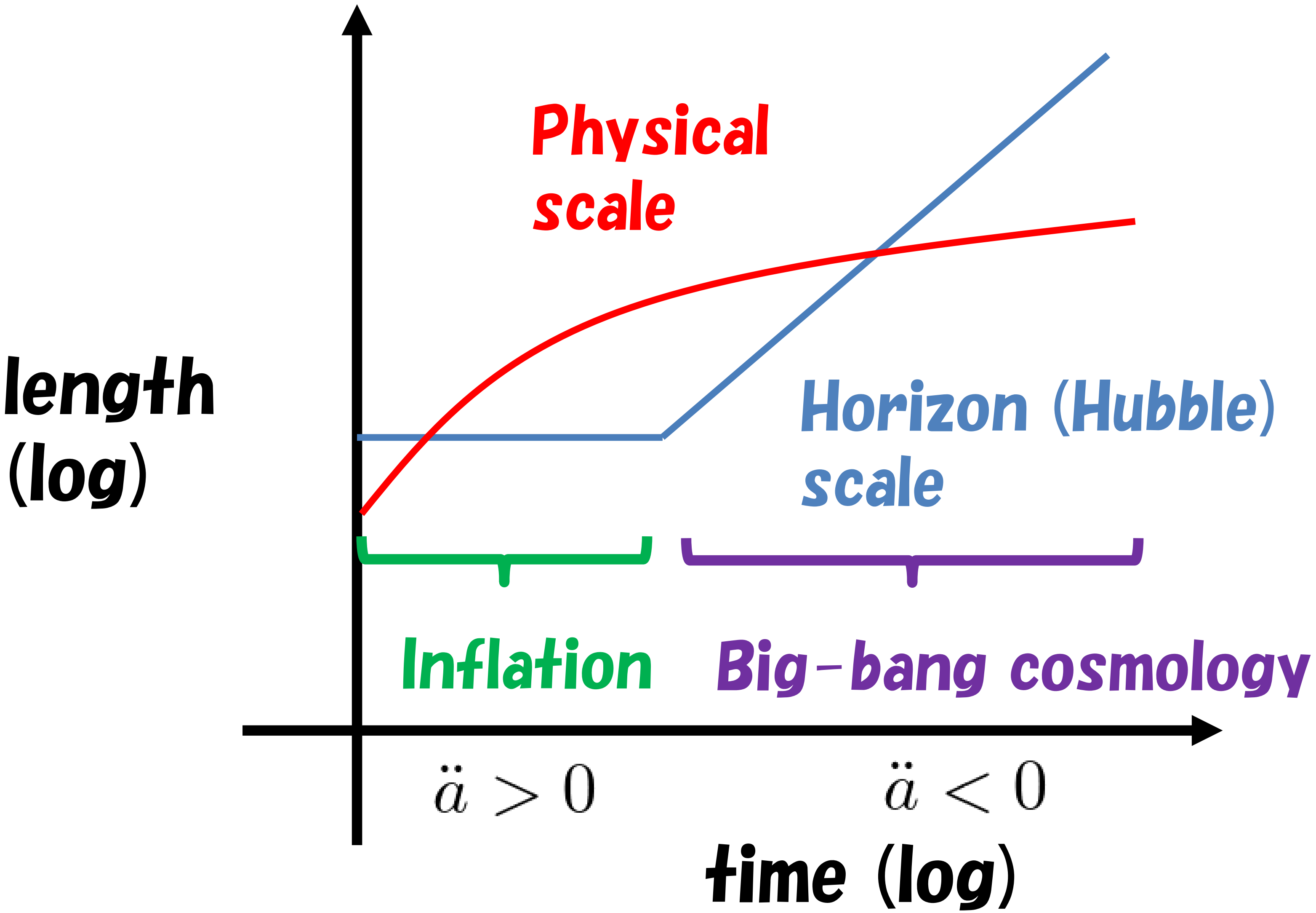
(3) Since the slow (fast)-roll parameter is not small during the inflation (ekpyrotic) phase, the explicit nature of the dynamics has made it impossible to realize the inflationary (ekpyrotic) scenario.

Ekpyrosis



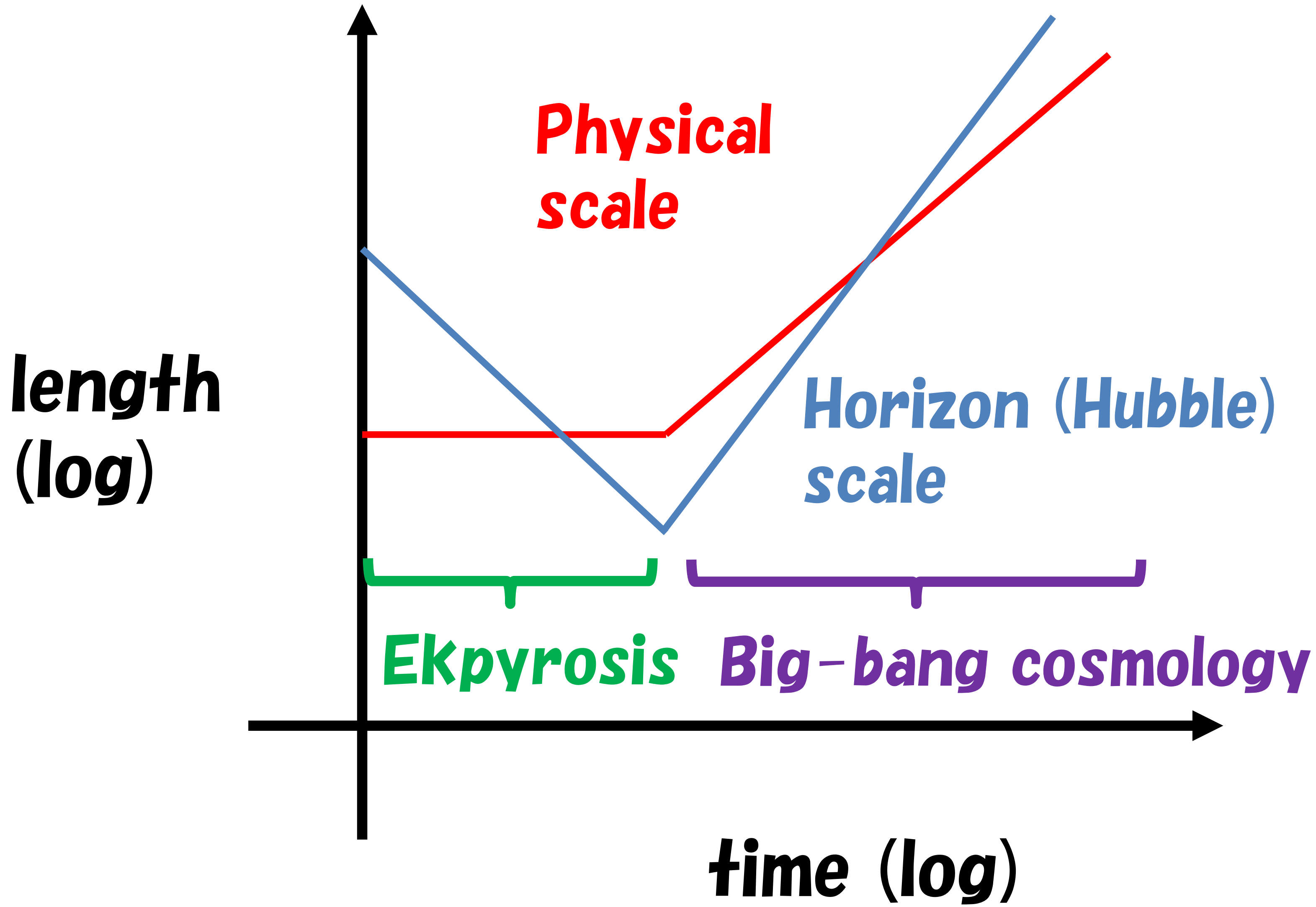
J. Khoury et at., hep-th/0103239

Inflation



Physical scale
length (log)
Horizon (Hubble) scale
Inflation
Big-bang cosmology
$\ddot{a} > 0$
$\ddot{a} < 0$
time (log)

Ekpyrosis


Physical scale
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