Spontaneous dark-matter mass generation along cosmological attractors in string theory

Thibaut Coudarchet[1], Lucien Heurtier[2] and Hervé Partouche[1]

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[1]Centre de physique théorique, École Polytechnique and CNRS
[2] Department of Physics, University of Arizona

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Introduction: Standard dark-matter freeze-out scenario

Boltzmann equation

Dark-matter number density n_{DM} is the result of two competitive effects:

- equilibrium through interactions $\mathrm{DM} + \mathrm{DM} \to \mathrm{SM} + \mathrm{SM}$
- dilution because of the universe expansion which slows the reaction

Boltzmann equation in d dimensions

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$$\frac{\mathrm{d}n_{\mathrm{DM}}}{\mathrm{d}t} + \underbrace{(d-1)Hn_{\mathrm{DM}}}_{\text{dilution}} = -\langle \sigma_{\mathrm{DM}\leftrightarrow\mathrm{SM}} v \rangle \left[n_{\mathrm{DM}}^2 - n_{\mathrm{DM,eq}}^2 \right]$$

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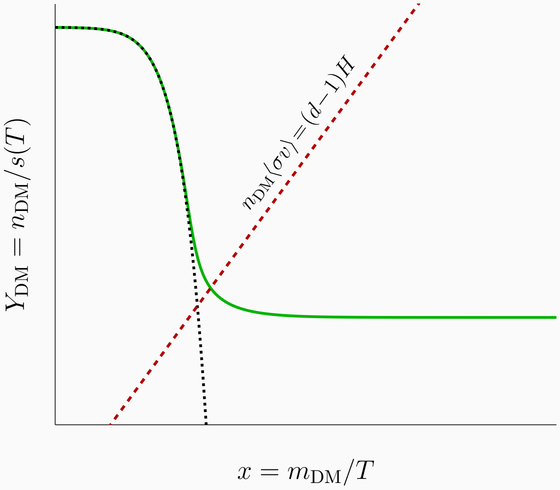
The yield

$$Y_{\mathsf{DM}} = n_{\mathsf{DM}}(T) \times \text{volume}$$

Freeze-out


$n_{\mathrm{DM}}\langle\sigma v\rangle=(d-1)H$
$Y_{\mathrm{DM}} = n_{\mathrm{DM}}/s(T)$
$x = m_{\mathrm{DM}}/T$

String theory setup

Purpose of the study

$E_8 \times E_8$ heterotic string at finite temperature with spontaneously supersymmetry (SUSY) breaking

- compute the one-loop free energy density, $\mathcal{F}$
- through the cosmological evolution: a modulus R_d first stabilized and then destabilized from self-dual point
- states initially massless acquire a mass

$$\rightarrow |R_d - 1/R_d|$$

- these states could play the role of dark matter (DM)

T drops below $m_{\text{DM}} \longleftrightarrow m_{\text{DM}}$ jumps above T

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Background manifold

$$S^1_E(R_0) \times \mathbb{R}^{d-1} \times T^2 \times T^{8-d}$$

- $S^1_E(R_0)$: compactified euclidean time, temperature $T = \frac{1}{2\pi R_0}$
- $\mathbb{R}^{d-1}$: together with time, d-dimensional spacetime
- T^2: torus with R_d and the Scherk-Schwarz radius R_9, SUSY breaking scale $M = \frac{1}{2\pi R_9}$
- T^{8-d}: rest of the internal space, volume ~ 1 in string units

T^2 metric and antisymmetric tensor

$$(G+B)_{ij} = \begin{pmatrix} R_d^2 & \epsilon \\ -\epsilon & 4R_9^2 \end{pmatrix}, \qquad i,j \in \{d, 9\},\ \epsilon \in \mathbb{Z}$$

$\to$ SUSY, $SU(2)$ enhancement at $R_d = 1$

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$(-1)^\epsilon = 1 \to 2$ fermions in $U(1)$

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Properties of the free energy $\mathcal{F}$

The radii R_0 and R_9 are large $\rightarrow$ non-trivial windings are heavy and exponentially suppressed
$\rightarrow$ only the Kaluza-Klein and Matsubara towers remain

Final result only depends on the light degrees of freedom

The mass term at one loop of $\zeta = \ln(R_d)$,

$$\frac{\zeta^2 T^{d-2}}{\pi}\left[(\tilde{n}_\mathrm{F}+\tilde{n}_\mathrm{B})\underbrace{f_\mathrm{T}(M/T)}_{\text{some function}} - (\tilde{n}_\mathrm{F}-\tilde{n}_\mathrm{B})\underbrace{f_\mathrm{V}(M/T)}_{\text{some function}}\right],$$

depends on the additional massless states

$(-1)^\epsilon = 0 \rightarrow \tilde{n}_\mathrm{B} f_\mathrm{T} + \tilde{n}_\mathrm{B} f_\mathrm{V}$ | $(-1)^\epsilon = 1 \rightarrow \tilde{n}_\mathrm{F} f_\mathrm{T} - \tilde{n}_\mathrm{F} f_\mathrm{V}$

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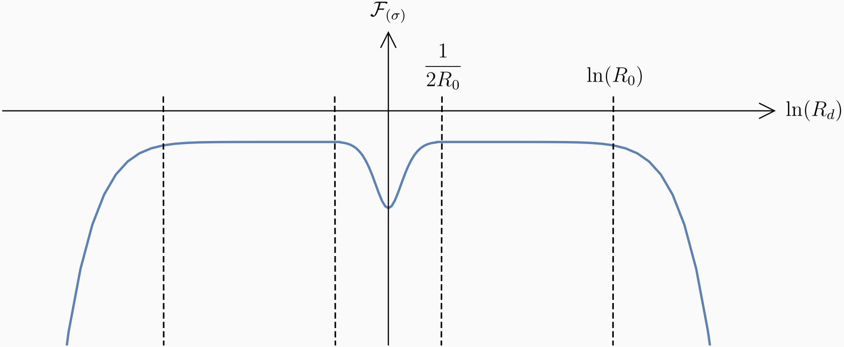
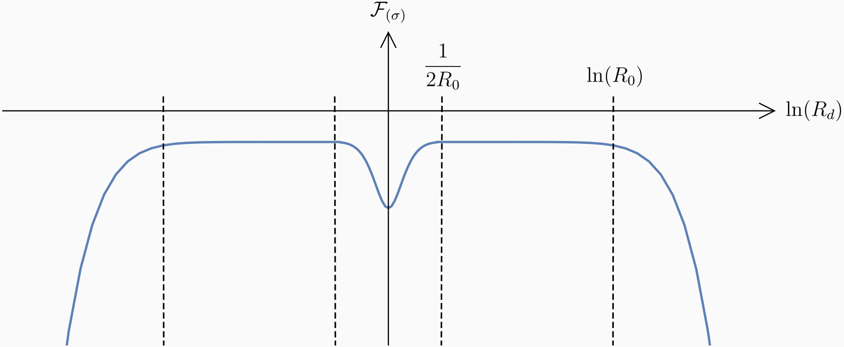
$(-1)^{\epsilon} = 0$



Properties of the free energy $\mathcal{F}$

$(-1)^{\epsilon} = 1, \frac{M}{T} < 1$



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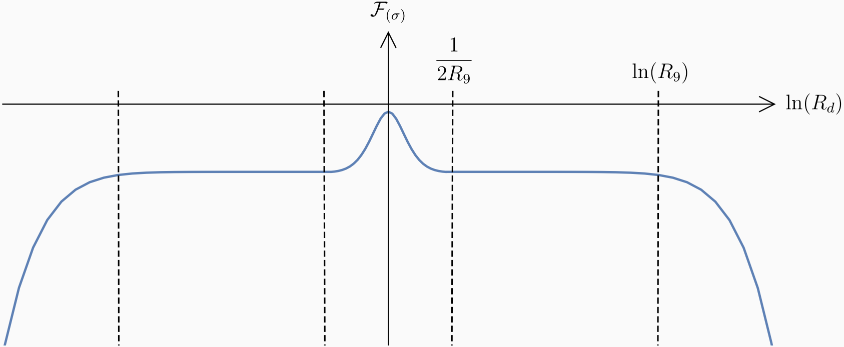

Destabilization process

- start with $M < T$
- start with R_d in the well

- R_d oscillates around 1 and the system is attracted by a critical solution with $M/T = \text{cst} = u_c$
- if $u_c < 1$, R_d stabilizes, the attractor is reached and corresponds to a radiation-like solution [Bourliot, C.K, H.P,'09] [Bourliot, J.E, C.K, H.P,'10]
- if $u_c > 1$, the well in the potential becomes a bump
- R_d becomes unstable and falls along its potential to freeze along a plateau

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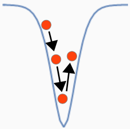

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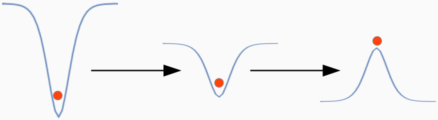

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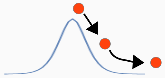

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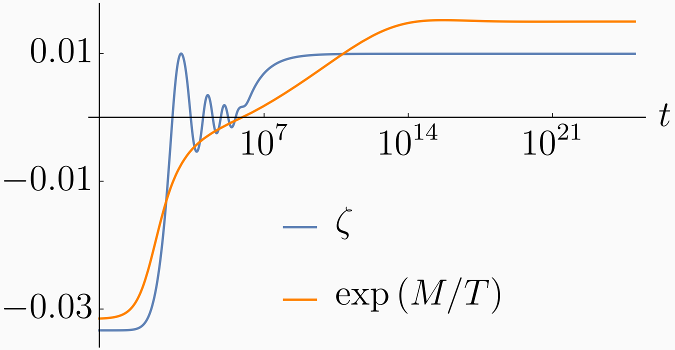
Numerical simulation



Radiation-like attractor at late times

$$\zeta \equiv \zeta_0, \quad \Phi_\perp \equiv \Phi_{\perp 0}, \quad M(t) \equiv T(t) \times u_{\mathrm{c}} \propto \frac{1}{a(t)},$$

where $a(t) \propto t^{\frac{2}{d}}$

Relic density evolution

Phase transition

m_{DM} is now driven by $\zeta = \ln(R_d)$. Before the transition, dark matter can be abundantly produced while still relativistic

Phase transition at T_c

Part of the spectrum "spontaneously" becomes non-relativistic and can freeze-out

Qualitatively:

$$m(T) = \begin{cases} 0 & \text{for } T > T_c \\ m_{DM} & \text{for } T < T_c \end{cases}$$

At the transition, $u_c = M_c/T_c$ but T_c is not determined

$\rightarrow$ different behaviors depending on $x_c = m_{DM}/T_c$

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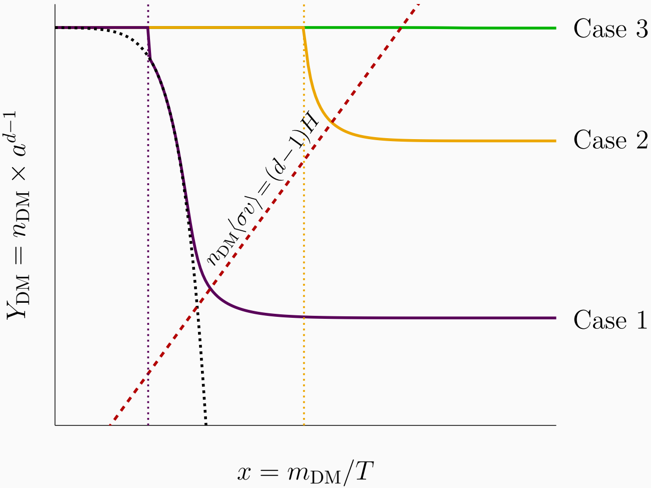
Relic density



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- we have built heterotic string models where a modulus is initially massive
- the universe is attracted by a first radiation-like evolution
- the properties of the modulus potential can make it switch from massive to tachyonic
- the destabilization of the modulus renders part of the light spectrum massive
- the universe then follows a second radiation-like solution
- the freshly created non-relativistic component of the universe density can play the role of dark matter, freeze-out and yield a relic density

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