Can you hear the shape of an axion potential?
Observing axion potentials through gravitational waves

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based on:
• JCAP 1905, no. 05, 057 (2019), with T. Fujita, M. Shiraishi

related work:
• JHEP 1612, 137 (2016), with P. Adshead, E. Martinec, M. Wyman
• JHEP 1708, 130 (2017), with P. Adshead
(Simple) Single field inflation:

- Solves horizon, flatness, monopole problems
- Explains fluctuations as stretched quantum mechanical perturbations
- Predicts a nearly scale invariant spectrum (of tunable amplitude)
- Predicts Gaussian perturbations

- Spectral index is $5\sigma$ away from flat
- Spectral index running is small
  \[ |f_{NL}| \lesssim \mathcal{O}(1) \]
Smoking Guns and Holy Grails

Can you hear an axion potential? 3/16
A pseudo-scalar (axion) field obeys a shift symmetry that protects the potential from UV sensitive terms ($\eta$ problem) 

Freese, Frieman, & Olinto 1990

A field with a shift symmetry can only couple derivatively to other degrees of freedom

$$\mathcal{L}_{\text{int}} \subset \frac{1}{f} \phi \epsilon^{\mu \nu \alpha \beta} F_{\mu \nu} F_{\alpha \beta} + \frac{C}{f} \partial_{\mu} \phi \bar{\psi} \gamma_5 \gamma^\mu \psi$$

Fermions: not efficiently produced (Pauli blocking) and quickly diluted & redshifted. However there is interesting phenomenology, e.g. leptogenesis

Adshead, EIS 2015 (JCAP & PRL)  
Adshead, Pearce, Peloso, Roberts, Sorbo 2018 & 2019

$U(1)$ gauge fields with a Chern-Simons coupling grow tachyonically towards the end of inflation.

Adshead, Giblin, Scully, EIS 2015 & 2016
Spectator Chromo-Natural Inflation

\[ S = \int d^4x \sqrt{-g} \left\{ \frac{M_{Pl}^2}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) - \frac{1}{2} (\partial \chi)^2 - U(\chi) - \frac{1}{2} Tr[F^2] - \frac{\lambda}{4f} \chi Tr[F \tilde{F}] \right\} \]

Dimastrogiovanni, Fasiello, Fujita 2016

The $SU(2)$ fields in the configuration

\[ A_0^\alpha = 0, \ A_i^\alpha = a(t) Q(t) \delta_i^\alpha \]

exhibit a scalar degree of freedom $Q(t)$

- Rotations: $A_i^\alpha \rightarrow R_{ij} \left( \hat{\theta} \right) A_j^\alpha = (\delta_{ij} + \epsilon_{ijk} \theta^k) A_j^\alpha$
- Gauge tr.: $A_i^\alpha \rightarrow U(\lambda) A_i U^{-1}(\lambda))^{\alpha} = (\delta_b^a + \epsilon_{bc}^{\alpha} \lambda^c) A_j^b$

Is this configuration stable? **YES!**

Maleknejad & Sheikh-Jabbari 2011
Background evolution

\[ \ddot{\chi} + 3H\dot{\chi} + U'(\chi) = -\frac{3g\lambda}{f}HQ^3 - \frac{3g\lambda}{f}Q^2\dot{Q} \]
\[ \ddot{Q} + 3H\dot{Q} + \dot{H}Q + 2H^2Q + 2g^2Q^3 = \frac{g\lambda}{f}Q^2\dot{\chi} \]

for \( \Lambda \equiv \frac{\lambda Q}{f} \gg 1 \) and \( m_Q \equiv \frac{gQ}{H} \gtrsim 1 \)

Introducing \( \xi \equiv \frac{\lambda \dot{\chi}}{2fH} \)

- \( \xi \sim m_Q + m_Q^{-1} \) and
- \( Q \sim \left( \frac{-fU'}{3\lambda gH} \right)^{1/3} \), \( m_Q \sim \left( \frac{-g^2fU'}{3\lambda H^4} \right)^{1/3} \)

The inflaton \( \phi \) is responsible for the scalar modes and defines \( n_s \).
The tensor modes in the metric (GW’s) are

\[ ds^2 = -dt^2 + a^2 e^{\gamma_{ij}} dx^i dx^j \]

In this case the **gauge sector has a tensor mode** too

\[ A_\mu = (0, a(t)Q(t)\delta_\alpha^i + t_\alpha^i(t, x)) \frac{\sigma_\alpha}{2} \]
Tensor fluctuations

\[ \Box h_{ij} = -16\pi G \pi_{ij} \]

- Usually we consider the **homogenous solution**, which is a manifestation of quantum gravity.
- The **inhomogenous solution** is given by sources of gravitational waves.

\[
\begin{align*}
\partial^2_\tau \gamma^\pm &+ \left( k^2 - \frac{2}{\tau^2} \right) \gamma^\pm = \mathcal{O}(\sqrt{\epsilon}) t^\pm \\
\partial_\tau^2 t^\pm &+ \left( k^2 + \frac{2m_Q\xi}{\tau^2} \right) t^\pm \pm \frac{k}{\tau} (m_Q + \xi) t^\pm = \mathcal{O}(\sqrt{\epsilon}) \gamma^\pm
\end{align*}
\]

\begin{itemize}
    
    
\end{itemize}

Parity violation in \( t^\pm \)

unstable for \( m_Q + \xi - \sqrt{m_Q^2 + \xi^2} < -k\tau < m_Q + \xi + \sqrt{m_Q^2 + \xi^2} \)
The GW’s are given by:

$$\mathcal{P}_h^{(s)} \simeq \frac{m_Q^4 H^4}{\pi^2 g^2 M_{Pl}^4} \exp[2\alpha m_Q]$$

controlled by $m_Q \simeq \left(\frac{-g^2 f U'}{3\lambda H^4}\right)^{1/3}$

<table>
<thead>
<tr>
<th>Potential type</th>
<th>Sample potential</th>
<th>GW template</th>
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<tbody>
<tr>
<td>I: convex / concave</td>
<td>$U(\chi) \propto \chi^p$</td>
<td>$\mathcal{P}<em>h^{(s)}(k) \propto \left(\frac{k}{k</em>*}\right)^{n_T}$</td>
</tr>
<tr>
<td>II: one inflection point</td>
<td>$\left[1 - \cos\left(\frac{\chi}{f}\right)\right]^{p/2}$</td>
<td>$\mathcal{P}<em>h^{(s)} \propto \exp\left[-\frac{\ln^2(k/k</em>*)}{2\sigma_h^2}\right]$</td>
</tr>
<tr>
<td>III: axion monodromy</td>
<td>$\chi^p + \delta \cos(\nu \chi)$</td>
<td>$1 + A \sin\left[C \ln\left(\frac{k}{k_*}\right) + \theta\right]$ for $p = 1$ &amp; $A \ll 1$</td>
</tr>
</tbody>
</table>
$U \propto \chi^p \Rightarrow n_T = (p - 1) \frac{2\alpha m_* + 4}{\Delta N}$

$U \propto [1 - \cos(\chi/f)]^{p/2}$

Around the inflection point the GW spectrum has a Gaussian shape

$$P_h^{(s)} \sim A_h \exp \left[ - \frac{\ln^2 (k/k_*)}{2\sigma_h^2} \right]$$

Kobayashi, Oikawa & Otsuka 2016
Consider a modulated potential $U(\chi) = \mu^4 \left[ \left| \frac{\chi}{f} \right|^p + \delta \cos \left( \frac{\kappa \chi}{\delta f} \right) \right]$. 

- Decompose the background quantities $\chi, Q, \ldots$ to extract the oscillatory part $\chi = \chi_0 + \chi_{\text{osc}}, \; Q = Q_0 + Q_{\text{osc}}$ etc.
Use the **oscillatory background functions** to compute the power spectrum.

We **analytically** capture the **period** and the **modulation amplitude** of GW’s for small / intermediate values of the modulation frequency \( \omega = \frac{2\kappa}{\delta \lambda} \xi_0 \).

\[
\mathcal{P}_h^{(s)}(k) \simeq \frac{H^4 m_0^4}{\pi^2 g^2 M_{Pl}^4} \left[ 1 + 2(\alpha m_0 + 2)\Delta \cdot \sin(\omega H t) \right] e^{2\alpha m_0}
\]
\[ \mathcal{P}_h^{(s)}(k) = \mathcal{P}_h^{(0)} \left[ 1 + A \sin(C \ln \frac{k}{k_*} + \theta) \right] \]

with \( A = 0.9, 0.3, 0.1 \)

In principle **detectable modulation** for \( A \gtrsim 0.3 \)
Is a blue tensor tilt a smoking gun for String Cosmology?

SU(2) fields can lead to blue-tilted tensor modes!

We did not ask for $n_T > 0$, it just happened . . .

We need better ballistics!
Standard Assumption:
A detection of tensor modes reveals the **energy scale** of inflation

Observable gravity waves can be produced at a (much) lower inflationary scale than naively estimated.
Experimental roadmap

First of all:
Discover **tensor modes** & make sure they’re truly **primordial**

1. Measure the **tensor spectrum**: scale-invariant, red or blue?
3. Measure tensor **non-Gaussianity**

<table>
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<th>If:</th>
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<tbody>
<tr>
<td>1. scale-invariant</td>
<td>1. scale-invariant or not</td>
</tr>
<tr>
<td>2. non-chiral</td>
<td>2. parity-violating</td>
</tr>
<tr>
<td>3. Gaussian</td>
<td>3. non-Gaussian</td>
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⇒ **vacuum modes**
⇒ **sourced primordial GW’s**

If in addition we detect **modulated** B-modes
⇒ **String Phenomenology** takes off
Thank you . . .

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Questions