Discrete Gauge Symmetries from Orbifold GUTs

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Motivation and Outline

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Discrete symmetries play an important role in successful (string) model building as they address

μ-problem, $\mathcal{CP}$-violation, proton decay ...

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Outline

- Toy model: six-dimensional gauge theories on $T^2/\mathbb{Z}_N$ orbifolds
- Compatibility of remnant symmetries and boundary conditions
- Examples:
  - D-parity in Pati–Salam models from $\mathrm{SO}(10)$
  - (early versions of) non-Abelian flavor symmetries
- Comment on systematic origin from the Weyl group

Six-dimensional gauge theory on $T^2/\mathbb{Z}_N$ orbifold

Decomposition of the gauge field into Cartans and ladder operators

$$
\begin{aligned}
V^M(x,y) \; &= \; \sum_I V_I^M(x,y)\, H_I + \sum_{w\in W} V_w^M(x,y)\, E_w \\
&= V^\mu(x,y) \; \oplus \; \chi^{1,2}(x,y)
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Choose geometric twist and an action in the gauge space

$$y \;=\; \vartheta\, y\;, \qquad P \;=\; \exp\left(2\pi\mathrm{i}\, V\cdot H\right)$$

Boundary conditions

$$V^\mu(x,\vartheta\, y) \;=\; P\, V^\mu(x,y)\, P^{-1}$$

In terms of the Lie algebra

$$\begin{aligned} V_I^\mu(x,\vartheta\, y) &= V_I^\mu(x,y)\;, \\ V_w^\mu(x,\vartheta\, y) &= \exp\left(2\pi\mathrm{i}\, V\cdot w\right)\, V_w^\mu(x,y) \end{aligned}$$

Residual symmetries and boundary conditions

Interchanging the order of a symmetry transformation and the orbifold action must not make a difference

$$
\begin{array}{ccc}
V_a^\mu(x,y)\,\mathsf{T}_a & \overset{O}{\longmapsto} & V_a^\mu(x,\vartheta^{-1}\,y)\,P\,\mathsf{T}_a\,P^{-1} \\
 & & \big\downarrow \mathcal{G} \\
\Big\downarrow \mathcal{G} & & V_a^\mu(x,\vartheta^{-1}\,y)\,U\,P\,\mathsf{T}_a\,P^{-1}\,U^{-1} \\
 & & \overset{!}{=} \\
V_a^\mu(x,y)\,U\,\mathsf{T}_a\,U^{-1} & \overset{O}{\longmapsto} & V_a^\mu(x,\vartheta^{-1}\,y)\,P\,U\,\mathsf{T}_a\,U^{-1}\,P^{-1}
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\end{array}
$$

Hence, we arrive at the condition for U

$$
P^{-1}\,U^{-1}\,P\,U\,\mathsf{T}_a \stackrel{!}{=} \mathsf{T}_a\,P^{-1}\,U^{-1}\,P\,U
$$

Consequences of Schur's Lemma

Schur's Lemma tells us that if a matrix commutes with all generators of an irreducible representation, it must be proportional to the identity

$$P^{-1}\,U^{-1}\,P\,U \;=: [P, U] \;= \omega^k \mathbb{1}$$

As $P^N = \mathbb{1}$, ω must be (a power of) an N-th root of unity.

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However, $P^{-1}\,U^{-1}\,P\,U$ must also be an element of the group $\mathcal{G}$ (and, as it's diagonal)

$$P^{-1}\,U^{-1}\,P\,U \;\in Z(\mathcal{G})$$

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  Example: $\mathbf{SU}(\boldsymbol{M})$ gauge theory on a $\mathbb{Z}_{\boldsymbol{N}}$ orbifold:
  $Z(\mathrm{SU}(M)) = \mathbb{Z}_M$, hence M and N must not be coprime

D-parity in Pati–Salam GUTs from SO(10)

Consider an $\mathrm{SO}(10)$ GUT on a $T^2/\mathbb{Z}_2$ orbifold with $P = \mathrm{diag}(-\mathbb{1}_6; \mathbb{1}_4)$

The action of P breaks $\mathrm{SO}(10)$ to the Pati–Salam group

$$\mathrm{SO}(10) \to \mathrm{SU}(4) \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$$

Observe

- no solutions for $k = 1$
- for $k = 0$, there is the element

$$D \;=\; \mathrm{diag}(-1, 1, 1, 1, 1, 1; 1, -1, -1, -1)$$

that fulfills $[P, D] = \mathbb{1}$ and acts as

$$D: \;(\mathbf{4}, \mathbf{2}, \mathbf{1}) \;\leftrightarrow\; (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$$

Hall, Harigaya '19

Weyl group

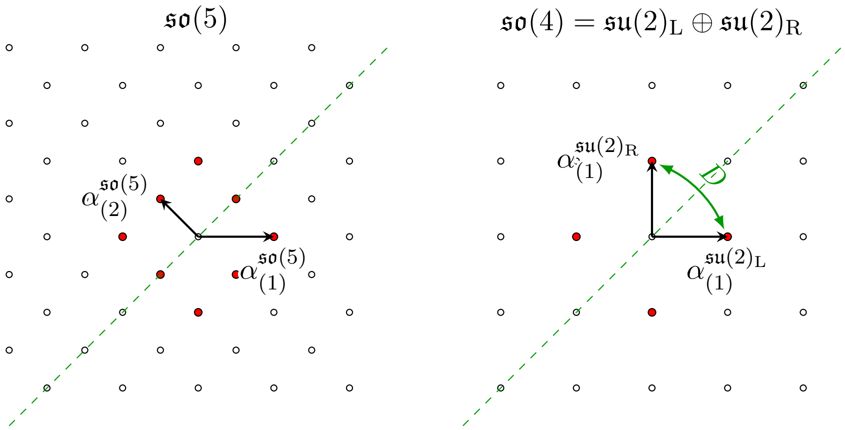
We'd like to understand where the residual discrete symmetries originate from. Possible candidates are the elements of the Weyl group of the upstairs gauge symmetry

$$w_\alpha(V) \;=\; V - 2\,\frac{V\cdot\alpha}{\alpha\cdot\alpha}\,\alpha\;,$$

but are *not* elements of the Weyl group of the residual gauge symmetry, hence

Weyl reflections at roots that are broken by the orbifolding.

D-parity: relation to Weyl group



Root lattices of $\mathfrak{so}(5)$ and its $\mathfrak{so}(4) = \mathfrak{su}(2)_{\mathrm{L}} \oplus \mathfrak{su}(2)_{\mathrm{R}}$ subalgebra.

Non-Abelian flavor symmetries – an $\mathrm{SU}(3)$ derived example

In a $T^2/\mathbb{Z}_3$ orbifold, choose $\mathcal{G} = \mathrm{SU}(3)$ and $P = \mathrm{diag}(\omega^2, \omega, 1)$.

$\boldsymbol{k = 0}$

One finds that $\begin{pmatrix} \mathrm{e}^{\mathrm{i}(\alpha+\beta)} & & \\ & \mathrm{e}^{\mathrm{i}(\alpha-\beta)} & \\ & & \mathrm{e}^{-2\mathrm{i}\alpha} \end{pmatrix}$ commutes with P,

$\Rightarrow \mathrm{U}(1) \times \mathrm{U}(1)$ gauge symmetry.

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$\boldsymbol{k = 1, 2}$

There are solutions for non-trivial k. In particular we find

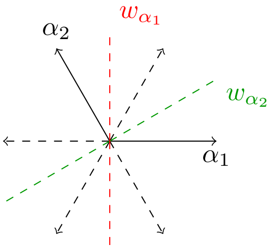
$$[P, U] = \omega \mathbb{1} \quad \text{for} \quad U = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

hence, $\mathrm{SU}(3)$ is broken to $[\mathrm{U}(1) \times \mathrm{U}(1)] \rtimes \mathbb{Z}_3$

$\mathrm{SU}(3) \to [\mathrm{U}(1) \times \mathrm{U}(1)] \rtimes \mathbb{Z}_3$

Again, the residual symmetry can be understood in terms of the $\mathrm{SU}(3)$ Weyl group

$$U \;=\; w_{\alpha_2} \circ w_{\alpha_1}$$



In a full theory containing twisted matter, the $\mathbb{Z}_3$ can be chosen to permute the fixed points $\to$ flavor symmetry

Beye-Kobayashi-Kuwakino '14

Conclusions

- we have studied discrete symmetries that arise as discrete remnants of a gauge symmetry that is broken by orbifolding
- for a boundary condition P, we have shown that a residual symmetry U must fulfill

$$P^{-1}\,U^{-1}\,P\,U \;=\; \omega^k\,\mathbb{1}$$

- we have discussed when and how discrete symmetries can survive the orbifold
- connection to the Weyl group of the unbroken gauge symmetry