Doublet-Triplet Splitting and Fertile Left-Right Symmetric Heterotic String Vacua

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Aims

- Plan for talk:
  1. Motivation
  2. Introduce free fermionic Left-Right Symmetric (LRS) models.
  3. Review results of LRS classification
  4. Outline fertility conditions in LRS models.
  5. Discuss Doublet-Triplet splitting in these models
  6. Present results from fertility analysis

Motivation

- String theory ought to reproduce Standard Model at low energies.
- Huge number of vacua in four dimensions.
- Classify vacua and identify general features of (quasi-)realistic vacua.
- Phenomenological requirements:
    1. $\mathcal{N} = 1$ SUSY
    2. $SO(10)$ GUT with 3 generations in **16** rep
    3. Higgs particles
    4. Top quark mass coupling
    5. Generation mass hierarchy, Seesaw mechanism, proton stability
    6. ...

Free Fermion Construction I

- $D = 4 \implies$ introduction of free fermions on worldsheet:

$$\{\underbrace{\psi^{\mu}}_{\substack{\text{S'partners}\\ \text{of } X^{\mu}}}, \underbrace{\chi^{i}}_{\substack{\text{S'partners to}\\ \text{six compactified}\\ \text{dimensions}}}, \underbrace{y^{i}, w^{i} \,\|\, \bar{y}^{i}, \bar{w}^{i}}_{\substack{\text{“compactified”}\\ \text{directions}}}, \underbrace{\bar{\psi}^{1,2,3,4,5}, \bar{\eta}^{1,2,3}}_{\text{observable G. G.}}, \underbrace{\bar{\phi}^{1,2,3,4,5,6,7,8}}_{\substack{\text{rank 8}\\ \text{Hidden G. G.}}}\} \tag{1}$$

$i = 1, \dots, 6$

- Partition function modular invariance sufficiently encoded at one-loop, i.e. a torus:

$$f_i \to -e^{i\pi\alpha(f_i)} f_i, \qquad \alpha(f) \in (-1, +1] \tag{2}$$

$$Z = \sum_{\substack{\text{All Spin}\\ \text{structures}}} C\begin{pmatrix} v_i \\ v_j \end{pmatrix} Z\begin{pmatrix} v_i \\ v_j \end{pmatrix} \tag{3}$$

Free Fermion Construction II

- Model defined through:
  1. Basis vectors:
  $$v_i = \{\alpha(f_1), \alpha(f_2), ..., \alpha(f_N)\}, \tag{4}$$
  2. GGSO phases:
  $$C\begin{pmatrix} v_i \\ v_j \end{pmatrix} = \pm 1 \text{ or } \pm i, \quad i > j \tag{5}$$

  $2^{\frac{N(N-1)}{2} - \#\text{constraints}}$: 'ABK rules'.

- GSO projection:
$$e^{i\pi v_i \cdot F_\alpha} |S_\alpha\rangle = \delta_\alpha C\begin{pmatrix} \alpha \\ v_i \end{pmatrix}^* |S_\alpha\rangle \tag{6}$$

References:

I. Antoniadis and C. Bachas, Nuclear Physics B, 298(3):586 - 612, 1988. I. Antoniadis and C. Bachas, and C. Kounnas, Nuclear Physics B, 289(0):87 - 108, 1987.

$\mathbb{Z}_2 \times \mathbb{Z}_2$ $SO(10)$ heterotic models

- Basis vectors:

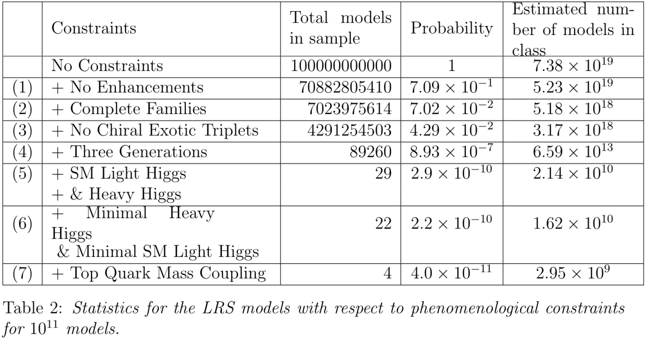
$$\begin{aligned}
&\mathbf{1} = \{\text{ALL}\}\} \quad \text{None transform}\\
&\mathbf{S} = \{\psi^\mu, \chi^{1,\dots,6}\}\} \quad \text{SUSY generator}\\
&\mathbf{e_i} = \{y^i, w^i | \bar{y}^i, \bar{w}^i\}, \quad i = 1,\dots 6\} \quad \text{Internal symmetric shifts}\\
&\left.\begin{aligned}
\mathbf{b_1} &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,..,5}\}\\
\mathbf{b_2} &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,..,5}\}
\end{aligned}\right\} \quad \mathbb{Z}_2 \text{ twists}\\
&\left.\begin{aligned}
\mathbf{z_1} &= \{\bar{\phi}^{1234}\}\\
\mathbf{z_2} &= \{\bar{\phi}^{5678}\}
\end{aligned}\right\} \quad SO(8) \times SO(8) \text{ hidden}
\end{aligned}$$

- $SO(10) \times U(1)^3 \times SO(8) \times SO(8)$ gauge group.

- α vector breaks $SO(10)$ to LRS:

$$\alpha(\bar{\psi}^{1,\dots,5}) = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}00\} \implies SU(3) \times SU(2)^2 \times U(1) \qquad (7)$$

Highlights Left-Right Symmetric Classification



	Constraints	Total models in sample	Probability	Estimated number of models in class
	No Constraints	100000000000	1	7.38×10^{19}
(1)	+ No Enhancements	70882805410	7.09×10^{-1}	5.23×10^{19}
(2)	+ Complete Families	7023975614	7.02×10^{-2}	5.18×10^{18}
(3)	+ No Chiral Exotic Triplets	4291254503	4.29×10^{-2}	3.17×10^{18}
(4)	+ Three Generations	89260	8.93×10^{-7}	6.59×10^{13}
(5)	+ SM Light Higgs + & Heavy Higgs	29	2.9×10^{-10}	2.14×10^{10}
(6)	+ Minimal Heavy Higgs & Minimal SM Light Higgs	22	2.2×10^{-10}	1.62×10^{10}
(7)	+ Top Quark Mass Coupling	4	4.0×10^{-11}	2.95×10^{9}

Table 2: *Statistics for the LRS models with respect to phenomenological constraints for* 10^{11} *models.*

- Proliferation of exotic states
- Good models rare (as in SLM) $\Longrightarrow$ fertility conditions

Reference:

A. E. Faraggi, G. Harries, J. Rizos (2018) arXiv:1806.04434

Classification of SO(10) Models: Spinorials

Observable Sectors (twisted)

- **16**'s / $\overline{\mathbf{16}}$'s of $SO(10)$ (48 possible sectors):

$$\begin{aligned} B^{(1)}_{pqrs} &= \mathbf{S} + \mathbf{b_1} + p\mathbf{e_3} + q\mathbf{e_4} + r\mathbf{e_5} + s\mathbf{e_6} && (8)\\ &= \{\psi^\mu, \chi^{1,2}, (1-p)y^3\bar{y}^3, pw^3\bar{w}^3, (1-q)y^4\bar{y}^4, qw^4\bar{w}^4, \\ &\quad (1-r)y^5\bar{y}^5, rw^5\bar{w}^5, (1-s)y^6\bar{y}^6, sw^6\bar{w}^6, \bar{\eta}^1, \bar{\psi}^{12345}\} \\ B^{(2)}_{pqrs} &= \mathbf{S} + \mathbf{b_2} + p\mathbf{e_1} + q\mathbf{e_2} + r\mathbf{e_5} + s\mathbf{e_6} && (9)\\ B^{(3)}_{pqrs} &= \mathbf{S} + \mathbf{b_3} + p\mathbf{e_1} + q\mathbf{e_2} + r\mathbf{e_3} + s\mathbf{e_4} && (10) \end{aligned}$$

where $p, q, r, s \in \{0, 1\}$ and:

$$\mathbf{b_3} = \mathbf{b_1} + \mathbf{b_2} + \mathbf{x} \text{ and } \mathbf{x} = \mathbf{1} + \mathbf{S} + \sum_{i=1}^{6} \mathbf{e_i} + \sum_{k=1}^{2} \mathbf{z_k} = \{\bar{\eta}^{123}, \bar{\psi}^{1,\ldots,5}\}. \quad (11)$$

Classification of SO(10) Models: Vectorials

- Vectorial **10**'s:

$$
\begin{aligned}
V^{(1)}_{pqrs} &= \mathbf{S} + \mathbf{b_1} + p\mathbf{e_3} + q\mathbf{e_4} + r\mathbf{e_5} + s\mathbf{e_6} + \mathbf{x} \\
&= \{\psi^{\mu}, \chi^{1,2}, (1-p)y^3\bar{y}^3, pw^3\bar{w}^3, (1-q)y^4\bar{y}^4, qw^4\bar{w}^4, \qquad (12) \\
&\quad (1-r)y^5\bar{y}^5, rw^5\bar{w}^5, (1-s)y^6\bar{y}^6, sw^6\bar{w}^6, \bar{\eta}^{2,3}\} \\
V^{(2,3)}_{pqrs} &= B^{(2,3)}_{pqrs} + \mathbf{x} \qquad (13)
\end{aligned}
$$

- $M = 0$ states require Neveu-Schwarz right-moving fermionic oscillator.

- Analyzing GSO projections allows us to determine $SO(10)$ numbers relating to the number and type of **16**'s, $\overline{\mathbf{16}}$'s and **10**'s.

Doublet-Triplet Splitting

- Can show:

$$C\begin{pmatrix} V^I \\ x \end{pmatrix} = \begin{cases} +1 & \implies \quad \{\overline{\psi}^{(*)4,5}\}\,|R\rangle \text{ bidoublet survives} \\ & \\ -1 & \implies \quad \{\overline{\psi}^{(*)1,2,3}\}\,|R\rangle \text{ triplet survives} \end{cases} \tag{14}$$

- Stringy doublet-triplet splitting mechanism from twisted sector
- $\{\overline{\psi}^{(*)1,2,3}\}\,|R\rangle_{V^I} \leftrightarrow \begin{cases} \text{Colour triplets (a.k.a 'leptoquarks') mediate} \\ \text{proton decay via dimension 5 operators} \end{cases}$
- $\implies$ Sufficiently heavy or must be projected out
- $\{\overline{\psi}^{(*)4,5}\}\,|R\rangle_{V^I} \leftrightarrow (\mathbf{1}, 0, \mathbf{2}, \mathbf{2})$: light Standard Model Higgs.

Fertility Conditions and TQMC

- At $SO(10)$ level we demand:

$$\begin{aligned} &\mathbf{N}_{\text{doublets}} > \mathbf{1} \\ &\mathbf{N_L} - \overline{\mathbf{N}}_\mathbf{L} \geq \mathbf{6}, \quad \mathbf{N_R} - \overline{\mathbf{N}}_\mathbf{R} \geq \mathbf{6}, \quad \overline{\mathbf{N}}_\mathbf{R} \geq \mathbf{1} \end{aligned} \tag{15}$$

- Implement Top Quark Mass Coupling conditions at $SO(10)$ level:

$$\begin{gathered} C\binom{b_1}{e_{1,2}} = C\binom{b_1}{e_2} = C\binom{b_2}{e_3} = C\binom{b_2}{e_4} = C\binom{b_1}{z_{1,2}} = C\binom{b_2}{z_{1,2}} = 1 \\ C\binom{b_1}{e_5} = C\binom{b_2}{e_5}, \quad C\binom{b_1}{e_6} = C\binom{b_2}{e_6} \end{gathered} \tag{16}$$

- $\implies$ $SO(10)$ ‘fertile cores’. Then do full classification in α phases

Reference:

J. Rizos, *Eur. Phys. Jour.* **C74** (2014) 2905

	Constraints	Total models in sample	Probability
	No Constraints	4915200	1
(1)	+ No Observable Enhancements	4539268	0.92
(2)	+ Anomaly Free	438913	8.9×10^{-2}
(3)	+ Three Generations	352561	7.2×10^{-2}
(4)	+ SM Light Higgs + & Heavy Higgs	257892	5.2×10^{-2}
(5)	+ Minimal Heavy Higgs & Minimal SM Light Higgs	72704	1.4×10^{-2}
(6)	+ Top Quark Mass Coupling	28672	5.8×10^{-3}

Table: *Statistics for the LRS models derived from fertile cores*

- Probability of a‘good’ model increased by ∼ 8 orders of magnitude compared to random classification methodology.

Thanks for listening! ☺