# Scaling Limits of dS Vacua

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# Overview

We investigated massive type IIA flux compactifications with different ingredients for the possibility of a realistic setup for dS vacua.

- In order to trust a dS solution in the SUGRA limit we need **large volume** and **weak coupling**.
- Solutions often have **remaining scaling symmetries** that, in principle, allow to move a solution from large coupling/small volume to a trustable regime.
- **Different ingredients**, like D-branes, O-planes, KK-monopoles, etc., have different scaling behavior.

### Motivation

Lately there has been ample discussion about dS Vacua in String Theory. (Danielsson, Van Riet: **arXiv:1804.01120**; Obied et.al.: **arXiv:1806.08362**)

Based on **arXiv:1811.07880** with Andreas Banlaki, Abhishek Chowdhury, Timm Wrase and on similar work by Daniel Junghans in **arXiv:1811.06990**.

# dS Vacua from String Theory

It is notoriously difficult to get dS vacua from String Theory.

- Best known construction: **KKLT** (Kachru et.al.: **arXiv:hep-th/0301240**). While well established there is no general consensus that these are completely consistent.
- Flux compactifications of massive type IIA don't allow for dS vacua (Maldacena, Núñez: **arXiv:hep-th/0007018**) unless we **include O6-planes**.
- **Even more no-go's exist!** Wrase, Zagermann: **arXiv:1003.0029**; Haque et.al.: **arXiv:0810.5328**; Andriot Blåbäck: **arXiv:1609.00385**; Andriot: **arXiv:1807.09698**
  $\rightarrow$ **All can be evaded!**
- Solutions often have strong coupling and small volume and thus should receive $\alpha'$ and/or string loop corrections.

**Question**

Can we in principle find parametrically controlled solutions of massive type IIA compactifications at weak coupling and large volume?

# The Scalar Potential

The full scalar potential is rather complicated but for the slice of the **string coupling** $e^{\phi}$ and **internal volume** $\mathcal{V}$ we can write it down using:

$$\rho = \mathcal{V}_6^{\frac{1}{3}} \qquad \text{and} \qquad \tau = e^{-\phi}\sqrt{\mathcal{V}_6}\,,$$

we get schematically:

$$V(\rho,\tau) = \frac{A_H}{\rho^3\tau^2} + \sum_{p=0,2,4,6} \frac{A_{F_p}}{\rho^{p-3}\tau^4} - \frac{A_{sources}}{\tau^3} + \frac{A_{R_6}}{\rho\tau^2}\ .$$

- $A_H$ arises from integrating the NSNS field strength $|H|^2$.
- $A_{F_p}$ corresponds to the RR-fluxes $|F_p|^2$.
- $A_{sources}$ includes contributions from $D6$, $\overline{D6}$ and $O6$. $A_{sources}$ depends on them like:

$$A_{sources} \propto 2N_{O6} - N_{D6} - N_{\overline{D6}}.$$

- $A_{R_6} \propto -R_6$ comes from internal Ricci scalar.

# A Maldacena-Núñez Type No-Go

In the case where $N_{O6} = 0$ or $2N_{O6} - N_{D6} - N_{\overline{D6}} < 0$ we minimize w.r.t. $\tau$:

$$\partial_\tau V = -2\frac{A_H}{\rho^3\tau^3} - 4\sum_{p=0,2,4,6}\frac{A_{F_p}}{\rho^{p-3}\tau^5} - 3\frac{|A_{sources}|}{\tau^4} - 2\frac{A_{R_6}}{\rho\tau^3} \stackrel{!}{=} 0.$$

For negatively curved manifolds $A_{R_6} \propto -R_6 > 0$ and there is no solution. For positively curved manifolds we find a dS no-go (Maldacena, Núñez: **arXiv:hep-th/0007018**):

$$V|_{min} = -\sum_{p=0,2,4,6}\frac{A_{F_p}}{\rho^{p-3}\tau^4} - \frac{1}{2}\frac{|A_{sources}|}{\tau^3} < 0.$$

$\Rightarrow$ No dS solutions exist for $N_{D6} + N_{\overline{D6}} > 2N_{O6}$.

**More No-Go's**

For de Sitter it is always required that $\mathbf{A_{R_6} < 0}$ and $\mathbf{A_{F_0} \neq 0}$!

Hertzberg et.al.: **arXiv:0711.2512**; Flauger et.al.: **arXiv:0812.3886**

# Controlled SUSY AdS

In **arXiv:hep-th/0505160** DeWolfe et.al. showed that $F_4$ flux can be made arbitrary large. If all $F_4$ fluxes ($f_4$) are large one finds a SUSY AdS solution with weak string couling $e^{-\phi} \propto (f_4)^{\frac{3}{4}}$ and large volume $\mathcal{V} \propto (f_4)^{\frac{3}{2}}$.

In our case ($A_{R_6} = 0$, $A_{F_4} = a_{F_4}(f_4)^2$, $\rho = \tilde{\rho}(f_4)^{\frac{1}{2}}$, $\tau = \tilde{\tau}(f_4)^{\frac{3}{2}}$):

$$V(\rho,\tau) = \frac{1}{(f_4)^{\frac{9}{2}}}\left(\frac{A_H}{\tilde{\rho}^3\tilde{\tau}^2} + \frac{A_{F_0}}{\tilde{\rho}^{-3}\tilde{\tau}^4} + \frac{a_{F_4}}{\tilde{\rho}\tilde{\tau}^4} - \frac{A_{sources}}{\tilde{\tau}^3}\right) + \frac{1}{(f_4)^{\frac{11}{2}}}\frac{A_{F_2}}{\tilde{\rho}^{-1}\tilde{\tau}^4} + \frac{1}{(f_4)^{\frac{15}{2}}}\frac{A_{F_6}}{\tilde{\rho}^3\tilde{\tau}^4}$$

$\Rightarrow$ $F_2$ and $F_6$ become irrelevant in this limit.

- All terms necessary to stabilize moduli survive.
- $N_{O6}$, $H$ and $F_0$ can be small and we still have SUSY AdS.
- Non-vanishing curvature will prohibit the large $F_4$ limit.

# Controlled dS Vacua

**Can we find a limit with dS vacua for $\rho, \tau \gg 1$?**

The large $F_4$ limit does not work due to non-vanishing curvature.
$\Rightarrow$ We need to investigate other limits of the scalar potential

$$V(\rho,\tau) = \frac{A_H}{\rho^3\tau^2} + \sum_{p=0,2,4,6} \frac{A_{F_p}}{\rho^{p-3}\tau^4} - \frac{A_{sources}}{\tau^3} + \frac{A_{R_6}}{\rho\tau^2} \ .$$

Let's make both moduli simultaneously large: $\rho \propto \lambda^{c_\rho}, \tau \propto \lambda^{c_\tau}$, $\lambda \to \infty$.

$$V(\rho,\tau) = \frac{A_H}{\lambda^{3c_\rho+4c_\tau}} + \sum_{p=0,2,4,6} \frac{A_{F_p}}{\lambda^{(p-3)c_\rho+4c_\tau}} - \frac{A_{sources}}{\lambda^{3c_\tau}} + \frac{A_{R_6}}{\lambda^{c_\rho+2c_\tau}} \ .$$

# Controlled dS Vacua

**Can we find a limit with dS vacua for $\rho, \tau \gg 1$?**

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In the $\lambda \to \infty$ limit, with fixed $A_{sources}, A_{F_0}$ and $A_{R_6}$, we find, in order to have these terms scale the same:

$$3c_\tau = 4c_\tau - 3c_\rho = 2c_\tau + c_\rho\,.$$

**Only solution: $\mathbf{c_\tau = c_\rho = 0}$. Volume and dilation do not scale!**

⇒ No dS vacua at parametically large volume and weak string coupling for $A_{sources}, A_{F_0}$ and $A_{R_6}$ fixed

# Controlled dS Vacua - Scaling Fluxes/Sources

**Scaling the flux/source terms in the potential might allow for dS.**

- Terms in the scalar potential are quadratic in flux quanta (Herraez et.al.: **arXiv:1802.05771**).
- Exception: $A_{sources} \propto -2N_{O6} + N_{D6} + N_{\overline{D6}}$.

The tadpole condition reads schematically:

$$\sqrt{2}\int(\omega \cdot F_2 + F_0 H) = -2N_{O6} + N_{D6} + N_{\overline{D6}}\,,$$

meaning that, since $N_{O6}$ is fixed by a particular orbifold projection, the fluxes cannot become arbitrary large.

**There is a loophole if the two terms on the left hand side almost cancel, even if they are very large!**

# Controlled dS Vacua - Scaling Fluxes/Sources

In our scalar potential:

$$V(\rho,\tau) = \frac{A_H}{\rho^3\tau^2} + \sum_{p=0,2,4,6} \frac{A_{F_p}}{\rho^{p-3}\tau^4} - \frac{A_{sources}}{\tau^3} + \frac{A_{R_6}}{\rho\tau^2} \ ,$$

we now also let $A_H \propto \lambda^{c_H}$, $A_{F_p} \propto \lambda^{c_{F_p}}$ and $A_{R_6} \propto \lambda^{c_{R_6}}$ and still require that the terms of $A_{sources}$, $A_{R_6}$ and $A_{F_0}$ in $V$ scale the same.

There is still no solution for dS vacua in this case!

Since the fluxes are bound from below the solution from this case implies that the volume has to shrink in the scaling limit.

# Controlled dS Vacua - Scaling Fluxes/Sources

**What if we allow for more O6-planes?**

In simple compactifications this number is usually small and fixed but one can imagine compactifications where the number could be large.
Letting $A_{sources} \propto \lambda^{c_s}$ we find from the scalar potential

$$V(\rho,\tau) = \frac{A_H}{\rho^3\tau^2} + \sum_{p=0,2,4,6} \frac{A_{F_p}}{\rho^{p-3}\tau^4} - \frac{A_{sources}}{\tau^3} + \frac{A_{R_6}}{\rho\tau^2}$$

$$\Rightarrow -3c_\rho + 4c_\tau - c_{F_0} = 3c_\tau - c_s = c_\rho + 2c_\tau - c_{R_6}$$

and from the tadpole condition $\sqrt{2}\int(\omega \cdot F_2 + F_0 \cdot H) = -2N_{O6} + N_{D6} + N_{\overline{D6}}$ we get

$$\frac{1}{2}\left(c_{R_6} + c_{F_2}\right) = \frac{1}{2}\left(c_{F_0} + c_H\right) = c_s\,.$$

This can be solved!

# Controlled dS Vacua - Scaling Fluxes/Sources

The volume and string coupling are related to $\rho$ and $\tau$ via

$$\rho = \mathcal{V}_6^{\frac{1}{3}} \qquad \text{and} \qquad \tau = e^{-\phi}\sqrt{\mathcal{V}_6}\,.$$

For the previous solution this leads to

$$\mathcal{V}_6 \propto N_{O6}^3 \qquad \text{and} \qquad e^{-\phi} \propto \sqrt{N_{O6}}.$$

Controlled dS vacua are in principle possible for compactifications with a large amount of $O6$-planes.

**Note:** Such compactifications are currently not know.

# Other Ingredients

There are many different ingredients one can add, for example:

- KK-monopoles: $V_{KK} = \frac{A_{KK}}{\rho\tau^2}$
- KKO-planes: $V_{KKO} = -\frac{A_{KKO}}{\rho\tau^2}$
- NS5-branes: $V_{NS5} = \frac{A_{NS5}}{\rho^2\tau^2}$
- NSO5-planes: $V_{NSO5} = -\frac{A_{NSO5}}{\rho^2\tau^2}$

Adding these will make the compactification more complicated, introducing new backreaction effects and potential new degrees of freedom.

# KK-monopoles

Adding KK-monopoles and their corresponding planes is trivial since they scale like the curvature term in our scalar potential. We simply define:

$$\bar{A}_{R_6} = A_{R_6} + A_{KK} - A_{KKO}\,.$$

Then our results from above apply and we require $\bar{A}_{R_6} > 0$ for dS solutions.

# NS5-branes

NS5-branes introduce genuinely new terms into the potential (Silverstein: **arXiv:0712.1196**):

$$V(\rho,\tau) = \frac{A_H}{\rho^3\tau^2} + \sum_{p=0,2,4,6} \frac{A_{F_p}}{\rho^{p-3}\tau^4} - \frac{A_{sources}}{\tau^3} + \frac{\bar{A}_{R_6}}{\rho\tau^2} + \frac{A_{NS5}}{\rho^2\tau^2} - \frac{A_{NSO5}}{\rho^2\tau^2} \,.$$

Minimizing with respect to $\tau$ leads to the relation

$$\frac{A_H}{\rho^3\tau^2} + \frac{\bar{A}_{R_6}}{\rho\tau^2} + \frac{A_{NS5}}{\rho^2\tau^2} - \frac{A_{NSO5}}{\rho^2\tau^2} = -2 \sum_{p=0,2,4,6} \frac{A_{F_p}}{\rho^{p-3}\tau^4} + \frac{3}{2}\frac{A_{sources}}{\tau^3}$$

and finally

$$V(\rho,\tau) = -\sum_{p=0,2,4,6} \frac{A_{F_p}}{\rho^{p-3}\tau^4} + \frac{1}{2}\frac{A_{sources}}{\tau^3} \,.$$

The contribution from the $NS5$-branes is thus not relevant and we are left with the same requirement for large $N_{O6}$.

# Conclusion

- Controlled AdS is possible in the large $F_4$ scaling limit.
- The same limit is not possible for dS.
- Allowing different ingredients to scale does not improve this situation.
- In type IIA flux compactifications controlled dS vacua seem to be only possible with a large number of $O6$-planes.

# Conclusion

- Controlled AdS is possible in the large $F_4$ scaling limit.
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**THANK YOU!**

# Known No-Go Theorems

- No dS vacuum for $\mathbf{A_{F_0} = 0}$:

$$(-\rho\partial_\rho - \tau\partial_\tau)V = 5\frac{A_H}{\rho^3\tau^2} + \sum_{p=2,4,6}(p+1)\frac{A_{F_p}}{\rho^{p-3}\tau^4} - 3\frac{A_{sources}}{\tau^3} + 3\frac{A_{R_6}}{\rho\tau^2} \geq 3V\,.$$

For a dS extremum we would need $V > 0$ and $\partial_\tau V = \partial_\rho V = 0$!
It is instructive to calculate the first slow-roll parameter $\epsilon$, which can be estimated to be:

$$\epsilon = \frac{1}{2}\frac{(\partial_{\hat{\rho}}V)^2 + (\partial_{\hat{\tau}}V)^2 + \ldots}{V^2} \geq \frac{1}{3}\left(\frac{\rho\partial_\rho V}{V}\right)^2 + \frac{1}{4}\left(\frac{\tau\partial_\tau V}{V}\right)^2\,.$$

Minimizing and using our result we get

$$\epsilon \geq \frac{9}{7}\,.$$

# Known No-Go Theorems

- No dS vacuum for $\mathbf{A_{F_0} = 0}$:

$$(-\rho\partial_\rho - \tau\partial_\tau)V \geq 3V \qquad and \qquad \epsilon \geq \frac{9}{7}\,.$$

  For a dS extremum we would need $V > 0$ and $\partial_\tau V = \partial_\rho V = 0$!
- For $\mathbf{A_{R_6} \leq 0}$ there exists an analogue theorem and we find via a similar calculation:

$$(-\rho\partial_\rho - 3\tau\partial_\tau)V \geq 9V,$$

  and, for the slow-roll parameter

$$\epsilon \geq \frac{27}{13}\,.$$

We always require $\mathbf{A_{F_0} > 0}$ and $\mathbf{A_{R_6} < 0}$ for dS vacua!

# $F_2$, $F_6$, the tadpole condition and curvature

$F_2$ and $F_6$ do affect the values of $B_2$ and $C_3$ axions but are not relevant for moduli stabilization.

$N_{O6}$, $H$ and $F_0$ are tied together via the tadpole condition

$$\sqrt{2}\int_{\Sigma_K} d(dC_1+F_0 B) = \sqrt{2}\int_{\Sigma_K} F_0 H = (-2N_{O6}+N_{D6}+N_{\overline{D6}})|_{\text{wrapped on }\Sigma_K}$$

For $R_6 \neq 0$ we find:

$$\begin{aligned} V(\rho,\tau) =& \frac{1}{(f_4)^{\frac{7}{2}}}\frac{A_{R_6}}{\tilde{\rho}\tilde{\tau}^2} + \frac{1}{(f_4)^{\frac{9}{2}}}\left(\frac{A_H}{\tilde{\rho}^3\tilde{\tau}^2} + \frac{A_{F_0}}{\tilde{\rho}^{-3}\tilde{\tau}^4} + \frac{a_{F_4}}{\tilde{\rho}\tilde{\tau}^4} - \frac{A_{sources}}{\tilde{\tau}^3}\right) \\ & + \frac{1}{(f_4)^{\frac{11}{2}}}\frac{A_{F_2}}{\tilde{\rho}^{-1}\tilde{\tau}^4} + \frac{1}{(f_4)^{\frac{15}{2}}}\frac{A_{F_6}}{\tilde{\rho}^3\tilde{\tau}^4}\ . \end{aligned}$$

The leading term is now only the curvature which leads to a runaway for $\tilde{\tau}$ and $\tilde{\rho}$.

# Vanishing Mass in Type IIA

In principle it seems like a large amount of $NSO5$-planes, such that $A_{NSO5} > A_{NS5}$ could allow for vanishin mass parameter $F_0$.
This is interesting because theories without mass parameter can in principle be lifted to M-theory.
Unfortunately a similar analysis to above shows that this would require that the curvature becomes small in the scaling limit, but $R_6$ is fixed for a given compactification. Thus a small number of $NSO5$-planes does not help our cause.