String Phenomenology 2019

Consistent truncation and de Sitter space from gravitational instantons

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RT and Dimitrios Tsimpis, 2019 (accepted)
RT and Dimitrios Tsimpis, JHEP 2019

Two main ingredients:

- Consistent truncation of IIA supergravity

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- Gravitini condensation

Supergravity IIA action

$$S_{IIA} = S_b + S_f + S_{\psi^4}$$

$$S_b = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x\sqrt{g}\Big(-R+\frac{1}{2}(\partial\phi)^2+\frac{1}{2}e^{3\phi/2}F^2+\frac{1}{2}e^{-\phi}H^2+\frac{1}{2}e^{\phi/2}G^2\Big)+\mathrm{CS}$$

$$S_f = \int \mathrm{d}^{10}x\sqrt{g}\left(\bar{\Psi}\nabla\Psi + \bar{\Psi}\not{F}\Psi\right)$$

$$S_{\psi^4} = 24 \text{ terms}\ldots$$

Calabi-Yau internal space

Spacetime of the form: $M_4 \times \mathrm{CY}_6$

CY_6 is eqquiped with:

- Kähler form J
- Holomorphic 3-form Ω

$$J \wedge \Omega = 0$$
$$\Omega \wedge \Omega^* = \frac{4i}{3} J^3$$

$$\mathrm{d}J, \mathrm{d}\Omega = 0$$

Truncation Ansatz

- Metric:

$$g_{10}(x,y) = e^{2A(x)}\left(e^{2B(x)}g_4(x) + g_6(y)\right)$$

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$$G = \varphi \mathrm{vol}_4 + \frac{1}{2} c_0 J \wedge J + J \wedge (\mathrm{d}\gamma - \alpha \wedge \mathrm{d}\chi) - \mathrm{Im}(\underbrace{\mathrm{d}\xi + b_0 \alpha}_{D\xi}) \wedge \Omega$$

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- Dilaton:

$$\phi(x)$$

What about the fermions

Decompose the gravitino:

$$\Psi_m = 0 \; ; \quad \Psi_{\mu+} = \psi_{\mu+} \otimes \eta - \psi_{\mu-} \otimes \eta^c \; ; \quad \Psi_{\mu-} = \psi'_{\mu+} \otimes \eta^c - \psi'_{\mu-} \otimes \eta$$

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$\rightarrow$ Wick rotate to Euclidean signature

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- Zero modes of the Dirac operator:
  - no spin-1/2
  - 2τ spin-3/2 (of positive chirality)

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$$\mathcal{A} := \left(\tilde{\psi}_{\mu+}\gamma^{\mu\nu}\psi'_{\nu+}\right) = -\left(\tilde{\psi}^{\mu}_{+}\psi'_{\mu+}\right)$$
$$\mathcal{B} := -\frac{3}{2}(\tilde{\psi}_{[\mu}\psi'_{\nu]})^2 + (\tilde{\psi}^{\mu}\gamma_{\rho\nu}\psi'_{\mu})(\tilde{\psi}^{\rho}\psi'^{\nu}) + 3(\tilde{\psi}_{[\mu_1}\gamma_{\mu_2\mu_3}\psi'_{\mu_4]})^2$$

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$\Rightarrow$ substitute in the Lagrangian and Wick rotate back to Lorentzian signature

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- External Einstein: $R^{(4)}_{\mu\nu} = \dots$
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$$\Box\chi = \dots$$
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- G-form:

$$\Box\xi = \dots$$
$$\mathrm{d}\star\mathrm{d}\gamma = \dots$$
$$\varphi = \dots$$

Lagrangian

$$S_4 = \int \mathrm{d}^4x\sqrt{g}\Big(R - 24(\partial A)^2 - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{3}{2}e^{-4A-\phi}(\partial\chi)^2 - \tfrac{1}{2}e^{-6A+\phi/2}|D\xi|^2$$
$$- \tfrac{1}{4}e^{3\phi/2+6A}\mathrm{d}\alpha^2 - \tfrac{3}{4}e^{\phi/2+2A}(\mathrm{d}\gamma - \alpha\wedge\mathrm{d}\chi)^2 - \tfrac{1}{2}e^{\phi-12A}(\mathrm{d}b + \tilde{\omega})^2 - V\Big)$$
$$+ \int 3\chi\ \mathrm{d}\gamma\wedge\mathrm{d}\gamma$$

Where:

$$\tilde{\omega} := \tfrac{1}{2}(\xi_1 D\xi_2 - \xi_2 D\xi_1) + 3c_0(\gamma - \chi\alpha)$$

And:

$$V(\chi,\phi,A) = \tfrac{9}{2}c_0^2\chi^2 e^{-\phi/2-18A} + \tfrac{3}{2}c_0^2 e^{\phi/2-14A} + \tfrac{1}{2}|b_0|^2 e^{-\phi-12A}$$
$$+ 3c_0\chi\mathcal{A}e^{-\phi/4-6A} - 3c_0\mathcal{A}e^{\phi/4-4A} + e^{6A}\big(\mathcal{B} + \tfrac{1}{2}\mathcal{A}^2\big)$$

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- Set

$$\alpha, \gamma = 0 \; ; \quad b, \xi = 0 \; ; \quad \phi, A, \chi = cst$$

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- Einstein equation

$$\begin{aligned} R =& 9c_0^2 \chi_0^2 e^{-\phi_0/2-18A_0} + 3c_0^2 e^{\phi_0/2-14A_0} \\ &+ |b_0|^2 e^{-\phi_0-12A_0} + 3c_0 \chi_0 \mathcal{A} e^{-\phi_0/4-6A_0} - 3c_0 \mathcal{A} e^{\phi_0/4-4A_0} \end{aligned}$$

de Sitter solution

- This system determines the background flux as function of ϕ_0, A_0 and the condensates:

$$\chi_0 = -\frac{\mathcal{A}}{3c_0}\, g_s^{1/4} e^{12A_0}$$

$$|b_0|^2 = \frac{3}{400}\, g_s e^{18A_0}\left(40\mathcal{B} - 21\mathcal{A}^2 \mp 3\mathcal{A}\sqrt{49\mathcal{A}^2 + 80\mathcal{B}}\right)$$

$$c_0 = \frac{1}{20}\, g_s^{-1/4} e^{10A_0}\left(7\mathcal{A} \pm \sqrt{49\mathcal{A}^2 + 80\mathcal{B}}\right)$$

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$\Rightarrow$ de Sitter solution if $\mathcal{B} > 0$

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- Mechanism for condensation: gravitational instanton
  - $\rightarrow$ relies on $\mathcal{B} > 0$
  - $\rightarrow$ computation in string theory ?

Thank you for your attention !