Infinite Distances and the Axion Weak Gravity Conjecture

Damian van de Heisteeg

Utrecht University

String Pheno 2019, June 27

based on: 1905.00901 with Thomas Grimm

Motivation

$$\mathcal{L} = -f^2(\partial_\mu a)^2 + \Lambda^4 \sum_n e^{-nS}(1 - \cos na) \qquad (a \sim a + 2\pi)$$

Transplanckian field ranges?
$\implies$ need **large** axion decay constant $f \gg M_p$

Weak Gravity Conjecture for axions [Arkani-Hamed et al. '06]

$$fS \leq qM_p$$

large $f \implies$ small S
$\implies$ **many** instanton corrections become relevant

Motivation

$$\mathcal{L} = -f^2(\partial_\mu a)^2 + \Lambda^4 \sum_n e^{-nS}(1 - \cos na) \qquad (a \sim a + 2\pi)$$

Transplanckian field ranges?
$\implies$ need **large** axion decay constant $f \gg M_p$

Weak Gravity Conjecture for axions [Arkani-Hamed et al. '06]

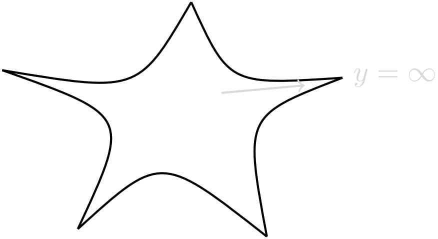
$$fS \leq qM_p$$

large $f \implies$ small S
$\implies$ **many** instanton corrections become relevant

Realization in string compactifications

This talk: Type IIA compactified on Y_3 (axions: $C_3 = \xi^{\mathcal{I}}\gamma_{\mathcal{I}}$)

$\Longrightarrow$ f and S vary **non-trivially** over moduli space of Y_3



In limits: **power-law behavior** of f and S in y

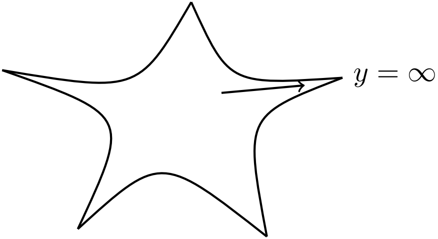
("*grow/decrease parametrically*")

Focus: **infinite distance limits** in $\mathcal{M}^{\mathrm{cs}}(Y_3)$

Realization in string compactifications

This talk: Type IIA compactified on Y_3 (axions: $C_3 = \xi^{\mathcal{I}} \gamma_{\mathcal{I}}$)

$\Longrightarrow$ f and S vary **non-trivially** over moduli space of Y_3



In limits: **power-law behavior** of f and S in y

(*"grow/decrease parametrically"*)

Focus: **infinite distance limits** in $\mathcal{M}^{\mathrm{cs}}(Y_3)$

Lessons from limiting mixed Hodge structures

[Deligne] [Schmid '73] [Cattani, Kaplan, Schmid '86]

Main points:

- **splitting** of three-form cohomology:

$$H^3(Y_3) = \oplus V_\ell \qquad (\ell = 0, \dots, 6)$$

- **asymptotic behavior** of metric $\boxed{\langle v, *w \rangle = \int_{Y_3} v \wedge *w}$

  - *growth* $\|v_\ell\|^2 \sim y^{\ell-3}$
  - *orthogonality* $\langle v_\ell, *_\infty v_{\ell'} \rangle = 0$ if $\ell \neq \ell'$

- **symplectic** property: $\langle v_\ell, v_{\ell'} \rangle = 0$ unless $\ell + \ell' = 6$

$\Longrightarrow$ split into $H^3(Y_3) = \underset{\text{axions} \nearrow}{V_{\text{heavy}}} \oplus \underset{\nwarrow \text{instantons}}{V_{\text{light}}} \oplus V_{\text{rest}}$

Lessons from limiting mixed Hodge structures

[Deligne] [Schmid '73] [Cattani, Kaplan, Schmid '86]

Main points:

- **splitting** of three-form cohomology:

$$H^3(Y_3) = \oplus V_\ell \qquad (\ell = 0, \ldots, 6)$$

- **asymptotic behavior** of metric $\boxed{\langle v, *w \rangle = \int_{Y_3} v \wedge *w}$

  - *growth* $\|v_\ell\|^2 \sim y^{\ell-3}$
  - *orthogonality* $\langle v_\ell, *_\infty v_{\ell'} \rangle = 0$ if $\ell \neq \ell'$

- **symplectic** property: $\langle v_\ell, v_{\ell'} \rangle = 0$ unless $\ell + \ell' = 6$

$\Longrightarrow$ split into $H^3(Y_3) = V_{\text{heavy}} \oplus V_{\text{light}} \oplus V_{\text{rest}}$

axions ↗ ↖ instantons

Lessons from limiting mixed Hodge structures

[Deligne] [Schmid '73] [Cattani, Kaplan, Schmid '86]

Main points:

- **splitting** of three-form cohomology:

$$H^3(Y_3) = \oplus V_\ell \qquad (\ell = 0, \ldots, 6)$$

- **asymptotic behavior** of metric $\boxed{\langle v, *w \rangle = \int_{Y_3} v \wedge *w}$

  - *growth* $\|v_\ell\|^2 \sim y^{\ell-3}$
  - *orthogonality* $\langle v_\ell, *_\infty v_{\ell'} \rangle = 0$ if $\ell \neq \ell'$

- **symplectic** property: $\langle v_\ell, v_{\ell'} \rangle = 0$ unless $\ell + \ell' = 6$

$\Longrightarrow$ split into $H^3(Y_3) = V_{\text{heavy}} \oplus V_{\text{light}} \oplus V_{\text{rest}}$

axions ↗ ↖ instantons

Lessons from limiting mixed Hodge structures

[Deligne] [Schmid '73] [Cattani, Kaplan, Schmid '86]

Main points:

- **splitting** of three-form cohomology:

$$H^3(Y_3) = \oplus V_{\boldsymbol{\ell}} \qquad (\ell_i = 0, \ldots, 6)$$

- **asymptotic behavior** of metric $\boxed{\langle v, *w \rangle = \int_{Y_3} v \wedge *w}$

  - *growth* $\|v_{\boldsymbol{\ell}}\|^2 \sim (\frac{y^1}{y^2})^{\ell_1 - 3} \cdots (\frac{y^{n-1}}{y^n})^{\ell_{n-1}-3} (y^n)^{\ell_n - 3}$

  - *orthogonality* $\langle v_{\boldsymbol{\ell}}, *_\infty v_{\boldsymbol{\ell}'} \rangle = 0$ if $\boldsymbol{\ell} \neq \boldsymbol{\ell}'$

- **symplectic** property: $\langle v_{\boldsymbol{\ell}}, v_{\boldsymbol{\ell}'} \rangle = 0$ unless $\boldsymbol{\ell} + \boldsymbol{\ell}' = \mathbf{6}$

$$\Longrightarrow \text{ split into } H^3(Y_3) = \underset{\text{axions}\nearrow}{V_{\text{heavy}}} \oplus \underset{\nwarrow\text{instantons}}{V_{\text{light}}} \oplus V_{\text{rest}}$$

Axion decay constants

Type IIA on Y_3: R-R axions $C_3 = \xi^{\mathcal{I}}\gamma_{\mathcal{I}}$ $(f^T \cdot f)_{\mathcal{I}\mathcal{J}}$

$$\mathcal{L}_{\text{kin}} = G_{\mathcal{I}\mathcal{J}}\,\partial_\mu \xi^{\mathcal{I}}\,\partial^\mu \xi^{\mathcal{J}}\,, \qquad \mathcal{G}_{\mathcal{I}\mathcal{J}} = \frac{1}{2}e^{2D}\int_{Y_3} \gamma_{\mathcal{I}} \wedge *\gamma_{\mathcal{J}}$$

LMHS: special choice of three-form basis $\gamma_{\mathcal{I}} \in V_\ell$

$\Longrightarrow$ kinetic terms **decouple** asymptotically into blocks

$$\Longrightarrow\; f_{\ell\ell'} \sim \begin{cases} 0 & \text{if } \ell \neq \ell' \\ e^D y^{(\ell-3)/2} & \text{if } \ell = \ell' \end{cases}$$

Axion decay constants

Type IIA on Y_3: R-R axions $C_3 = \xi^{\mathcal{I}}\gamma_{\mathcal{I}}$

$$\mathcal{L}_{\text{kin}} = G_{\mathcal{I}\mathcal{J}}\,\partial_\mu \xi^{\mathcal{I}}\,\partial^\mu \xi^{\mathcal{J}}\,, \qquad \overset{(f^T \cdot f)_{\mathcal{I}\mathcal{J}}}{\overset{\swarrow}{\mathcal{G}_{\mathcal{I}\mathcal{J}}}} = \frac{1}{2}e^{2D}\int_{Y_3} \gamma_{\mathcal{I}} \wedge *\gamma_{\mathcal{J}}$$

LMHS: special choice of three-form basis $\gamma_{\mathcal{I}} \in V_\ell$

$\Longrightarrow$ kinetic terms **decouple** asymptotically into blocks

$$\Longrightarrow\; f_{\ell\ell'} \sim \begin{cases} 0 & \text{if } \ell \neq \ell' \\ e^D y^{(\ell-3)/2} & \text{if } \ell = \ell' \end{cases}$$

Axion decay constants

Type IIA on Y_3: R-R axions $C_3 = \xi^{\mathcal{I}}\gamma_{\mathcal{I}}$ $\qquad$ $(f^T \cdot f)_{\mathcal{I}\mathcal{J}}$

$$\mathcal{L}_{\text{kin}} = G_{\mathcal{I}\mathcal{J}}\, \partial_\mu \xi^{\mathcal{I}}\, \partial^\mu \xi^{\mathcal{J}}\,, \qquad \mathcal{G}_{\mathcal{I}\mathcal{J}} = \frac{1}{2} e^{2D} \int_{Y_3} \gamma_{\mathcal{I}} \wedge *\gamma_{\mathcal{J}}$$

LMHS: special choice of three-form basis $\gamma_{\mathcal{I}} \in V_{\boldsymbol{\ell}}$

$\implies$ kinetic terms **decouple** asymptotically into blocks

$$\implies f_{\boldsymbol{\ell}\boldsymbol{\ell}'} \sim \begin{cases} 0 & \text{if } \boldsymbol{\ell} \neq \boldsymbol{\ell}' \\ e^D \big(\frac{y^1}{y^2}\big)^{\frac{\ell_1 - 3}{2}} \cdots \big(\frac{y^{n-1}}{y^n}\big)^{\frac{\ell_{n-1}-3}{2}} \big(y^n\big)^{\frac{\ell_n - 3}{2}} & \text{if } \boldsymbol{\ell} = \boldsymbol{\ell}' \end{cases}$$

Infinite distances in $\mathcal{M}^{\mathrm{cs}}(Y_3)$

Swampland Distance Conjecture [Ooguri, Vafa '06]

Infinite distance in field space $\implies$ infinite tower of massless states

- Type IIB: Tower of D3-branes wrapping **3-cycles**
  $\implies$ Particles with masses $M(Q) \to 0$

  [Grimm, Palti, Valenzuela '18] [Grimm, Li, Palti '18]

- Type IIA: Euclidean D2-branes wrapping **3-cycles**
  $\implies$ Instantons with actions $S(Q) \to 0$

  [Grimm, DH '19], see also [Marchesano, Wiesner '19]

$S(Q), M(Q) \leq \|Q\| \to 0$ (via $S, M = e^{K/2} |\int_{Y_3} \Omega \wedge * Q|$)

Crucial feature: growth of number of (stable) states $m_{\mathrm{crit}} \sim y$

[Grimm, Palti, Valenzuela '18], multi-parameter version in [Grimm, DH '19]

Weak Gravity Conjecture and R-R axions

So far:

- Axions $\xi^\ell \in V_{\text{heavy}}$ with **increasing** $f \sim y^{(\ell-3)/2}$
- D2-brane instantons $Q \in V_{\text{light}}$ with **decreasing** $S(Q) \to 0$

Quick look at one-parameter results:

Singularity Type	$S(Q)$	$\max(\ell)$	fS bounded
II	$y^{-1/2}$	4	✓
III	y^{-1}	5	✓
IV	$y^{-1/2}$	6	?

Recall $fS \leq qM_p$

$\implies$ **Increase in charge** plays crucial role ($m_{\text{crit}} \sim y$)

Underlying reason: $Q(m) = q_0 + mNq_0$

$\implies$ this axion couples to charge Nq_0 that generates tower

Weak Gravity Conjecture and R-R axions

So far:

- Axions $\xi^\ell \in V_{\text{heavy}}$ with **increasing** $f \sim y^{(\ell-3)/2}$
- D2-brane instantons $Q \in V_{\text{light}}$ with **decreasing** $S(Q) \to 0$

Quick look at one-parameter results:

Singularity Type	$S(Q)$	$\max(\ell)$	fS bounded
II	$y^{-1/2}$	4	✓
III	y^{-1}	5	✓
IV	$y^{-1/2}$	6	?

Recall $fS \leq qM_p$

$\implies$ **Increase in charge** plays crucial role ($m_{\text{crit}} \sim y$)

Underlying reason: $Q(m) = q_0 + mNq_0$

$\implies$ this axion couples to charge Nq_0 that generates tower

Weak Gravity Conjecture and R-R axions

So far:

- Axions $\xi^\ell \in V_{\text{heavy}}$ with **increasing** $f \sim y^{(\ell-3)/2}$
- D2-brane instantons $Q \in V_{\text{light}}$ with **decreasing** $S(Q) \to 0$

Quick look at one-parameter results:

Singularity Type	$S(Q)$	$\max(\ell)$	fS bounded
II	$y^{-1/2}$	4	✓
III	y^{-1}	5	✓
IV	$y^{-1/2}$	6	?

Recall $fS \leq qM_p$

$\implies$ **Increase in charge** plays crucial role ($m_{\text{crit}} \sim y$)

Underlying reason: $Q(m) = q_0 + mNq_0$

$\implies$ this axion couples to charge Nq_0 that generates tower

What about multi-parameter limits?

Some two-parameter cases:

space	decomposition	dimensions
V_{light}	$V_{21} \oplus V_{22} \oplus V_{32}$	$2 + (b-2) + r$
V_{heavy}	$V_{45} \oplus V_{44} \oplus V_{34}$	$2 + (b-2) + r$
V_{rest}	$V_{43} \oplus V_{33} \oplus V_{23}$	$2 + 2(h^{2,1} - b - r - 1) + 2$

space	decomposition	dimensions
V_{light}	$V_{10} \oplus V_{12} \oplus V_{22} \oplus V_{32}$	$1 + 1 + c + (r+1)$
V_{heavy}	$V_{56} \oplus V_{54} \oplus V_{44} \oplus V_{34}$	$1 + 1 + c + (r+1)$
V_{rest}	V_{33}	$2(h^{2,1} - c - r - 2)$

space	decomposition	dimensions
V_{light}	$V_{30} \oplus V_{22} \oplus V_{32}$	$1 + a + r$
V_{heavy}	$V_{36} \oplus V_{44} \oplus V_{34}$	$1 + a + r$
V_{rest}	V_{33}	$2(h^{2,1} - a - r)$

space	decomposition	dimensions
V_{light}	$V_{20} \oplus V_{22} \oplus V_{32}$	$1 + b + r$
V_{heavy}	$V_{46} \oplus V_{44} \oplus V_{34}$	$1 + b + r$
V_{rest}	$V_{42} \oplus V_{33} \oplus V_{24}$	$1 + 2(h^{2,1} - b - r - 1) + 1$

$\Longrightarrow$ similar arguments work ✓

More than two parameters?

Short answer: yes ✓

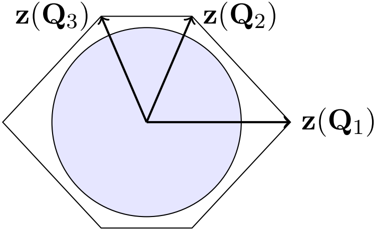
Weak Gravity Conjecture for multiple axions

[Cheung, Remmen '14]

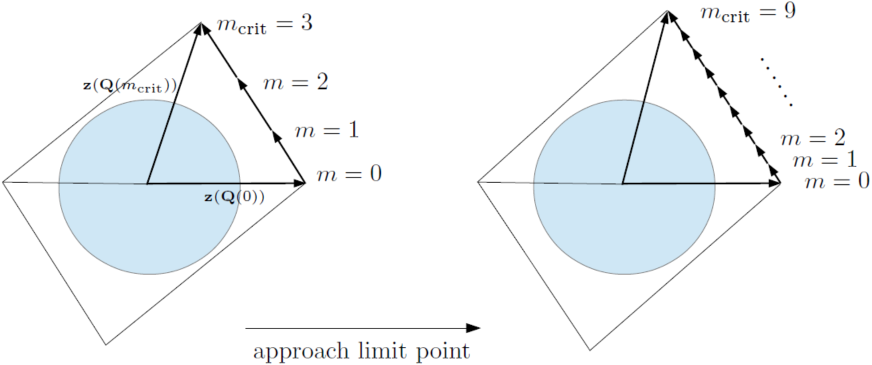
$$fS \leq qM_p \qquad \Longrightarrow \qquad \mathbf{z}^{\mathcal{I}}(\mathbf{Q}_a) = \frac{f^{\mathcal{I}\mathcal{J}}\mathbf{Q}_{a\mathcal{J}}}{S(\mathbf{Q}_a)}$$

Convex Hull Condition

There exist instantons $\mathbf{Q}_a$ such that the vectors $\pm\mathbf{z}(\mathbf{Q}_a)$ span a convex hull that contains the unit ball. (here: ball of finite size)



R-R axions and the Convex Hull Condition



Conclusions

Summary:

- Studied infinite distances in $\mathcal{M}^{cs}(Y_3)$ for Type IIA on Y_3

  $\Longrightarrow$ infinite towers of D2-brane instantons

- Studied parametrically growing axion decay constants for the axion Weak Gravity Conjecture

  $\Longrightarrow$ (growth of) same tower of instantons plays a crucial role

Note: no control over precise coefficients in the WGC
(i.e. radius of the ball)

Thanks for your attention!

Conclusions

Summary:

- Studied infinite distances in $\mathcal{M}^{cs}(Y_3)$ for Type IIA on Y_3

  $\Longrightarrow$ infinite towers of D2-brane instantons

- Studied parametrically growing axion decay constants for the axion Weak Gravity Conjecture

  $\Longrightarrow$ (growth of) same tower of instantons plays a crucial role

Note: no control over precise coefficients in the WGC
(i.e. radius of the ball)

Thanks for your attention!