Cosmological constant at finite string coupling in F-theory flux compactifications

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(Perturbative) superstring theory predicts the extra 6D space $10=4+6$

- The geometric parameters of extra 6D dimensions → 4D scalar fields (called moduli)
- Stabilization of the extra dimensional space → Moduli stabilization (creating a moduli potential)
- Realization of de Sitter spacetime (dark energy and inflation)
D3 sits at the bottom of a warped throat of CY
→ Tiny positive energy

\[ V_{D3} \sim \frac{e^{4A_{\text{min}}}}{V_{\text{CY}}^2} \]  

\( V_{\text{CY}} \): CY volume
Recently, Maldacena-Nunez type no-go theorem for the KKLT uplift was strongly debated.

- Gaugino condensation on D7-branes in the analysis of the 10D Einstein equation, starting from [Moritz-Retolaza-Westphal ’17,...]

- Reliability of the KKLT minimum:
  A finite number of KK modes are lighter than UV cut-off.
  [Blumenhagen-Klawer-Schlechter, ’19]

- Existence of the runaway behavior for the conifold modulus, parametrizing the length of the throat [Bena-Dudas-Grana-Lust, ’18]

Can we construct the dS models in a general moduli space of CY?
Non-supersymmetric fluxes (in contrast to the GKP construction)

\[ iG_3 \neq *_6 G_3 \]

Non-supersymmetric fluxes also lead to the positive energy demonstrated in toroidal orientifold

No a priori need for warped throats

[A. Saltman and E. Silverstein’04]

This talk:

We discuss non-SUSY minima in F-theory compactifications

with Y. Honma
Outline

✔ Introduction

◯ Flux compactification in F-theory
  - SUSY minimum
  - SUSY-breaking (de Sitter) minimum

◯ Conclusion
F-theory compactification on CY4

- Elliptically fibered CY4
- $\tau(u)$: dilaton
- $u_0$, $u$ points
- 3D Kähler manifold

Left:
- CY3+branes

Right:
- Elliptically fibered CY4
- Complex structure moduli of CY3
- Dilaton
- Open string (brane) moduli

Complex structure moduli of CY4
Flux compactification in F-theory on CY4

○The orientifold limit of F-theory
  [Dasgupta-Rajesh-Sethi ‘99, Denef-Douglas-Florea-Grassi-Kachru ‘05]

○K3 × K3 background [Berglund-Mayr ‘13]

○Elliptically fibered CY4 in the large complex structure limit
  [Honma-Otsuka ‘17,19]
  We focus on
  Elliptically fibered CY4 = mirror-dual to Quintic CY3 over $CP^1$
  [Berglund-Mayr ‘98, Jockers-Mayr-Walcher ‘09]
In the toric language, 

A-model : Quintic CY3 over $CP^1$ 

\[ l_1 = (-4,0,1,1,1,1,-1,-1,0) \]
\[ l_2 = (-1,1,0,0,0,0,1,-1,0) \]
\[ l_3 = (0,-2,0,0,0,0,0,1,1) \]

$\ l_1 + l_2$ : Quintic CY3
$\ l_2$ : brane deformation
$\ l_3$ : base $CP^1$

B-model : Elliptically fibered CY4
F-theory on elliptically fibered CY4 $\rightarrow$ 4D N=1 supergravity

In 4D N=1 effective SUGRA (in $M_{\text{pl}} = 1$ unit)

Kähler potential:

$$
K = -\ln \int_{\text{CY4}} \Omega \wedge \overline{\Omega} - 2\ln V_{\text{vol}}
$$

$\Omega$ : hol. 4-form of CY4

$V_{\text{vol}}$ : Volume of 3D Kähler base

Superpotential:

$$
W = \int_{\text{CY4}} G_4 \wedge \Omega
$$

[Gucko-Vafa-Witten, ’99]

Scalar potential:

$$
V = e^K \left( \sum_{I,J} K^{IJ} D_I W D_J \right)
$$

$D_I W = \partial_I W + W \partial_I K$

$I, J$ : CS moduli of CY4
F-theory compactification on elliptically fibered CY4

(mirror-dual to Quintic CY3 over $CP^1$)

$z$: Closed string modulus
$S$: Dilaton
$z_1$: Brane modulus

$n_i = \int_{\gamma^i} G_4$: Quantized fluxes $\gamma^i \in H^4_H(CY4, Z)$ ($i = 1, 2, \ldots, 11$)

Kähler potential:

$$K = -\ln \left[-i(S - \overline{S})\right] - \ln \left[\frac{5i}{6}(z - \overline{z})^3 + \frac{i}{S - \overline{S}} \left(-\frac{1}{6}(z_1 - \overline{z_1})^4 + \frac{5}{12}(z - \overline{z})^4\right)\right] - 2\ln \mathcal{V}$$

NLO in $g_s$ correction

Superpotential:

$$W = n_{11} + n_{10}S + n_8z + n_6Sz + \frac{5}{2} \left(\frac{n_5}{5} + \frac{2n_6}{5}\right)z^2 - \frac{5n_4}{6}z^3 - n_2 \left(\frac{5}{2}Sz^2 + \frac{5}{3}z^3\right) - n_9z_1 - \frac{n_7}{2}z_1^2$$

$$- \frac{2n_3}{3}z_1^3 + n_1 \left(\frac{5}{6}Sz_1^3 + \frac{5}{12}z^4 - \frac{1}{6}z_1^4\right)$$
As a consequence of the self-dual condition to $G_4$ fluxes, all the moduli fields are stabilized at

$$D_S W = D_z W = D_{z_1} W = 0$$

**VEVs:**

\[
\begin{align*}
\text{Re} z &= \text{Re} z_1 = \text{Re} S = 0 \\
\text{Im} z &= \left( \frac{6n_{11}}{5n_1} \right)^{1/4} \frac{2\sqrt{n_6}}{(8n_6(n_5 + n_6) - 5n_7^2)^{1/4}}, \\
\text{Im} z_1 &= \left( \frac{30n_{11}}{n_1} \right)^{1/4} \frac{\sqrt{n_7}}{(8n_6(n_5 + n_6) - 5n_7^2)^{1/4}}, \\
\text{Im} S &= \left( \frac{6n_{11}}{5n_1} \right)^{1/4} \frac{n_5}{\sqrt{n_6}(8n_6(n_5 + n_6) - 5n_7^2)^{1/4}}
\end{align*}
\]

$z$: Closed string modulus
$S$: Dilaton
$z_1$: Brane modulus
$n_i$: Quantized fluxes
Result : SUSY fluxes

- Taking into account the tadpole condition,

\[
\frac{\chi}{24} = n_{D3} + \frac{1}{2} \int_{CY4} G_4 \land G_4
\]

\( \chi = 1860 \): Euler number of CY4
\( n_{D3} \): # of D3

we find the consistent F-theory SUSY vacuum, e.g.,

\[
\begin{align*}
n_1 &= 1, 
 n_5 &= 15, 
 n_6 &= 10, 
 n_7 &= 2, 
 n_{11} &= 28 \\
n_{D3} &= 0
\end{align*}
\]

- All the moduli fields can be stabilized at

\[
\begin{align*}
\text{Re} z &= \text{Re} z_1 = \text{Re} S = 0, \\
\text{Im} z &\approx 2.28, 
\text{Im} z_1 &\approx 1.14, 
\text{Im} S &\approx 1.71
\end{align*}
\]

- The masses of all the moduli fields are positive definite.
We consider full $G_4$-fluxes, leading to the stabilization of moduli

\[ V > 0 \quad \partial_I V = 0 \quad \partial_I \partial_J V > 0 \]

\[ I = z, S, z_1 \]

Fluxes are constrained by the tadpole cancellation condition:

\[ \chi = \frac{1860}{24} = 0 + \frac{1}{2} \int_{CY_4} G_4 \wedge G_4 \]

\[ \chi = 1860: \text{Euler number of CY4} \]

\[ n_{D3} = 0 \]

We numerically search for fluxes to realize dS minimum with

\[ V = \frac{C}{V_{vol}^2} M_{pl}^4 > 0 \quad C < 1 \]

$C$: constant determined by fluxes
Result: SUSY-breaking fluxes

At present, dS minima exist within the following set of fluxes:

\[ n_1 = n_2 = 0 \quad -6 \leq n_7 \leq 6 \quad -10 \leq n_5, n_6 \leq 10 \]

\[ -8 \leq n_3, n_4, n_8, n_9, n_{10}, n_{11} \leq 8 \]

Each dot corresponds to the dS minima.

\[ C = \frac{V_{vol}^2 \times V}{M_{pl}^4} \]

\[ g_s = (\text{Im}S)^{-1} \]
- Finite $g_s$ leads to smaller vacuum energy!
- At present, smallest dS potential is given by

$$V \simeq \frac{10^{-8}}{V_{\text{vol}}^2} \times M_{\text{pl}}^4$$

$$C = \frac{V_{\text{vol}}^2 \times V}{M_{\text{pl}}^4}$$

$$g_s = (\text{Im} S)^{-1}$$

**Result: SUSY-breaking fluxes**
Comment on the dS solution

- No a priori need for D3-brane
- Tiny dS potential could be a candidate of uplifting potential

\[ V \simeq \frac{10^{-8}}{V_{\text{vol}}} \times M_{\text{pl}}^4 \]

- The richness of the set of possible fluxes
  \( \rightarrow \) A more tiny dS energy, similar to toroidal orientifold model (type IB)  
  [A. Saltman and E. Silverstein’04]
- Full \( G_4 \)-fluxes and stringy corrections are necessary to obtain the 
  dS solution (in the M-theory context)  
  [Dasgupta-Gwyn-McDonough-Mia-Tatar’14]
- Our model includes the leading stringy \( (g_s) \)-corrections
  (It is straightforward to include instanton corrections)
- It is interesting to check such dS solutions from the perspective of
  the E.O.M., using M/F-duality
Conclusion

- We explicitly demonstrate the moduli stabilization around the LCS point of the elliptically-fibered F-theory fourfold

- $G_4$-fluxes lead to the stabilization of all the complex structure moduli at the Minkowski minimum or dS minimum

- In the dS case, small but finite string coupling prefers tiny dS potential (No a priori need for $\overline{D3}$ -brane)

- “Swampland conjecture” in the F-theory context