Branes, quiver gauge theories, and discrete symmetries

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Based on: J.R., R. Tatar [1807.01754]
J.R., R. Tatar [1902.10019]
and works in progress.
We want to get a grasp on every symmetry we could encounter. Quiver gauge theories can display some unusual symmetries in obvious ways.

Why $3d \, \mathcal{N} = 4$ quivers (for now)?
- Brane configurations can be relatively simple.
- Good understanding of moduli space geometry $\rightarrow$ good consistency checking for developing new procedures.
- Can realize novel symmetries and procedures of the field theory.
- Manipulation at the level of quivers is simple but powerful.
- Clues as to interesting phenomena for gauge theories in higher dimensions, lower supersymmetry, etc.
- $3d \, \mathcal{N} = 4$ quivers are being used very effectively in the study of 5d theories at infinite coupling and 6d theories to investigate tensionless strings.
Anatomy of a unitary 3d $\mathcal{N} = 4$ quiver

**Circular Node**

$U(m)$ gauge group, adjoint vectormultiplet $V = (A_\mu, 2\lambda, \eta, \phi)$.

**Edge between circular nodes**

$U(m) \times U(n)$ bifundamental hypermultiplet, $H = (2\psi, 2\phi)$.

**Square node with an edge**

Flavoured $U(m)$ fundamental hypermultiplet(s).
A stringy interpretation

Standard Hanany-Witten configuration

Quiver gauge theory lives on the D3 branes → moving branes around we can manipulate the field theory.
The Higgsing of certain subsets of field can be realized using the Kraft–Procesi transition \cite{Cabrera, Hanany, '16}
Moduli space of vacua

Higgs and Coulomb branches, Higgs is classically exact but Coulomb is quantum corrected.

HyperKähler manifolds but the Coulomb branch metric is hard.

Describe as highly singular, highly nested, complex algebraic varieties (Nilpotent varieties in Lie algebras, etc ...).

The nesting structure is generally very complicated, denoted using a Hasse diagram

Can locally analyze the geometry by slicing transversely
- Kraft-Procesi transition
- Quiver addition and subtraction
Local analysis without global description

J.R., R. Tatar, [1807.01754]
Dynkin quivers

Quiver with gauge nodes in the shape of a Dynkin diagram

$D$-type completely locally analyzed for good quivers, but nilpotent varieties in $\mathfrak{so}_{2n}$ are only sometimes realised.

J.R., R. Tatar [1902.10019]
Dynkin diagrams have folding symmetry corresponding to outer automorphisms of the gauge group.

Dynkin quivers correspond to product gauge groups → Dynkin quiver folding symmetry is an outer automorphism of the product gauge group and the folding procedure would be the quotient → $\mathbb{Z}_2$ for A to C folding.
Future work

Folding
- What would it mean to fold by this symmetry of the gauge symmetry?
- How would the rep theory and field content change?
- A field interpretation to non-simply laced edges?

Other directions just with $3d \; \mathcal{N} = 4$
- Bad and ugly quivers
- Good Abelian trees