



# Branes, quiver gauge theories, and discrete symmetries

Branes,  
quiver gauge  
theories, and  
discrete  
symmetries

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Based on: J.R., R. Tatar [1807.01754]  
J.R., R. Tatar [1902.10019]  
and works in progress.

Motivation

Quivers

Branes

Vacua

Circular  
quivers

Dynkin quivers

Folding and  
novel  
symmetries



# Introduction

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We want to get a grasp on every symmetry we could encounter. Quiver gauge theories can display some unusual symmetries in obvious ways.

Why  $3d \mathcal{N} = 4$  quivers (for now)?

- Brane configurations can be relatively simple.
- Good understanding of moduli space geometry  $\rightarrow$  good consistency checking for developing new procedures.
- Can realize novel symmetries and procedures of the field theory.
- Manipulation at the level of quivers is simple but powerful.
- Clues as to interesting phenomena for gauge theories in higher dimensions, lower supersymmetry, etc.
- $3d \mathcal{N} = 4$  quivers are being used very effectively in the study of 5d theories at infinite coupling and 6d theories to investigate tensionless strings.



# Anatomy of a unitary $3d \mathcal{N} = 4$ quiver

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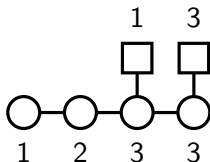
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## Circular Node

$U(m)$  gauge group, adjoint vectormultiplet  $V = (A_\mu, 2\lambda, \eta, \phi)$ .

## Edge between circular nodes

$U(m) \times U(n)$  bifundamental hypermultiplet,  $H = (2\psi, 2\phi)$ .

## Square node with an edge

Flavoured  $U(m)$  fundamental hypermultiplet(s).



# A stringy interpretation

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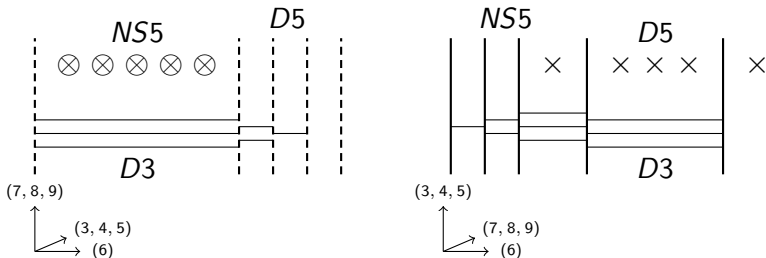
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## Standard Hanany-Witten configuration



Quiver gauge theory lives on the D3 branes  $\rightarrow$  moving branes around we can manipulate the field theory.



# Kraft–Procesi transitions

The Higgsing of certain subsets of field can be realized using the Kraft–Procesi transition [Cabrera, Hanany, '16]

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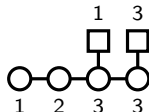
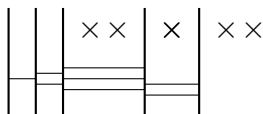
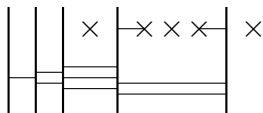
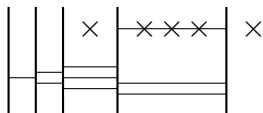
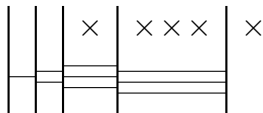
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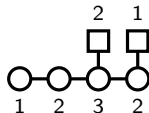
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$A_2$





# Moduli space of vacua

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Higgs and Coulomb branches, Higgs is classically exact but Coulomb is quantum corrected.

HyperKähler manifolds but the Coulomb branch metric is hard.

Describe as highly singular, highly nested, complex algebraic varieties (Nilpotent varieties in Lie algebras, etc ...).

The nesting structure is generally very complicated, denoted using a Hasse diagram

Can locally analyze the geometry by slicing transversely

- Kraft-Procesi transition
- Quiver addition and subtraction



# Local analysis without global description

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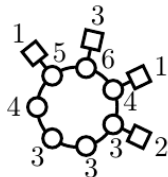
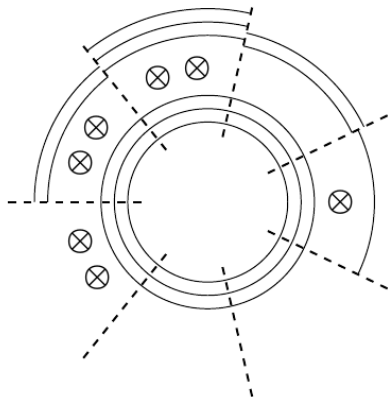
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# Dynkin quivers

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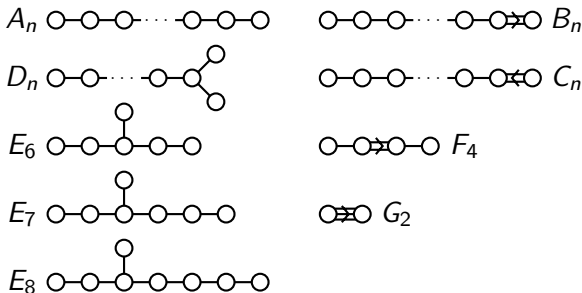
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Quiver with gauge nodes in the shape of a Dynkin diagram

$D$ -type completely locally analyzed for good quivers, but nilpotent varieties in  $\mathfrak{so}_{2n}$  are only sometimes realised.

J.R., R. Tatar [1902.10019]





# Folding symmetry

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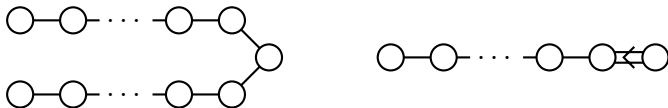
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Dynkin diagrams have folding symmetry corresponding to outer automorphisms of the gauge group.

Dynkin quivers correspond to product gauge groups  $\rightarrow$  Dynkin quiver folding symmetry is an outer automorphism of the product gauge group and the folding procedure would be the quotient  $\rightarrow \mathbb{Z}_2$  for  $A$  to  $C$  folding.



# Future work

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## Folding

-What would it mean to fold by this symmetry of the gauge symmetry?

-How would the rep theory and field content change?

-A field interpretation to non-simply laced edges?

Other directions just with  $3d \mathcal{N} = 4$

-Bad and ugly quivers

-Good Abelian trees