Triangulating the Landscape

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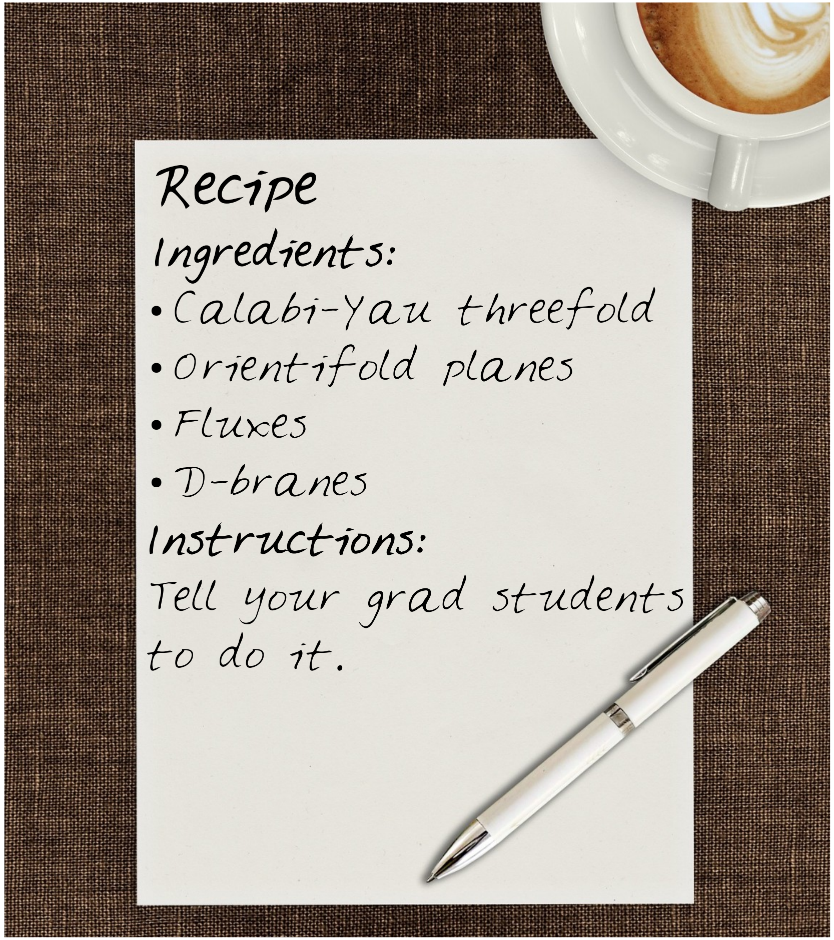
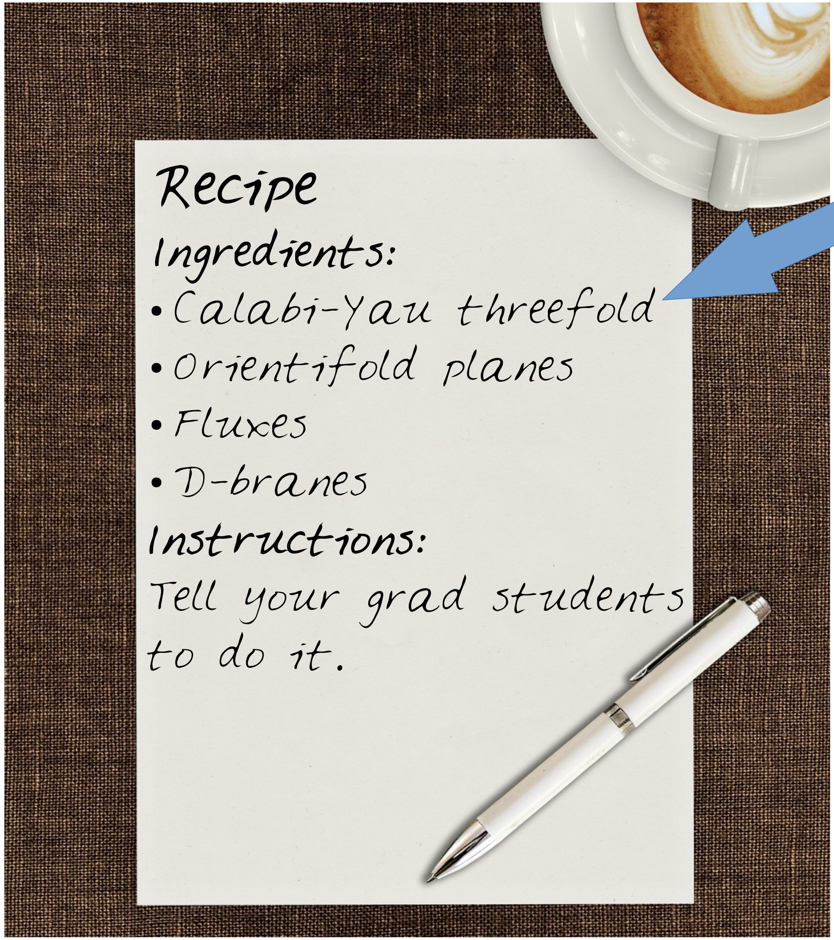
Based on: M. Demirtas, L. McAllister, A. Rios-Tascon. 1908.xxxxx

Constructing 4D EFTs from String Theory



Constructing 4D EFTs from String Theory



Already a huge number of choices.
(Finite?)

Can we make any statistical statements about CY manifolds, and their physical implications? e.g. typical axion masses and field ranges

We first focus in the case of CYs obtained as hypersurfaces in toric varieties:

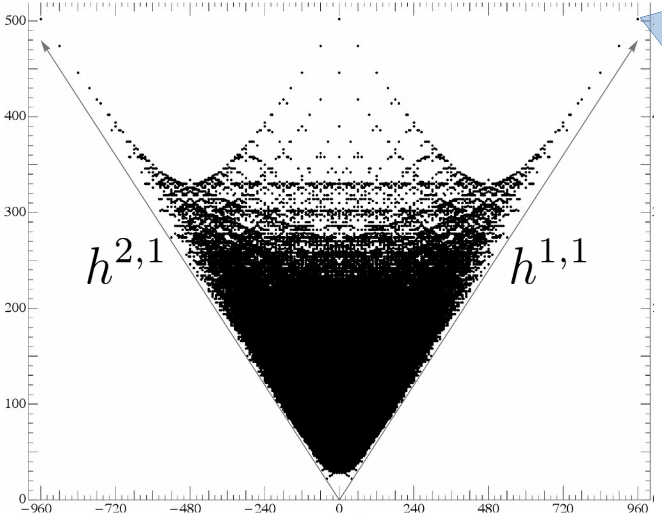
- Estimate/bound number of such CYs.
- Obtain representative samples.

Hypersurface CY Construction

- Generic anticanonical divisor X of toric variety V defined by the face fan of a 4D reflexive polyope Δ is CY threefold. [Batyrev alg-geom/9310003]

[hep-th/0002240]

- Complete list of 473,800,776 such polytopes was found by Kreuzer-Skarke.
- Polytope determines hodge numbers $h^{1,1}$ and $h^{2,1}$ of resulting threefold, with $h^{1,1}$ being the number of boundary points not interior to facets minus 4 (when favorable).



- Largest polytope has $h^{1,1} = 491$, and it is of particular interest to us because, as you will see, it might give rise to most of the CY hypersurfaces.

Resolving CY Singularities

Resolving the singularities can be done by finding a fine, regular, star triangulation (FRST) of the polytope Δ, with each FRST giving rise to a different toric variety and potentially a different CY.

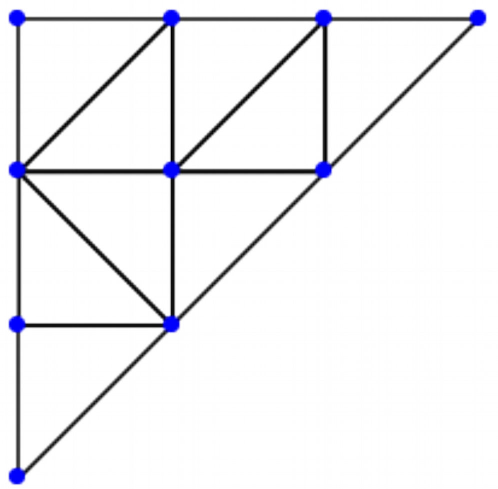
Star: Origin is vertex of all full-dim simplices $\longrightarrow$ Simplices define cones of a fan

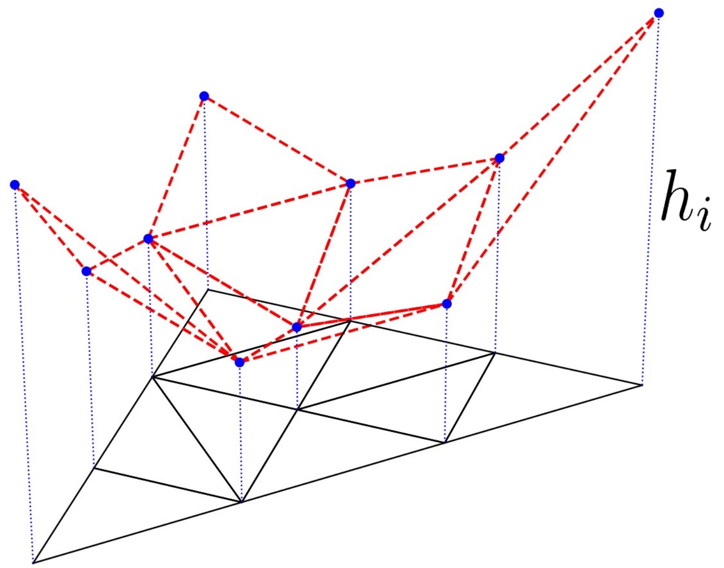
Fine: Uses all points $\longrightarrow$ All singularities are resolved

Regular: Is needed so that variety is projective and Kähler, and will be key for our discussion. Let's see what it means in detail.

Regular Triangulations are Special

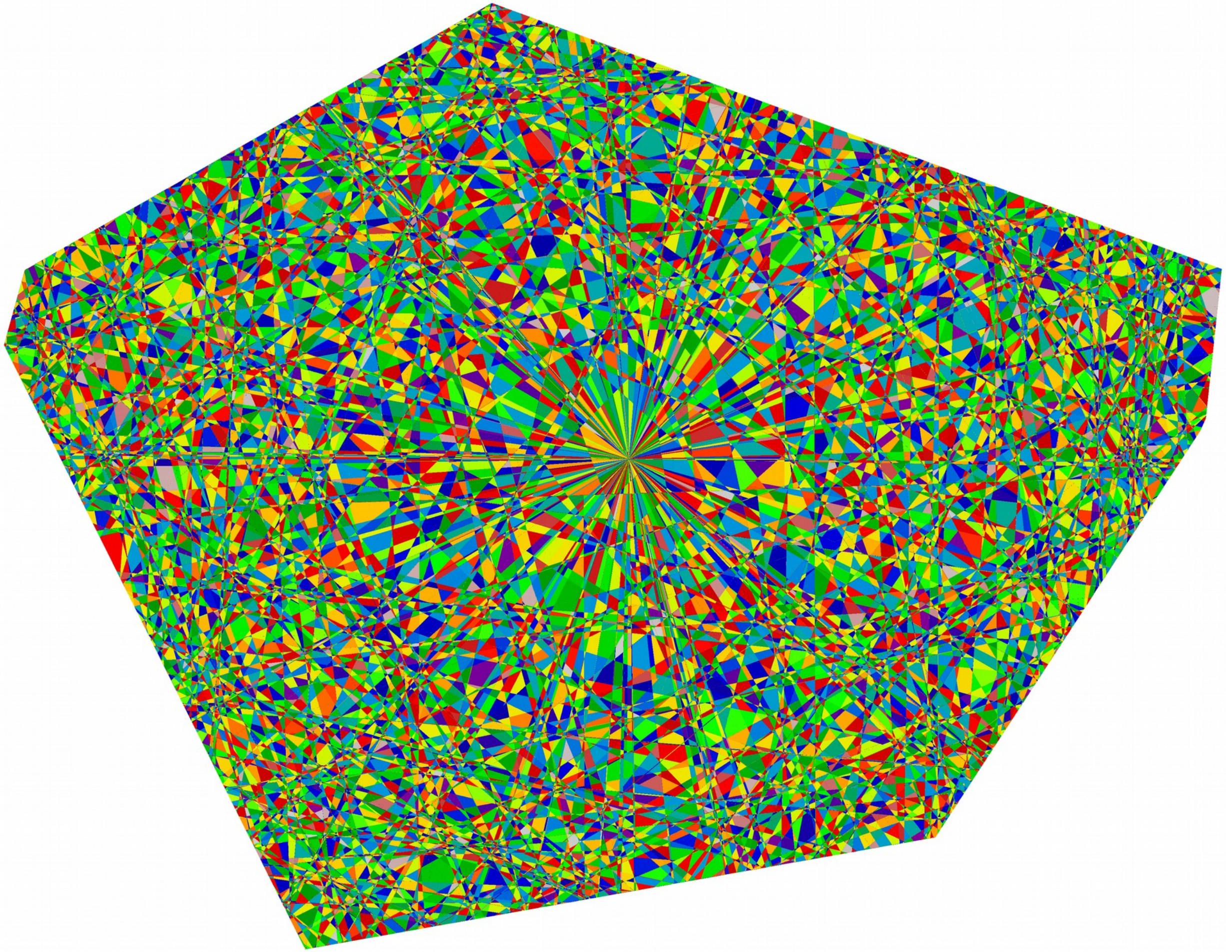
A d-dimensional triangulation is regular if the points can be lifted by heights h_i to form a d+1-dimensional convex hull with the lower faces projecting down to the simplices of the triangulation.





Set of heights that result in a particular triangulation form a cone. Full set of regular triangulations form a complete fan, called the ***secondary fan***. [Gelfand,Kapranov,Zelevinsky]

2D Cross Section of FRST Secondary Fan of 491 polytope



This Image Highlights our Computational Advances

- Most studies have only studied polytopes with $h^{1,1} \lesssim 10$.
- Up until now, it was only possible to find a few FRSTs of the $h^{1,1} = 491$ polytope.
- For the previous image we computed 16 million triangulations of this polytope.
- We can obtain each FRST in $\lesssim$ 20 ms, even for the largest polytope.
- Altough we cannot find all triangulations, we can produce as many as storage allows.

The Secondary Polytope

The secondary fan is the normal fan of a polytope called the ***secondary polytope***, which was first studied by Gelfand, Kapranov, and Zelevinsky (GKZ) in the '90s.

Vertices of secondary polytope are in one-to-one correspondence with regular triangulations of original polytope.

Constructing the Secondary Polytope

Define the GKZ vector of a triangulation $\mathcal{T}$ by

$$\varphi_i^{\mathcal{T}} := \sum_{\{\sigma \in \mathcal{T} | p_i \in \mathrm{vert}(\sigma)\}} \mathrm{vol}(\sigma)$$

Secondary polytope is given by

$$P(\Delta) := \mathrm{conv}\{\varphi^{\mathcal{T}} | \mathcal{T} \text{ is a triangulation of } \Delta\}$$

Can be shown that <u>FRSTs form a convex subfan</u> of secondary fan, and thus a convex subpolytope of the secondary polytope.

Bounding the Number of FRSTs

- First attempt to estimate the number of FRSTs was done by Altman, Carifio, Halverson, and Nelson using machine learning. [1811.06490]

- Previously shown embedding can be used to constrain the region where the secondary polytope resides.

- Can find further constraints for fine and star triangulations.

- Results in simple combinatorial problem which yields

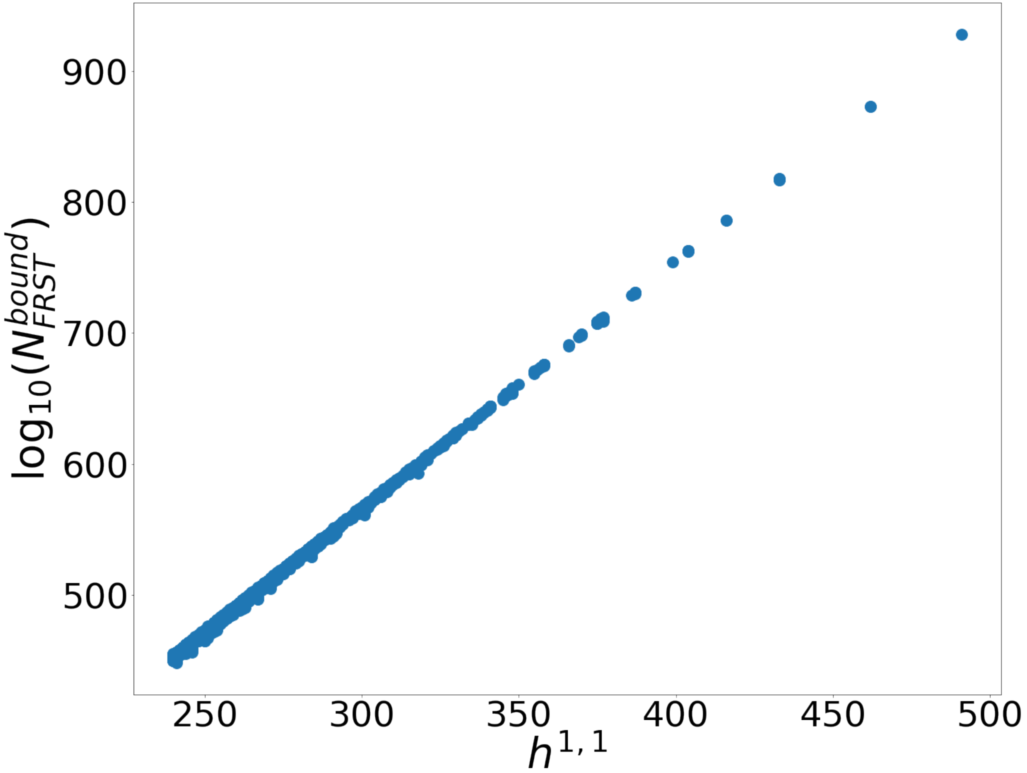
$$N_{\text{FRST}} \leq \binom{4V-1}{h^{1,1}+3}$$

Kreuzer-Skarke Database

- 491 polytope dominates database with

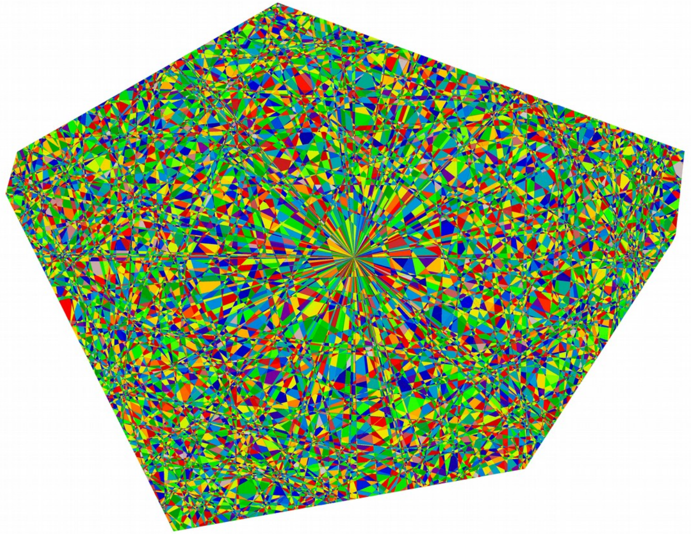
$$N_{\text{FRST}} \leq \binom{14,111}{494} \approx 1.53 \times 10^{928}$$

- Bound for subleading polytope is 55 orders of magnitude smaller.
- Studying small polytopes suggests that real number of FRSTs is much smaller than this bound.



Obtaining representative samples

- Typical method of obtaining random triangulations is by performing random bistellar flips to initial triangulation.
  - Slow because regularity must be checked at each step.
  - Biased towards initial triangulation.
- We produce random triangulations by doing random walks on the height space of FRSTs.
- Using small polytopes we have shown that this is not only much faster, but also produces a better sample.



Other Directions we are Exploring

- Machine learning triangulations. (see Mehmet Demirtas' talk at 17:30)
- Database containing a "typical" triangulation along with other data. (e.g. intersection numbers, Mori cone, ...)
- 5D reflexive polytopes / CY fourfolds:
  - Not fully classified.
  - More than 185,269,499,015. [Schöller and Skarke 1808.02422]
  - Largest $h^{1,1}$ is 303,148.
    Can obtain an FRST in ~1 minute.
  - $N_{\mathrm{FRST}} \leq 1.05 \times 10^{746,977}$

The Largest 2-face of the 303,148 5D Polytope

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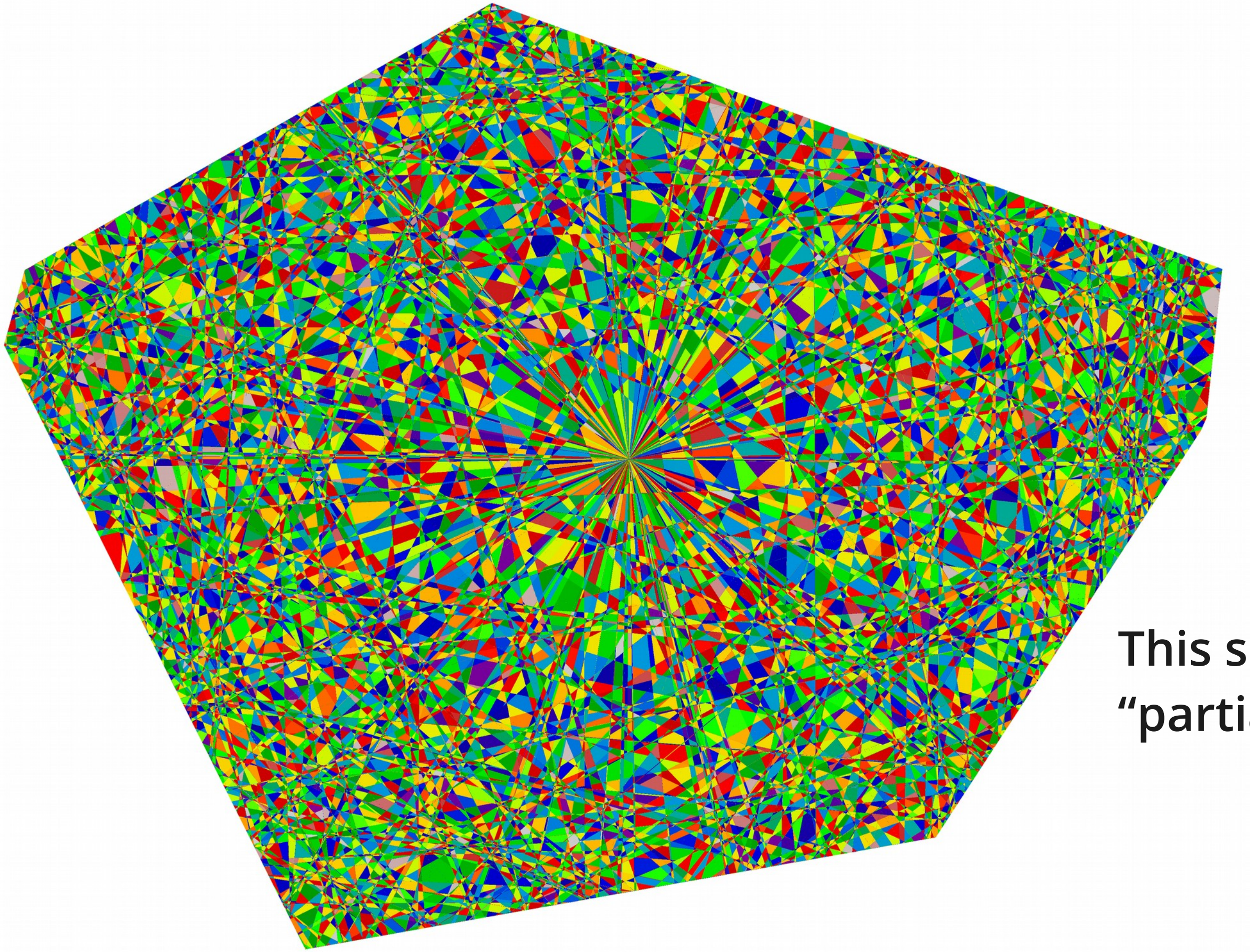
It’s been there all along!

Here is the largest 2-face of the 491 4D polytope for comparison.

A Beautiful Connection



This secondary fan of FRSTs is the "partially enlarged" Kähler cone!

[Gelfand, Kapranov, Zelevinsky; Oda, Park]

Conclusions

- We aim to make statistical studies of CY hypersurfaces in toric varieties.
- We are starting to exploit the special structure of regular triangulations by using tools developed by GKZ.
- We derived a rigorous upper bound for the number of FRSTs of reflexive polytopes.
- These tools have multiple potential uses, such as the generation of representative samples.

Thank you!

Questions?