INSTANTONS AWAY FROM EQUILIBRIUM

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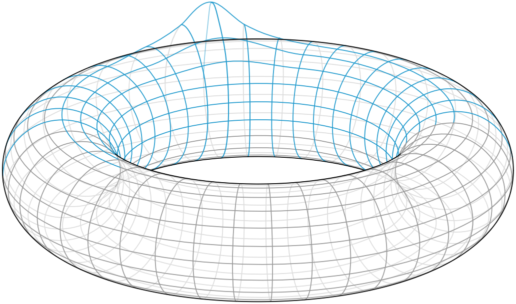
in collaboration with

Guilherme Pimentel

[1905.00219]



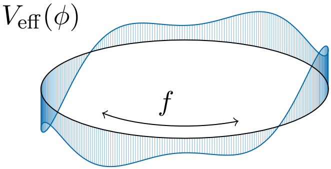
UNIVERSITY OF AMSTERDAM

String Pheno 2019

Motivation

Non-perturbatively generated effective potentials
are an often crucial ingredient in string phenomenology:

Axion Inflation, Axionic Dark Matter, Moduli Stabilization, dS, Strong CP, …



The **gauge symmetry** $\phi \sim \phi + f$ protects effective potential $V_{\text{eff}}(\phi)$, the vacuum energy with ϕ a fixed parameter. Useful QG lamppost!

However, $V_{\text{eff}}(\phi)$ often used off-shell or **out-of-equilibrium**.

Main Question | When does $V_{\text{eff}}(\phi)$ fully capture the dynamics?

Mainly interested in **axion** or **natural inflation** [Freese, Frieman, Olinto '90]

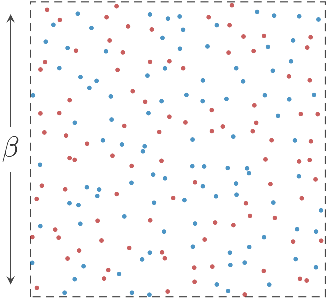
$$S = \int \mathrm{d}^4 x \left(-\frac{f^2}{2} (\partial \varphi)^2 - \frac{1}{4g^2} F^2 + \frac{\varphi}{8\pi} F \tilde{F} + \mathcal{L}_{\mathrm{cm}} \right)$$

Key Ingredient | Both φ and A have topologically non-trivial field spaces:

$$\varphi \sim \varphi + 1 \qquad A \sim A + \omega$$

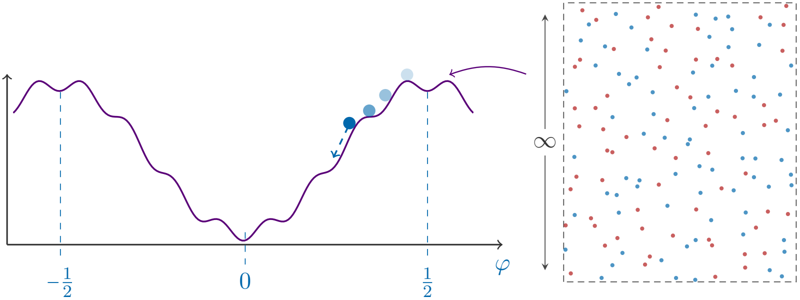
Classically, $\varphi \to \varphi + \epsilon$ is a symmetry but broken by **instantons**,

$$\int \mathcal{D}A\, e^{-S_A[\varphi]} \sim$$



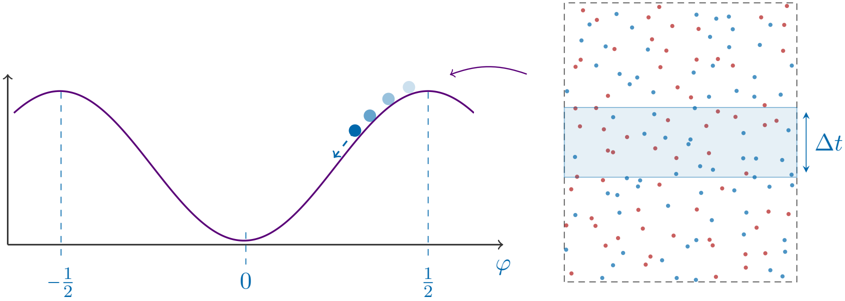
$$\sim \exp(-\beta V_{\mathrm{eff}}(\varphi))$$

This potential drives inflation, determines φ's semi-classical dynamics!



Main Question | When does $V_{\text{eff}}(\varphi)$ fully capture the dynamics?

But, at short times the system may not see all instantons!



Main Question | When does $V_{\text{eff}}(\varphi)$ fully capture the dynamics?

How slowly must φ move to see the effective potential?

Does it ever?

Toy Model

We want to mimic

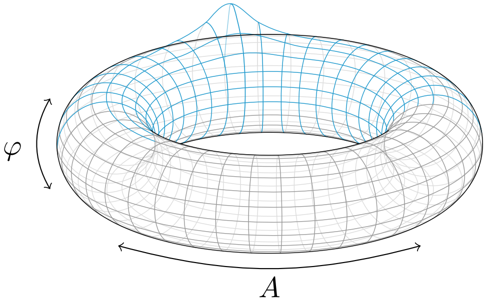
$$S = \int \mathrm{d}^4x \left[-\frac{f^2}{2} (\partial\varphi)^2 - \frac{1}{4g^2} F^2 + \frac{\varphi}{8\pi} F\tilde{F} + \mathcal{L}_{\mathrm{cm}} \right]$$

$\varphi \sim \varphi + 1$
and
$A \sim A + \omega$

Consider a particle on a torus in an electromagnetic field,

$$S = \int \mathrm{d}t \left[\frac{f^2}{2}\dot{\varphi}^2 + \frac{1}{2g^2}\dot{A}^2 + 2\pi\hbar\varphi\dot{A} - \underbrace{V_{\mathrm{c}}(A)}_{\text{periodic}} \right]$$

$\varphi \sim \varphi + 1$
and
$A \sim A + 1$



Interested in zero modes:

Like $\varphi(t, \boldsymbol{x}) = \varphi(t)$ and

$C_3 = A(t)\,\mathrm{d}x \wedge \mathrm{d}y \wedge \mathrm{d}z$,

where $F \wedge F = \mathrm{d}C_3$.

When A remains "near" its ground state, how does φ behave?

Described by an **effective** Schrödinger equation,

$$i\,\partial_t \psi(\varphi, t) = \left(\frac{1}{2f^2} \left(p_\varphi - \mathcal{A}_{\rm B}\right)^2 + V_{\rm eff}(\varphi) + V_{\rm B}(\varphi) \right) \psi(\varphi, t)$$
$$+ \sum_{\substack{\text{excited} \\ \text{states,n}}} \left(\mathcal{F}_{0,n}(\varphi) \left(p_\varphi - \mathcal{A}_{\rm B}\right) + V_{0,n}(\varphi) \right) \psi_n(\varphi, t)$$

The gauge field's vacuum energy acts as an
effective potential, $V_{\rm eff}(\varphi)$

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Non-trivial **Berry potential** $\mathcal{A}_\mathrm{B}$ can act like a magnetic field, and can (sometimes) be gauged away

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Berry scalar potential $V_{\mathrm{B}}(\varphi)$ corrects $V_{\mathrm{eff}}(\varphi)$

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Can not be seen from standard vacuum Euclidean path integral!

[Moody, Shapere, Wilczek '89]

$$V_{\mathrm{B}}(\varphi) = \frac{1}{f^2} \sum_{\substack{\text{excited}\\ \text{states},n}} \left| \frac{\langle 0 | \partial_\varphi \mathcal{H}_{\mathrm{A}} | n \rangle}{V_{\mathrm{eff}}(\varphi) - E_n(\varphi)} \right|^2$$

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Coupling to other modes imposes an **axion speed limit**

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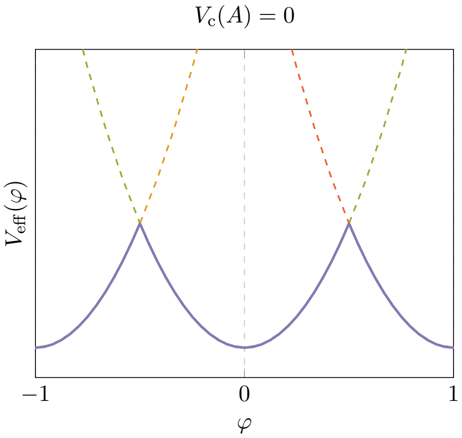
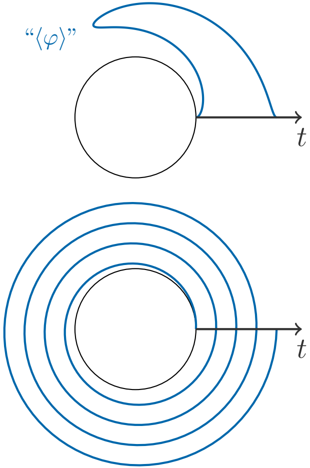
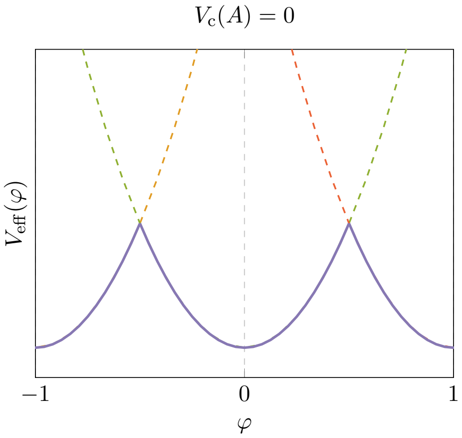
Coupling to other modes, **backreacts** on the gauge field


$V_c(A) = 0$
$V_{\text{eff}}(\varphi)$
-1
0
1
φ


"$\langle\varphi\rangle$"
t
t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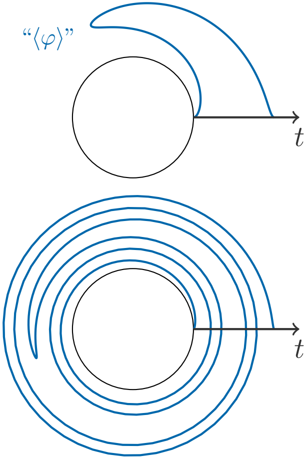
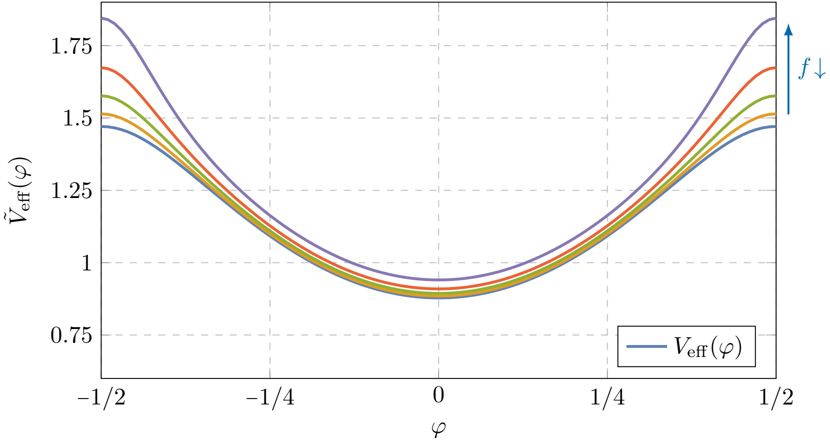

“$\langle\varphi\rangle$”
t
t

$V_c(A) \propto 1 - \cos(2\pi A)$



Conclusions

Main Question | When does $V_{\text{eff}}(\varphi)$ fully capture the dynamics?

- In this model, almost never
- Quantum adiabatic limit can be richer than classical, in ways that cannot be seen from the standard vacuum-to-vacuum Euclidean path integral
- These corrections can compete with the quantum mechanically generated effective potential

Future | How do we consistently incorporate non-perturbative effects in an EFT off-shell and away from equilibrium?

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Future | How do we consistently incorporate non-perturbative effects in an EFT off-shell and away from equilibrium?

Thanks!