Effective Multi-Axion Cosmology and Constraints from Black Hole Superradiance

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String Phenomenology 2019
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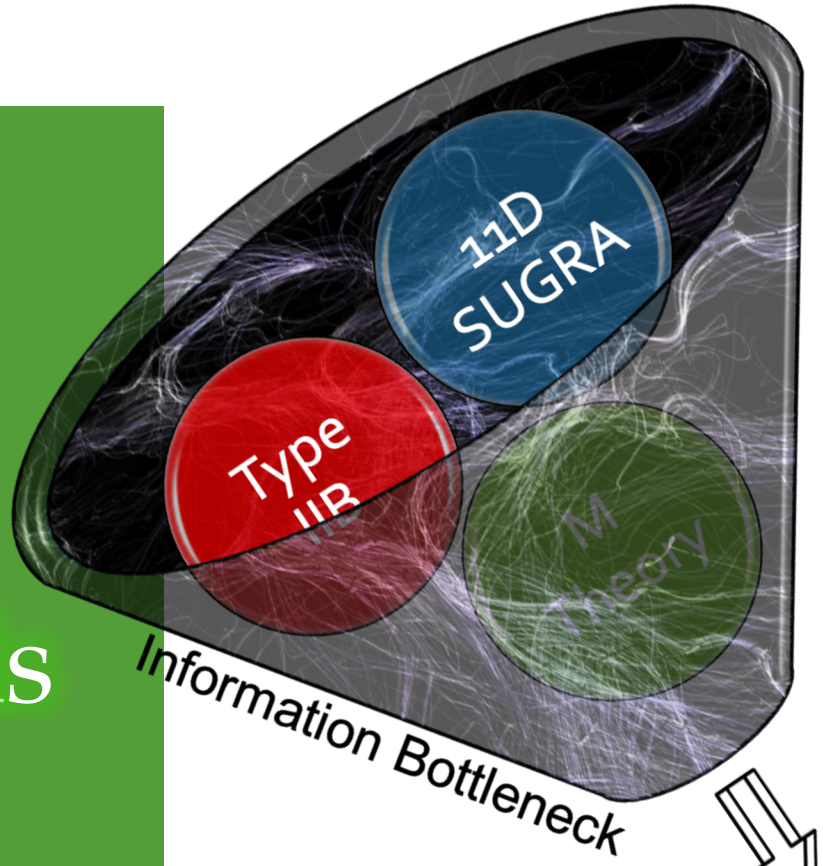
Effective Model - ***arXiv:1706.03236***

Phenomenology - ***arXiv:1805.02016***

W/ David J. E. Marsh

Toy Models

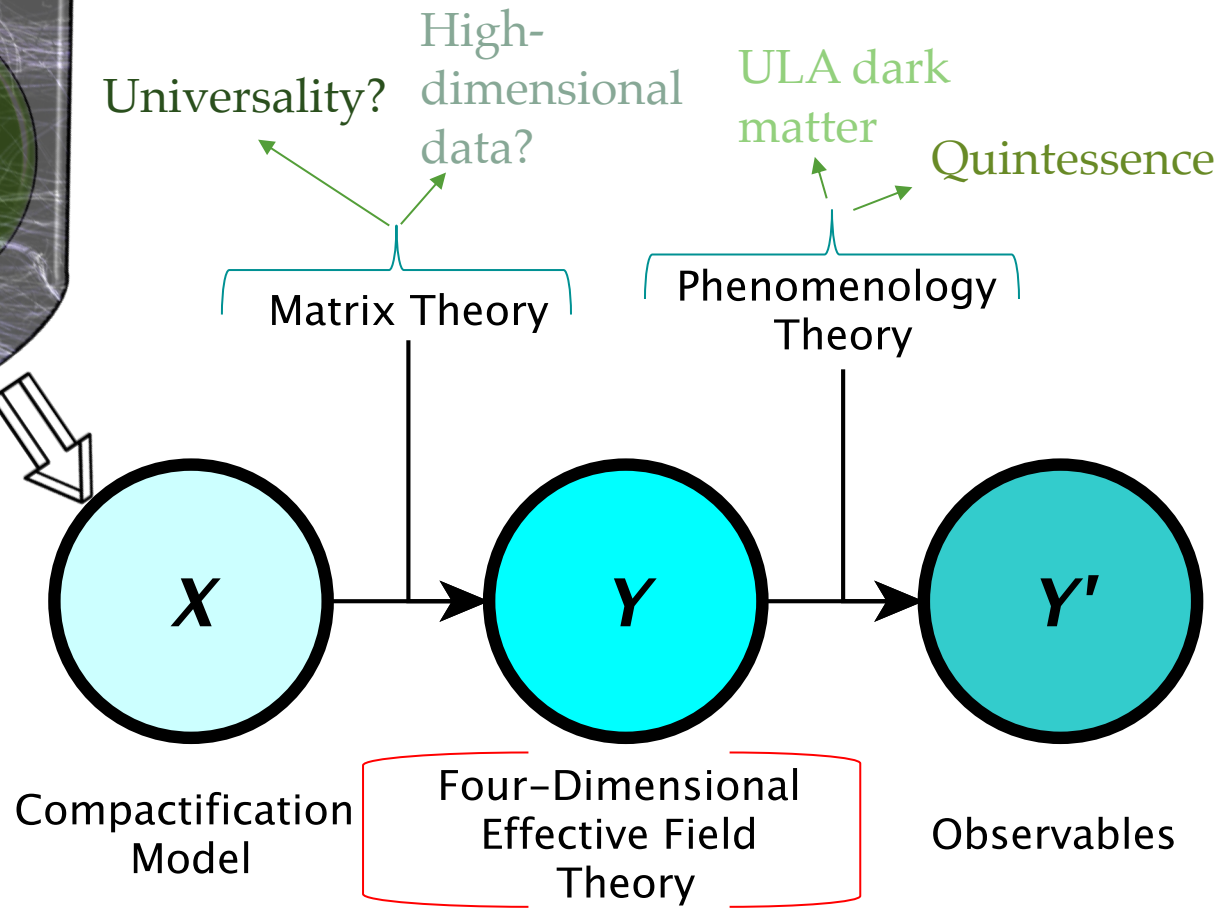


‘True data’ is inherently large

Curse of dimensionality

How can we approach this statistically?

String Axiverse



See [Mafalda Dias et al! : arXiv:1810.05199]

AXIONS

Multiple Fields

SINGLE

$$\mathcal{L} = -\frac{f_a^2}{2}\partial_\mu\theta^\mu\theta - \mu^4 e^{-S}\left(1-\cos(\theta)\right)$$

$$\phi = f_a\theta \qquad m_a = \frac{\Lambda_a^2}{f_a} \qquad f_a \sim \frac{M_{pl}}{S}$$

MULTIPLE

$$\mathcal{L} = -\sum_{i,j=1}^{N_{\rm ax}} \mathcal{K}_{ij}\partial_\mu\theta_i\partial^\mu\theta_j - \sum_{\alpha=1}^{n_{\rm inst}}\sum_{j=1}^{N_{\rm ax}} \Lambda_\alpha U_\alpha(\mathcal{Q}_{j,\alpha}\theta_j + \delta_\alpha)$$

TREATMENT OF MIXED KINETIC TERMS AND GENERAL ``*WELL ALIGNED*'' POTENTIAL

CANONICAL BASIS : $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_i - \frac{1}{2}\mathrm{diag}(\mu_{\rm ax}^2)\phi_i\phi_i$

SIMPLE EFFECTIVE MODEL

$$\mathcal{L} = -\mathcal{K}_{ij}\partial_\mu\theta_i\partial^\mu\theta_j - \mathcal{M}_{ij}\theta_i\theta_j$$

Canonical Normalisation → Multiplicative Matrix Perturbation Theory

$$\tilde{\phi}_i = M_{pl}\mathrm{diag}(f_a)U_{ij}\theta_j$$

$$\tilde{\mathcal{M}} = 2\mathrm{diag}(1/f_a)U\mathcal{M}U^T\mathrm{diag}(1/f_a)$$

$$\tilde{\mathcal{X}}_{ij} = \mathbb{P}_{ik}\mathcal{X}_{kj}$$

$$\tilde{\mathcal{X}}_{ij} = \mathbb{P}^{\frac{1}{2}}_{ik}\mathcal{X}_{kl}\mathbb{P}^{\frac{1}{2}}_{lj}$$

General Covariance Matrix

$$\mu_{\tilde{\mathcal{M}}} \sim \mu_{\mathcal{K}} \boxtimes \mu_{\mathcal{M}}$$

UV/GUT Scale

Ultralight sector

Free Convolution Operations

Traceability – moments/regularity etc.

Multivariate Analysis

Random Matrix Theory

[Easter & McAllister (2006)]

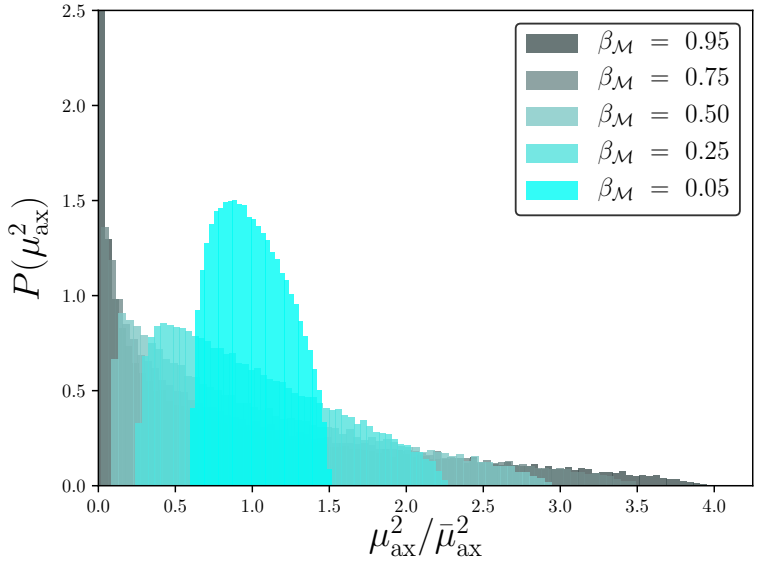
Null Hypothesis $\rightarrow$ $\Omega(\mu = 0, \sigma) \rightarrow \mathcal{N}(0,1) \rightarrow \Sigma_{lk} \propto \mathbb{I}$ $\rightarrow$ Classical Universal Measure Space

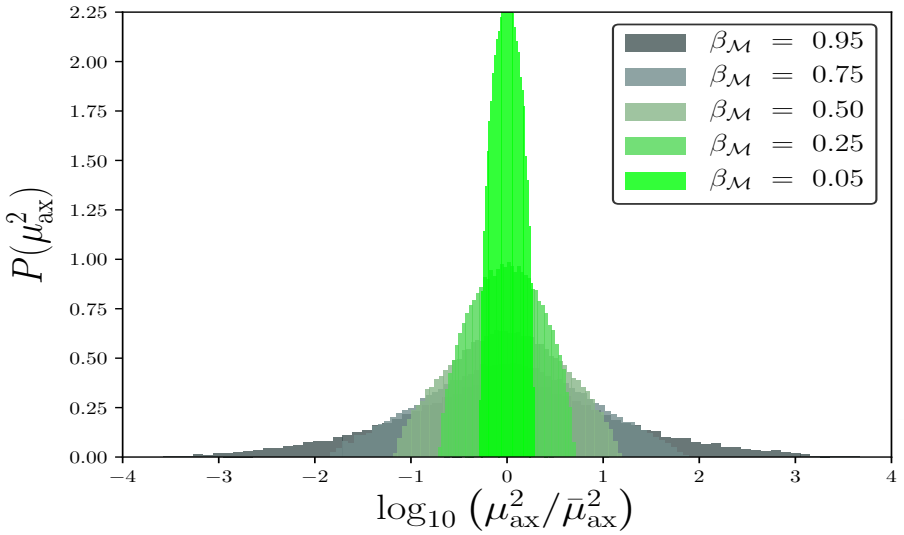
$$\mathcal{M}_{ij} \leftarrow \mathcal{X}_{ij} = \frac{1}{\mathcal{V}} \mathbb{Y}_{il} \Sigma_{lk} \mathbb{Y}^{\mathrm{T}}_{kj} \in \mathbb{R}^{N \times \mathcal{V}}$$

Population covariance matrix

Sample covariance matrix

Alternative Hypothesis $\rightarrow$ $\Omega(\mu \neq 0, \sigma) \rightarrow \Sigma_{lk} \neq \mathbb{I}$ $\rightarrow$ Perturbed population models





[B. S. Acharya et al. (2007)], [B. S. Acharya et al. (2008)], [B. S. Acharya & K. Bobkov (2010)]

M-theory

Stochastic Superpotential

$$\mathcal{M}_{ij} = \sum_{k=1}^{N_{\rm ax}} \sum_{r=1}^{N} \frac{4F\widetilde{\Lambda}_r^3 b_r N_r^k}{M_S^3} e^{-b_r \sum_m^{N_{\rm ax}} N_r^m s_m b_r N_r^i b_r N_r^j}$$

$$\mathcal{M}_{ij} = \sum_{r=1}^{N} 4F\widetilde{\Lambda}_r^3 C_r e^{-S_r} \tilde{N}_r^i \tilde{N}_r^j$$

$$\alpha_{\rm GUT} = \frac{1}{V_X} \approx \frac{1}{25}$$

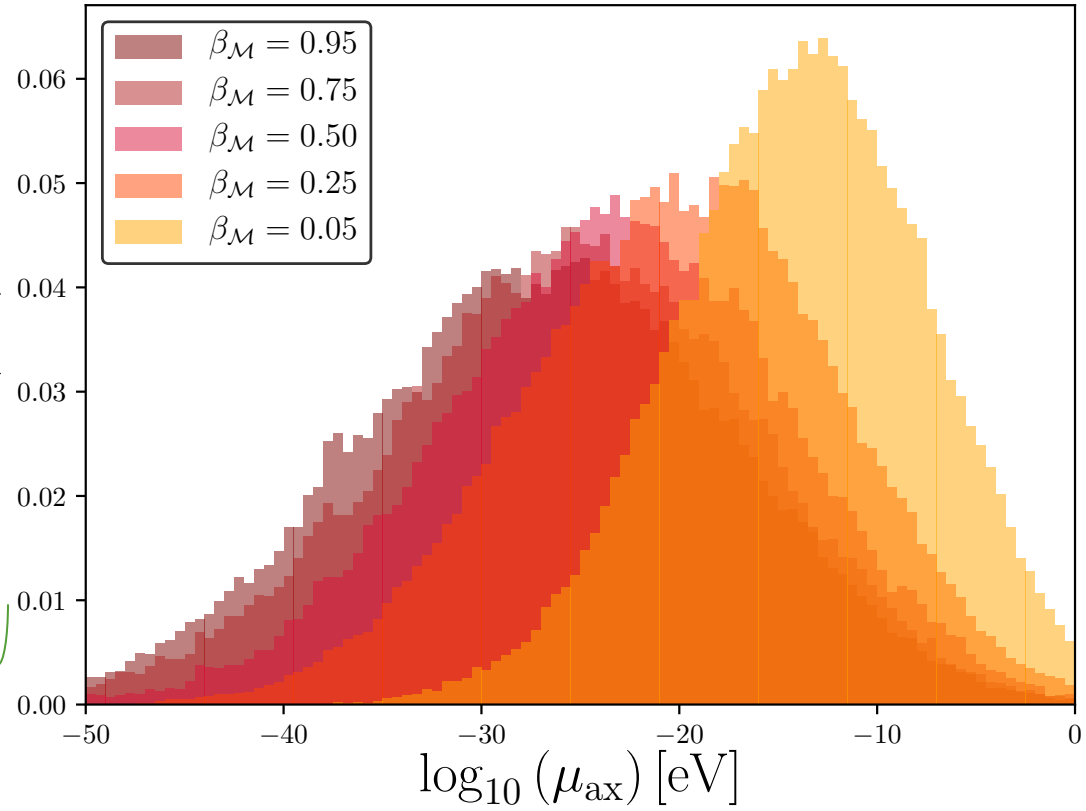
→ Lower Moments

RMT Factorisation

$$\mathcal{M}_{ij} = \frac{1}{N} A_{ir} A_{jr}$$

$$A_{ir} = \left(2\sqrt{F\widetilde{\Lambda}_r^3 C_r}\right) e^{-S_r/2} \widetilde{N}_r^i$$

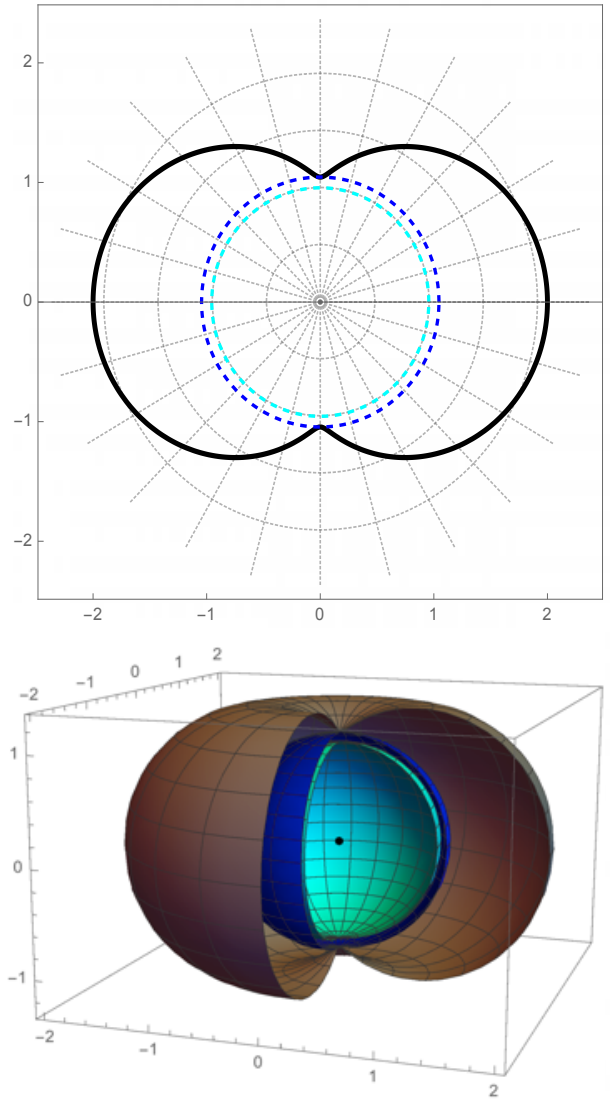
Stochastic Variables

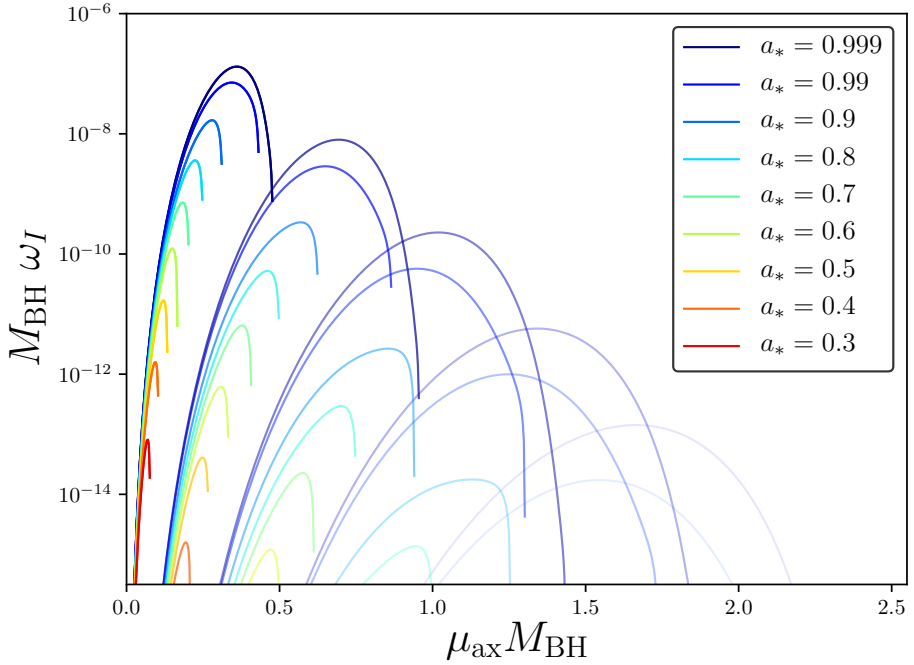


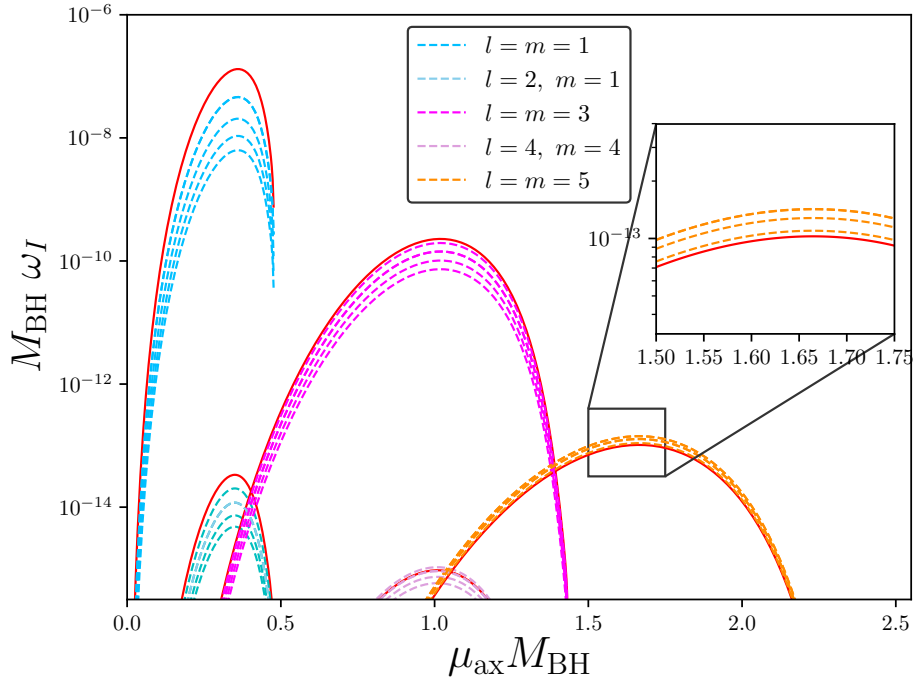
1805.02016

SUPERRADIANCE

Limits on the number of axion-like fields







Axion Instability

$$\omega_{nlm} < m\Omega_H \;\rightarrow\; \text{Unstable}$$

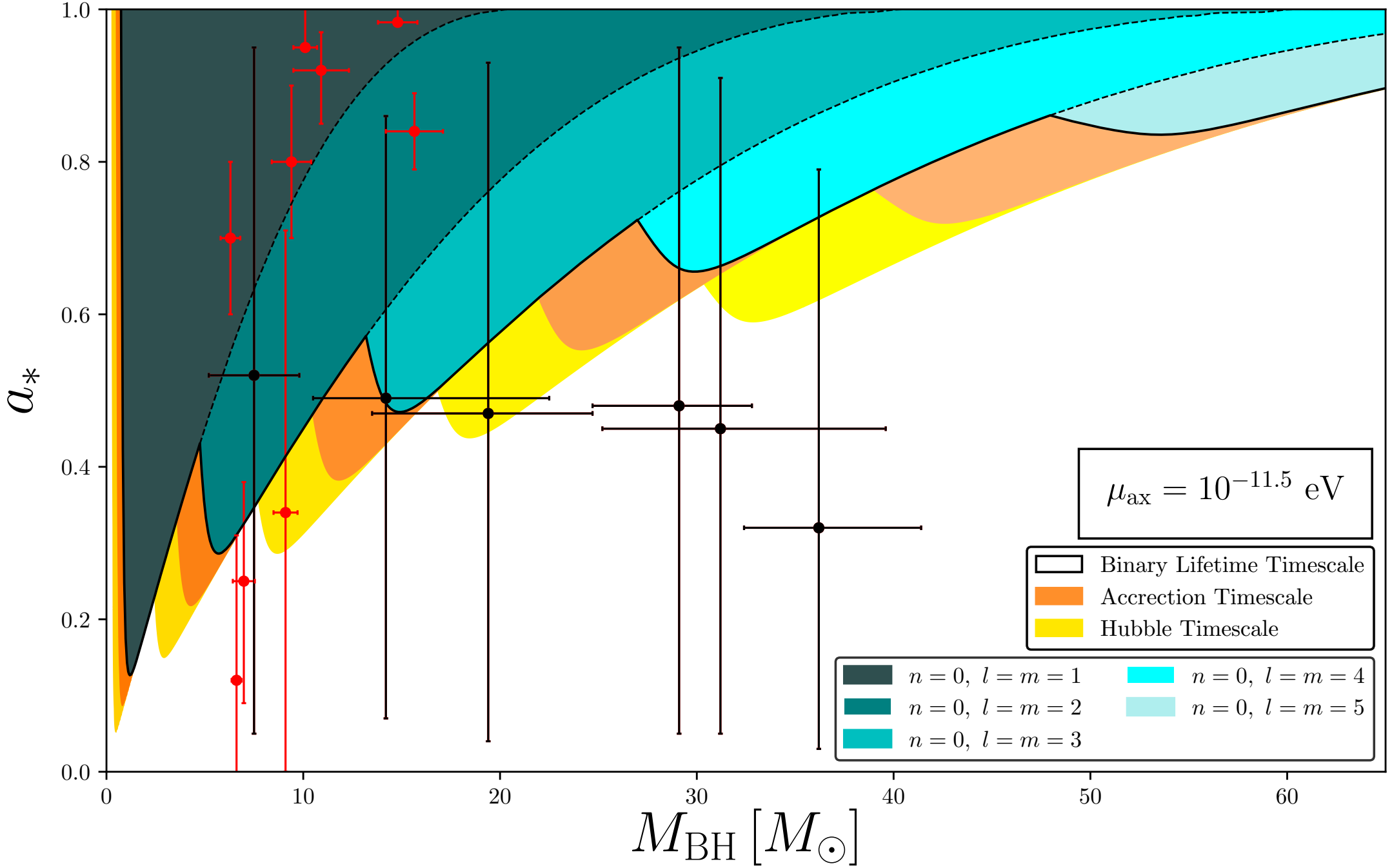
$$\omega_{nlm} = \omega_R + i\omega_I$$

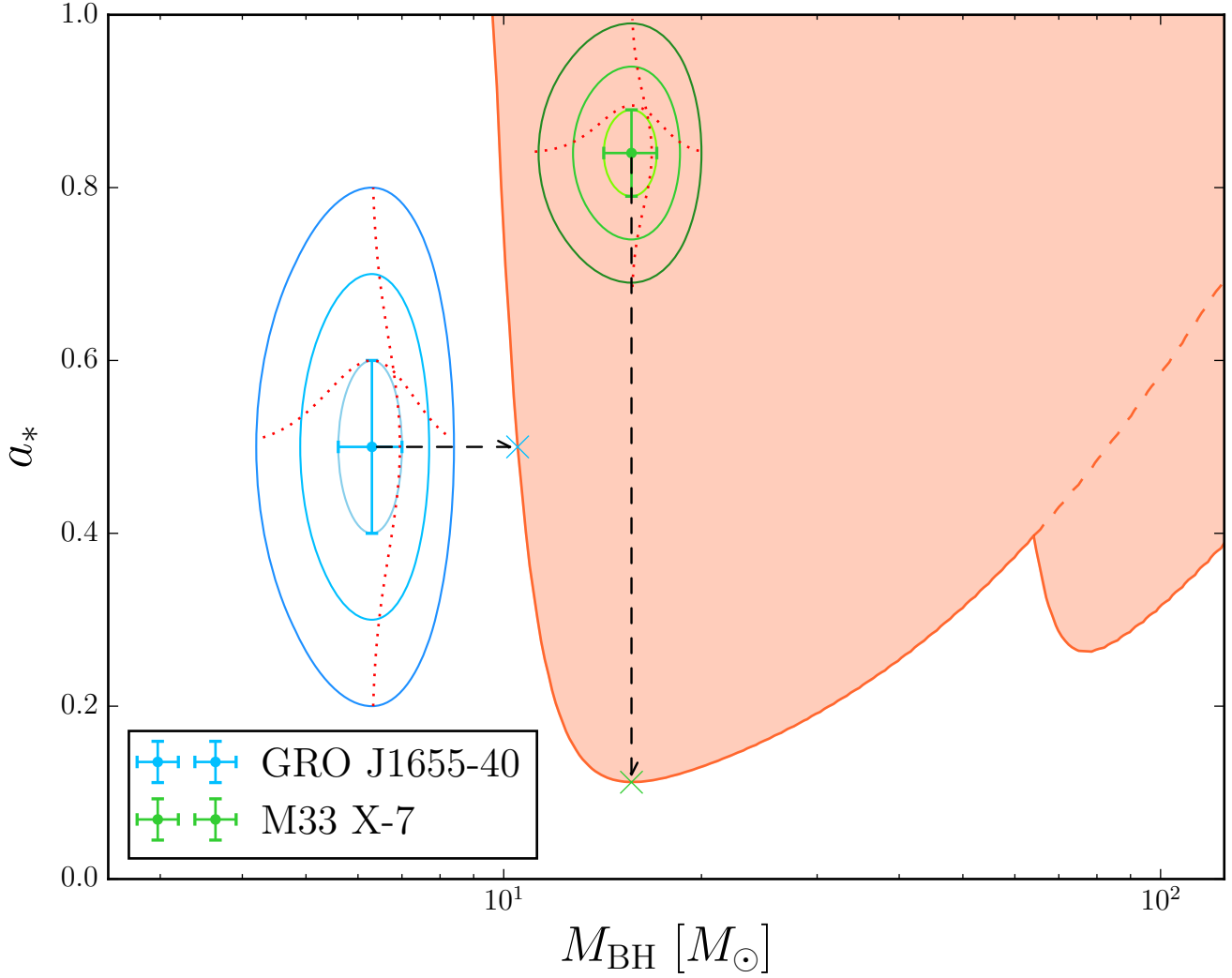
$$\omega_I \equiv \Gamma_{nlm}$$

$$\alpha = r_g \mu_{\rm ax} \equiv \mu_{\rm ax} M_{\rm BH}$$

$$\Gamma_{nlm} \propto \alpha^{4l+4} \mu_{\rm ax}$$

$$\tau_{\rm Salpeter} = \frac{\sigma_{\rm T}}{4\pi m_{\rm P}} \sim 4.5 \times 10^7 \text{ yrs}$$


1.0
0.8
0.6
0.4
0.2
0.0
a_*
0
10
20
30
40
50
60
$M_{\rm BH}\,[M_\odot]$
$\mu_{\rm ax} = 10^{-11.5}$ eV
Binary Lifetime Timescale
Accretion Timescale
Hubble Timescale
$n = 0,\ l = m = 1$
$n = 0,\ l = m = 2$
$n = 0,\ l = m = 3$
$n = 0,\ l = m = 4$
$n = 0,\ l = m = 5$



$$\mu_{\rm ax} = 10^{-12.75}\ {\rm eV}$$

SINGLE

$$P_{\rm allowed}(\mathcal{M}|\{d_i\}) = \prod_i P_{\rm allowed}(\mathcal{M}|d_i)$$

$$P_{\rm ex}(\mathcal{M}|\{d_i\}) = 1 - P_{\rm allowed}(\mathcal{M}|\{d_i\})$$

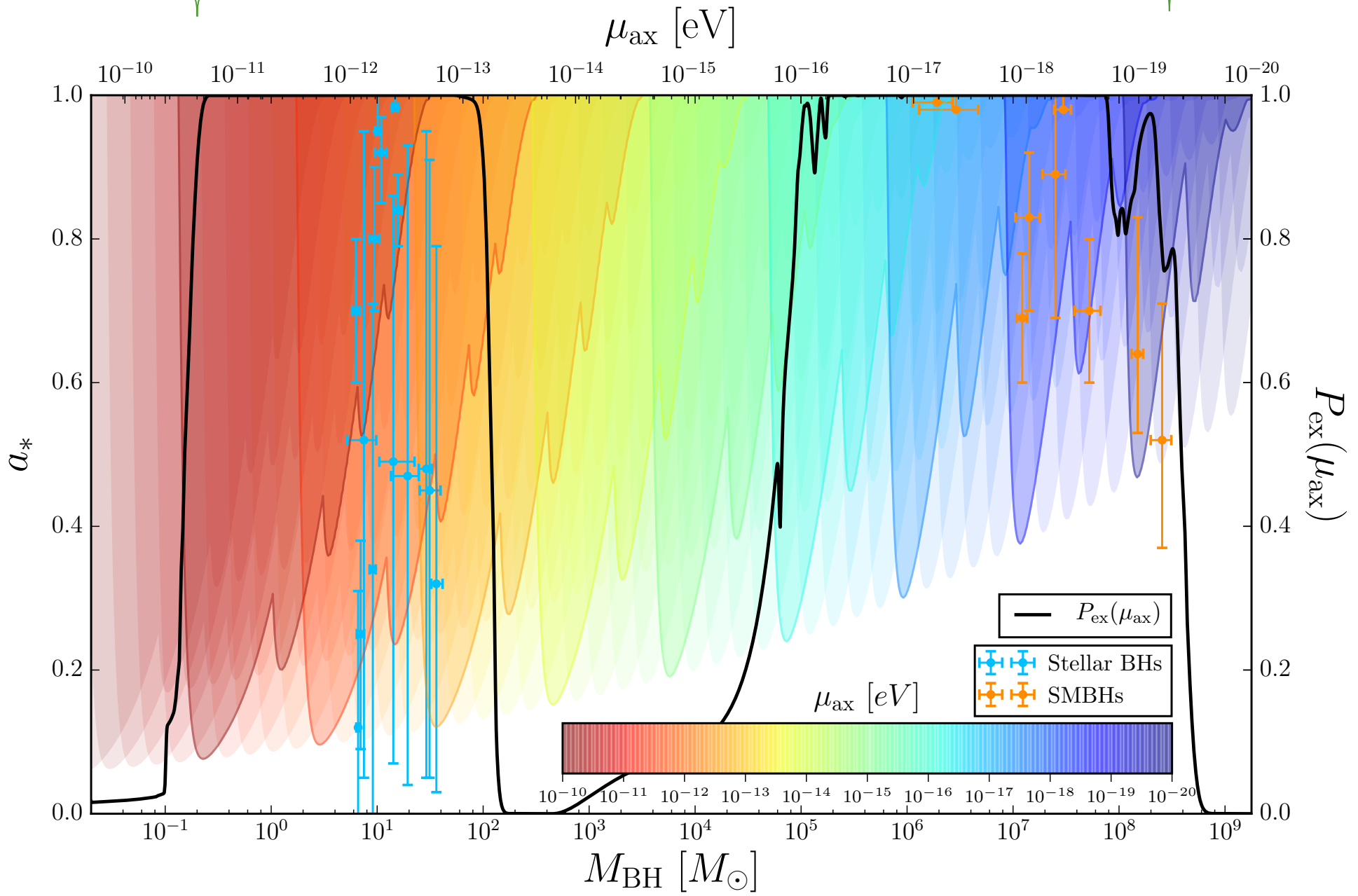
$$\Sigma_y^2 = \sigma_y^2 + f'(\bar{x})^2\sigma_x^2\,.$$

$$\Sigma_x^2 = \sigma_x^2 + g'(\bar{y})^2\sigma_y^2\,.$$

MULTIPLE

$7 \times 10^{-14}\ \mathrm{eV} \lesssim \mu_{\mathrm{ax}} \lesssim 2 \times 10^{-11}\ \mathrm{eV}$

$7 \times 10^{-20}\ \mathrm{eV} \lesssim \mu_{\mathrm{ax}} \lesssim 1 \times 10^{-16}\ \mathrm{eV}$



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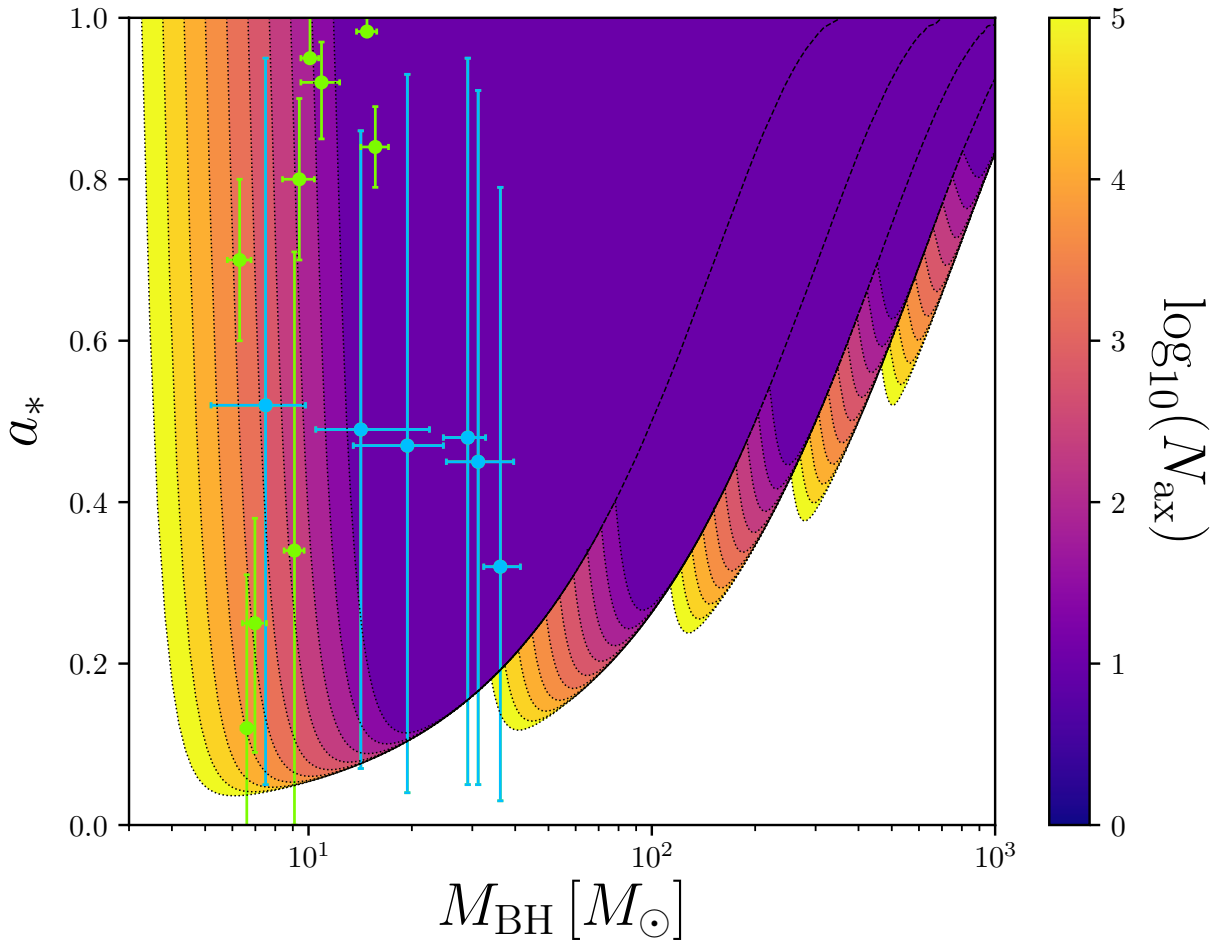


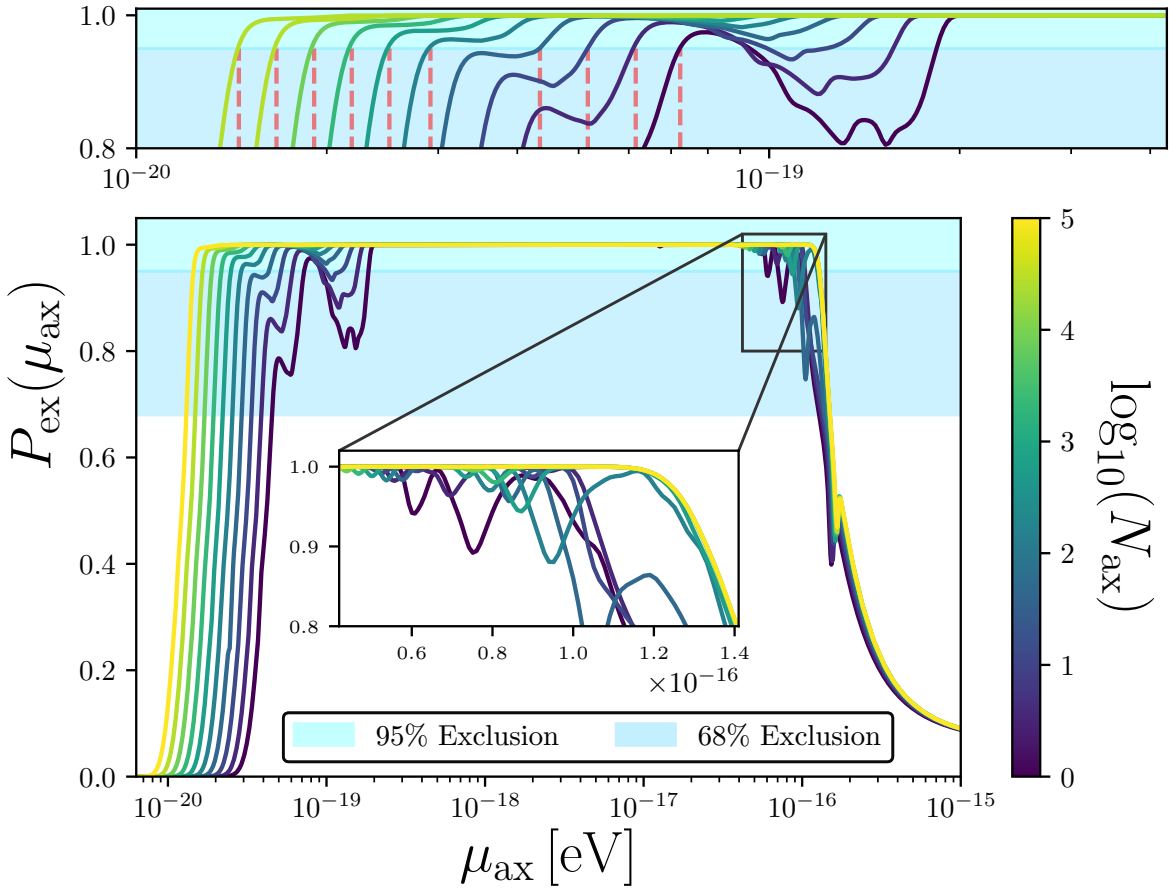
$$f_a \geq 10^{14-16}\ {\rm GeV}$$

GENERAL

$$\Gamma_{\rm tot} = \sum_i^{N_{\rm ax}} \Gamma_i$$

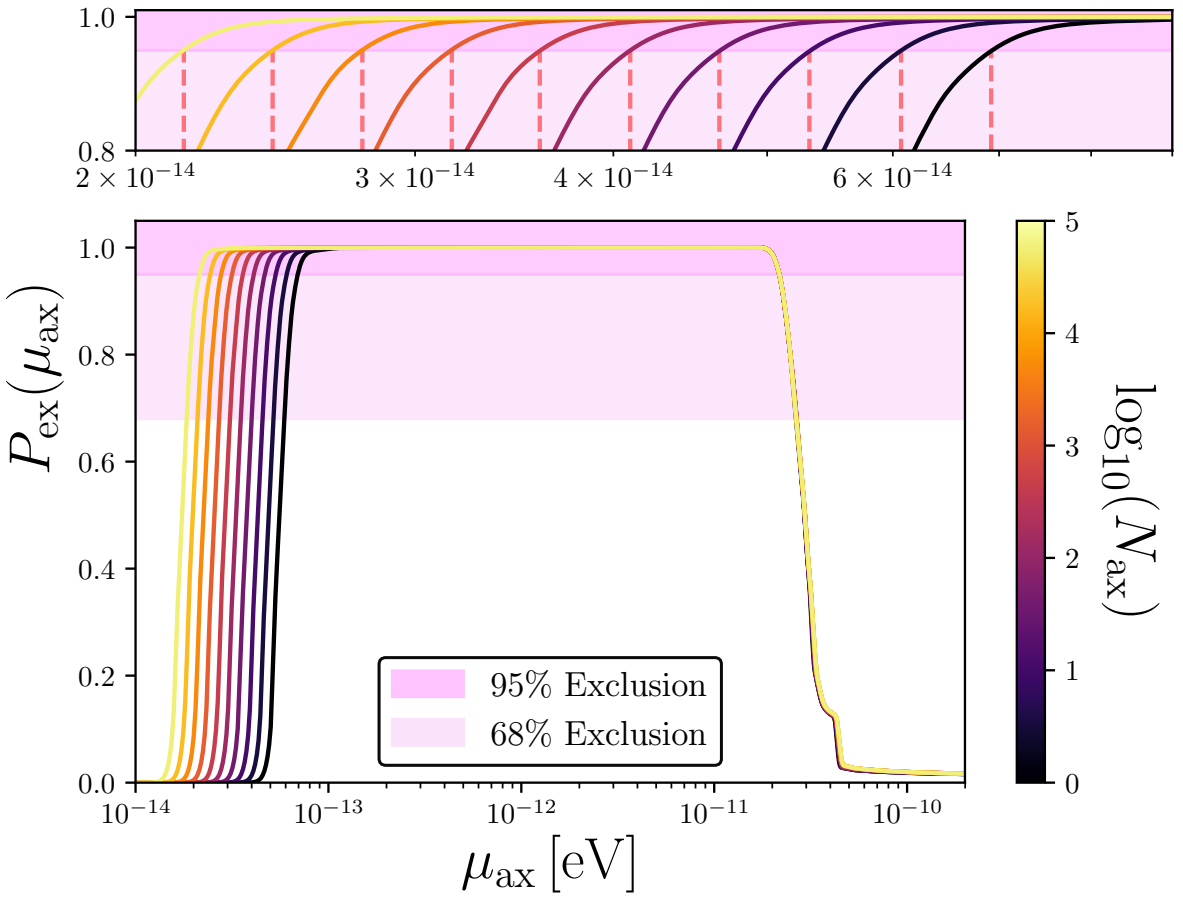
$$\Gamma_{\rm tot} = N_{\rm ax}\Gamma$$

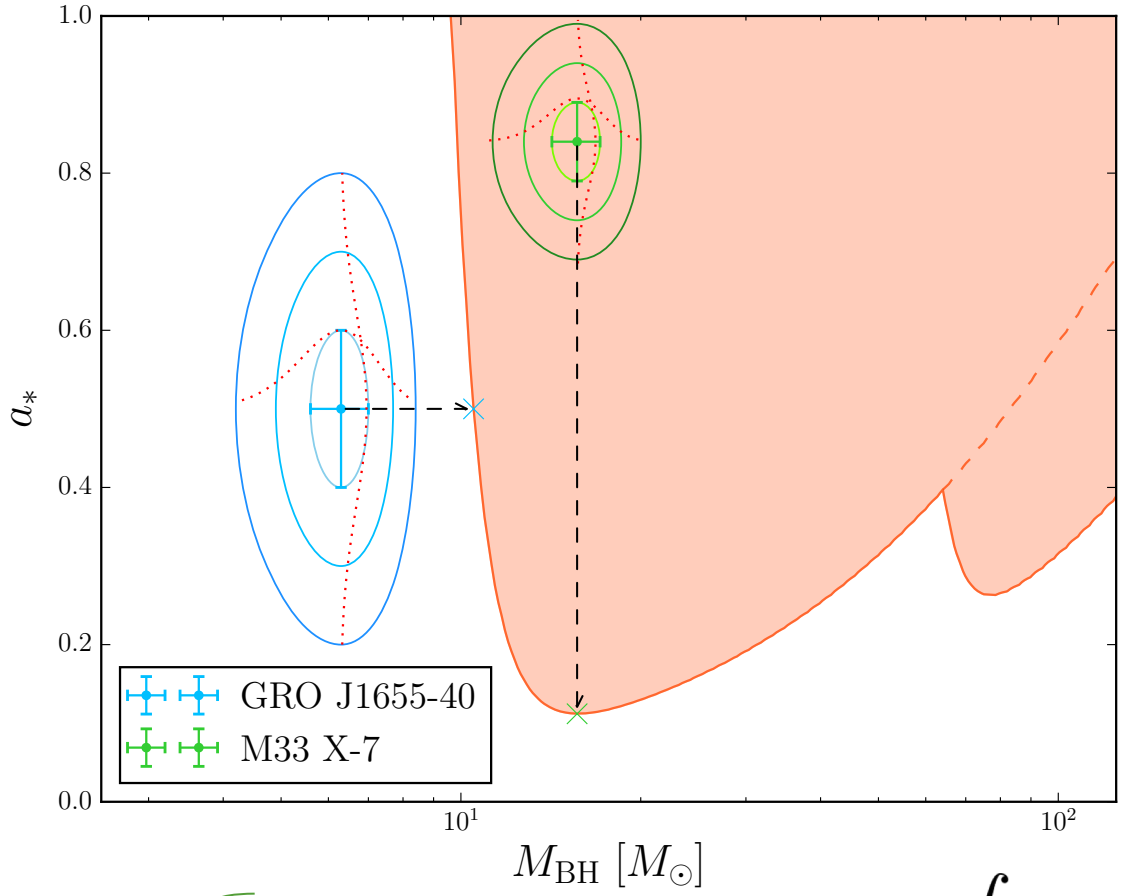
DEGENERATE



SUPERMASSIVE BLACK HOLES

STELLAR BLACK HOLES





$$P_{\rm ex}(\mathcal{M}|\{d_i\}) = 1 - P_{\rm allowed}(\mathcal{M}|\{d_i\}) \qquad \underline{\textit{SINGLE}}$$

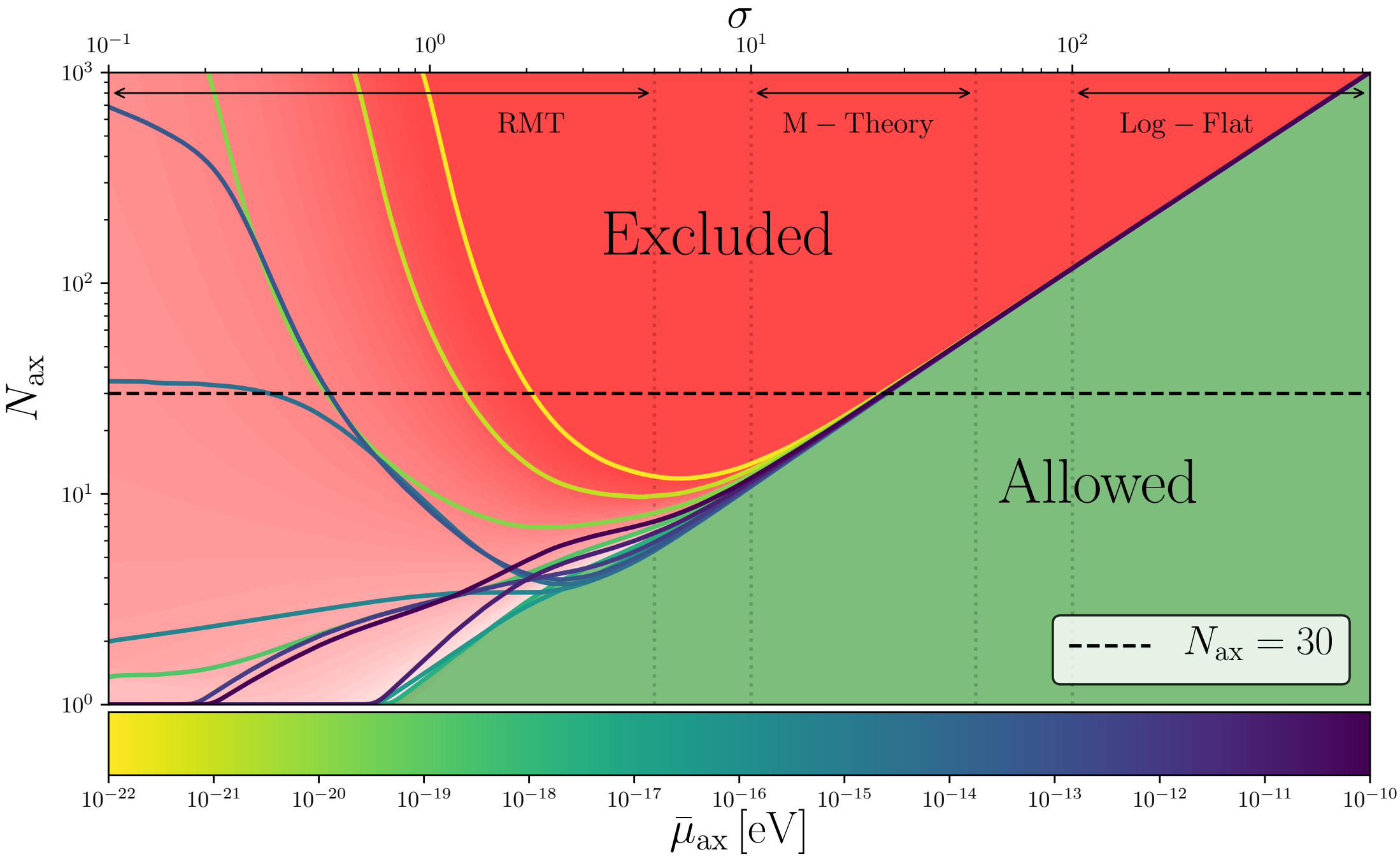
$$P_{\rm allowed}(\mathcal{M}|\{d_i\}) = \prod_i P_{\rm allowed}(\mathcal{M}|d_i)$$

$$\Sigma_y^2 = \sigma_y^2 + f'(\bar{x})^2\sigma_x^2\,.$$

$$\Sigma_x^2 = \sigma_x^2 + g'(\bar{y})^2\sigma_y^2\,.$$

$\underline{\textit{MULTIPLE}}$

$$N_{\rm ax}\begin{cases} P_{\rm al}(\theta, N_{\rm ax}=1|\mathcal{M}) = \int {\rm d}\mu_{\rm ax} p(\mu_{\rm ax}|\theta,\mathcal{M}) P_{\rm al}(\mu_{\rm ax}|N_{\rm ax}=1) \\ P_{\rm al}(\theta, N_{\rm ax}|\mathcal{M}) = \left[\int {\rm d}\mu_{\rm ax}\ p(\mu_{\rm ax}|\theta,\mathcal{M}) P_{\rm al}(\mu_{\rm ax}|N_{\rm ax}=1)\right]^{N_{\rm ax}} \\ P_{\rm ex}(\theta, N_{\rm ax}|\mathcal{M}) = 1 - P_{\rm al}(\theta, N_{\rm ax}|\mathcal{M}) \end{cases}$$


σ
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{2}
10^{1}
10^{0}
$N_{\rm ax}$
RMT
M − Theory
Log − Flat
Excluded
Allowed
$N_{\rm ax} = 30$
10^{-22}
10^{-21}
10^{-20}
10^{-19}
10^{-18}
10^{-17}
10^{-16}
10^{-15}
10^{-14}
10^{-13}
10^{-12}
10^{-11}
10^{-10}
$\bar{\mu}_{\rm ax}$ [eV]

[see Hooman Davoudiasl, Peter B. Denton]

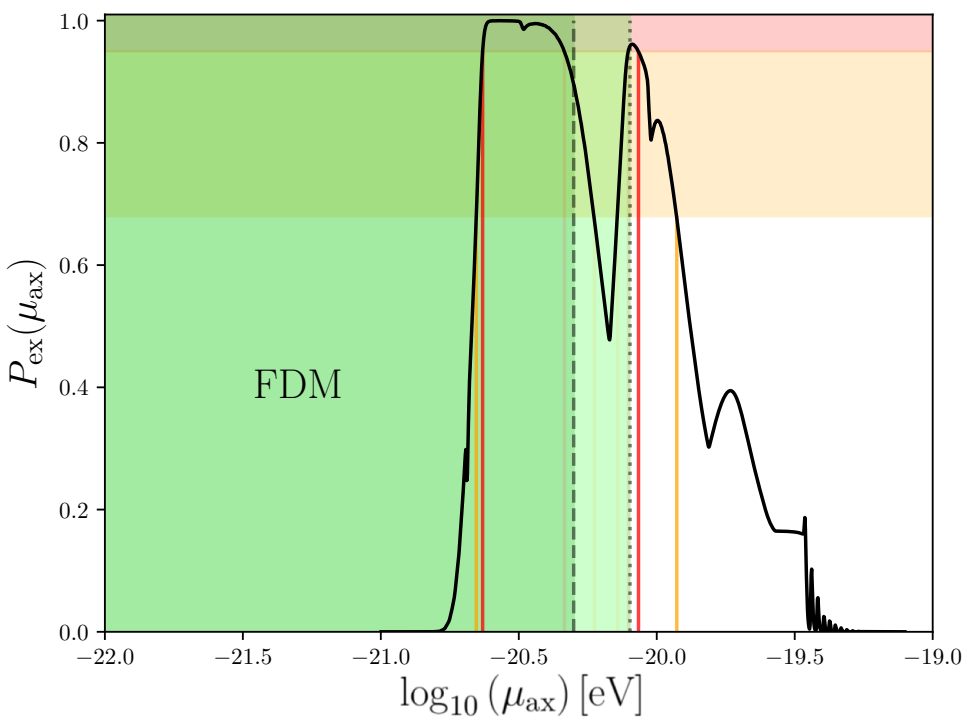
M87*

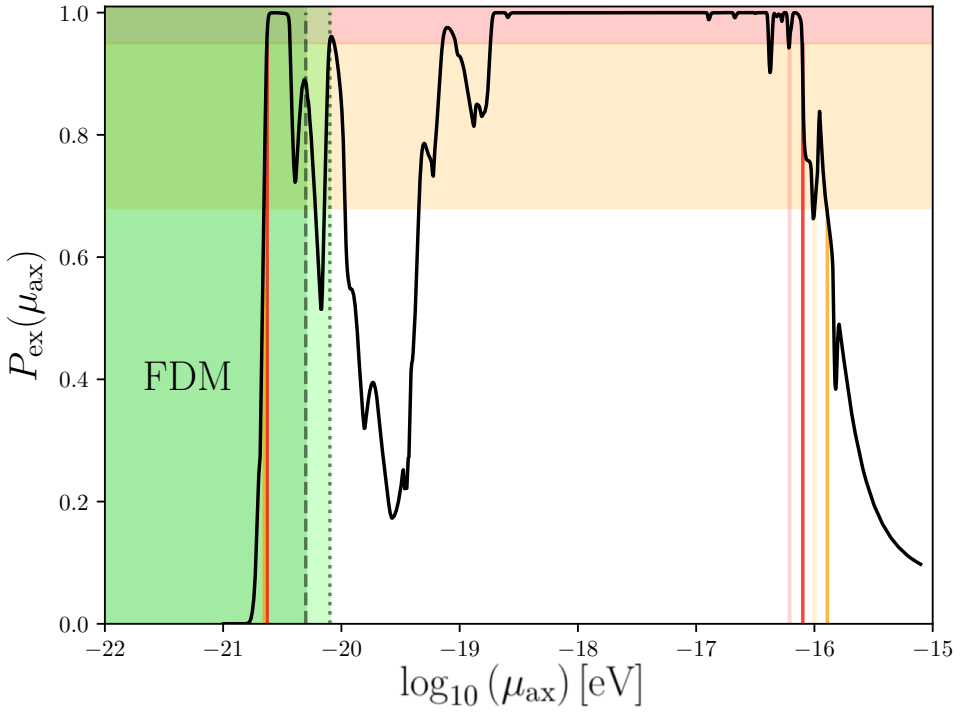
Initial Fiducial Priors

[A. Lidz and L. Hui, 2018] 21 cm FDM limits
[A. Schneider, 2018]

[K. Akiyama et al.] BH Mass

[Fabrizio Tamburini, Bo Thidé, Massimo Della Valle] BH Spin





THANK YOU !