Local Heterotic Reductions

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Based on work in collaboration with Bobby Acharya, Alex Kinsella
1907.xxxxx, etc

June 27, CERN

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Introduction and Motivation

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Increased recent interest in considering M-theory on local G_2 manifolds:

- Mathematics: Improve our understanding of G_2 structures through local description.
- Physics: Local understanding of *chiral matter* and higher co-dimension singularities.

A natural reduction of the seven-dimensional geometry results in a three-dimensional Hitchin system:

- Abelian: [Acharya-Witten ’01, Pantev-Wijnholt ’09, Braun-Cizel-Hubner-SchferNameki ’18].
- Non-Anbelian: [Barbosa-Cvetic-Heckman-Lawrie-Torres-Zoccarato ’19]; relations to T-branes and systematic approach initiated.

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This talk: I will consider reductions of the *dual heterotic system*:

- α'-corrections back-react on the geometry.
- Some explicit solutions.
- Reduced moduli problem and coupling between matter and gravitational degrees of freedom.

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The reduction of the geometry on the heterotic side derives from reducing on the T^3-fiber of the conjectured SYZ-fibration [Strominger-Yau-Zaslow '96].

A reduction of the Hermitian Yang-Mills equations again results in a three-dimensional Hitchin system

$$\begin{aligned} F &= A^\phi \wedge A^\phi \\ \mathrm{d}_\nabla A^\phi &= 0 \\ \mathrm{d}_\nabla^\dagger A^\phi &= 0\,, \end{aligned}$$

where $A^\phi \in \Omega^1(\mathrm{End}(V))$ is a one-form Higgs field. The first two equations derive form requiring the six-dimensional bundle to be *holomorphic*, while the last equation derives from the Yang-Mills *D-term*, or *stability constraint*.

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An SYZ reduction

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Note that the first two equations can be re-parametrised as a flatness condition for a complexified connection $\mathcal{A} = A + iA^\phi$.

Working on flat space, such solutions only makes sense *locally*. In particular, non-trivial solutions will always have infinite energy [Gagliardo-Uhlenbeck '14]. Related to introduction of *localised sources*. Reflect breakdown of local description.

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The zeroth order $SU(3)$ structure is given by a flat metric and holomorphic top-form

$$\Omega^{(0)} = \mathrm{d}z^1 \wedge \mathrm{d}z^2 \wedge \mathrm{d}z^3$$
$$\omega^{(0)} = i \sum \mathrm{d}z^i \wedge \mathrm{d}\overline{z}^i ,$$

where $\mathrm{d}z^i = \mathrm{d}x^i + i\mathrm{d}y^i$ and y^i correspond to T^3 coordinates.

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where $\mathrm{d}z^i = \mathrm{d}x^i + i\mathrm{d}y^i$ and y^i correspond to T^3 coordinates.

The heterotic gauge sector back-reacts on the geometry through the six-dimensional heterotic Bianchi-identity

$$\partial\overline{\partial}\omega = -i\frac{\alpha'}{8}\,\mathrm{tr}\,F^{(6)} \wedge F^{(6)} + \mathcal{O}(\alpha'^2)$$

For “most” solutions, the back-reaction on the reduced geometry can be explicitly calculated:

$$\delta_{\alpha'} g_{ij} \propto \alpha'\,\mathrm{tr} A_i^\phi A_j^\phi$$
$$\partial_i \phi \propto \alpha'\,\nabla^j \left(\mathrm{tr} A_i^\phi A_j^\phi\right) \ ,$$

so that the corresponding reduced system again defines an $SU(3)$ structure satisfying the Hull-Strominger system.

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For Abelian bundles, the connection is flat, and we can write the Higgs field as

$$A^{\phi} = \mathrm{d}h \; ,$$

where h is a harmonic function.

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$$A^\phi = \mathrm{d}h \,,$$

where h is a harmonic function.

Standard Monopole: Here

$$h \propto \tfrac{1}{r} \,.$$

The back-reaction on the geometry may be absorbed into the dilaton as $\delta_{\alpha'}(e^\phi) \propto \alpha'/r^4$ and $g_{ij} = e^\phi \delta_{ij}$.

Note: If treated as an exact solution this can be shown to have *finite energy*.

Cylindrical Solution: Here

$$h \propto \log(r_2) \,,$$

Back-reaction on geometry: $\delta_{\alpha'}(e^\phi) \propto \alpha'/r_2^2$ and $g_{ij} = e^\phi \delta_{ij}$.

Saddle Point Solution [Pantev-Wijnholt '09, Braun etal '18]: Here e.g.

$$h \propto x_1 x_2 + x_2 x_3 + x_3 x_1 \,.$$

Back-reaction: $\delta_{\alpha'}(e^\phi) \propto \alpha' \, h$ and $g_{ij} = e^\phi \delta_{ij}$.

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To find non-Abelian solution, utilise that we have a complex flat connection (See also [Barbosa etal '19]):

$$\mathcal{A} = -i\, G^{-1}\mathrm{d}G \ ,$$

where g lives in *the complexified* Lie algebra.

Consider an $SU(2)$ gauge group $\Rightarrow$ $G \in \Gamma\left(SL(2,\mathbb{C})\right)$.

"Radial" example: Ansatz inspired by the t'Hooft-Polyakov monopole:

$$G = \sinh(g(r))\hat{x}_i\sigma_i + \cosh(g(r))\mathrm{I}_2 \ , \qquad \hat{x}_i = \frac{x_i}{r} \ .$$

Stability equation $\mathrm{d}_{\nabla}^{\dagger} A^{\phi} = 0$ results in the equation for $g(r)$:

$$\sinh(4g(t)) = 2t^2 g''(t) \ , \qquad t = \frac{1}{r} \ .$$

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- (Numerical) solutions have one or more divergencies at finite r (sources).
- Find solutions with localised matter. Investigate further using systematic methods of [Barbosa etal '19]).
- Equations for $SU(2)$ system very often of (Euclidean) Sinh-Gordon type. Classification in terms of n-solitons possible?

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Six-dimensional heterotic moduli are governed by a holomorphic Chern-Simons action derived from the superpotential [Ashmore-delaOssa-Minasian-StricklandConstable-ES '18]:

$$\Delta W = \int_{X_6} \left(\langle y, \overline{D} y \rangle - \tfrac{2}{3} \langle y, [y, y] \rangle \right) \wedge \Omega \, ,$$

where $y \in \Omega^{(0,1)} \left(T^{*(1,0)} \oplus \mathrm{End}(V) \oplus T^{(1,0)} \right)$. EOM gives Maurer-Cartan equation for heterotic moduli problem (Should be supplemented with D-term conditions).

Reduces to an interesting generalisation of complex CS theory

$$S = \int_{M_3} \left(\langle x, \mathcal{D} x \rangle - \tfrac{2}{3} \langle x, [x, x] \rangle \right)$$

where now $x \in \Omega^1_{\mathbb{C}}(T^* M \oplus \mathrm{End}(V) \oplus TM)$.

- Study explicit local realisations of moduli structure, e.g. Atiyah mechamisn?
- Supersymmetric version? Can we localise theory given the explicit nature of the 3d system?
- Get CS theory coupled to gravitational dof's. Relation to gravitational corrections to open string?

Thank you!

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Thank you for your attention!