Local Heterotic Reductions

Eirik Eik Svanes (King’s College London, ICTP Trieste)
Based on work in collaboration with Bobby Acharya, Alex Kinsella
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Introduction and Motivation
Increased recent interest in considering M-theory on local $G_2$ manifolds:

- **Mathematics**: Improve our understanding of $G_2$ structures through local description.
- **Physics**: Local understanding of *chiral matter* and higher co-dimension singularities.

A natural reduction of the seven-dimensional geometry results in a three-dimensional Hitchin system:

- **Abelian**: [Acharya-Witten ’01, Pantev-Wijnholt ’09, Braun-Cizel-Hubner-SchferNameki ’18].
- **Non-Abelian**: [Barbosa-Cvetic-Heckman-Lawrie-Torres-Zoccarato ’19]; relations to T-branes and systematic approach initiated.
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This talk: I will consider reductions of the *dual heterotic system*:

- $\alpha'$-corrections back-react on the geometry.
- Some explicit solutions.
- Reduced moduli problem and coupling between matter and gravitational degrees of freedom.
Heterotic Hitchin System
The reduction of the geometry on the heterotic side derives from reducing on the $T^3$-fiber of the conjectured SYZ-fibration [Strominger-Yau-Zaslow ’96].

A reduction of the Hermitian Yang-Mills equations again results in a three-dimensional Hitchin system

\[
F = A^\phi \wedge A^\phi
\]

\[
d\nabla A^\phi = 0
\]

\[
d^{\dagger} \nabla A^\phi = 0,
\]

where $A^\phi \in \Omega^1(\text{End}(V))$ is a one-form Higgs field. The first two equations derive form requiring the six-dimensional bundle to be \textit{holomorphic}, while the last equation derives from the Yang-Mills \textit{D-term}, or \textit{stability constraint}. 
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$$\begin{align*}
F &= A^\phi \wedge A^\phi \\
\text{d} \nabla A^\phi &= 0 \\
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\end{align*}$$

where $A^\phi \in \Omega^1(\text{End}(V))$ is a one-form Higgs field. The first two equations derive form requiring the six-dimensional bundle to be holomorphic, while the last equation derives from the Yang-Mills D-term, or stability constraint.

Note that the first two equations can be re-parametrised as a flatness condition for a complexified connection $\mathcal{A} = A + iA^\phi$.

Working on flat space, such solutions only makes sense locally. In particular, non-trivial solutions will always have infinite energy [Gagliardo-Uhlenbeck ’14]. Related to introduction of localised sources. Reflect breakdown of local description.
The zeroth order $SU(3)$ structure is given by a flat metric and holomorphic top-form

$$\Omega^{(0)} = dz^1 \wedge dz^2 \wedge dz^3$$

$$\omega^{(0)} = i \sum dz^i \wedge d\bar{z}^i,$$

where $dz^i = dx^i + i dy^i$ and $y^i$ correspond to $T^3$ coordinates.
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The heterotic gauge sector back-reacts on the geometry through the six-dimensional heterotic Bianchi-identity

\[ \partial \overline{\partial} \omega = -i \frac{\alpha'}{8} \text{tr} F^{(6)} \wedge F^{(6)} + O(\alpha'^2) \]

For “most” solutions, the back-reaction on the reduced geometry can be explicitly calculated:

\[ \delta_{\alpha'} g_{ij} \propto \alpha' \text{tr} A_i^\phi A_j^\phi \]
\[ \partial_i \phi \propto \alpha' \nabla^j \left( \text{tr} A_i^\phi A_j^\phi \right), \]

so that the corresponding reduced system again defines an $SU(3)$ structure satisfying the Hull-Strominger system.
Examples

Abelian examples
Non-Abelian examples

Moduli
For Abelian bundles, the connection is flat, and we can write the Higgs field as

$$A^\phi = dh,$$

where $h$ is a harmonic function.
Abelian examples

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\[ A^\phi = dh , \]

where \( h \) is a harmonic function.

**Standard Monopole:** Here

\[ h \propto \frac{1}{r} . \]

The back-reaction on the geometry may be absorbed into the dilaton as \( \delta_{\alpha'} (e^\phi) \propto \alpha' / r^4 \) and \( g_{ij} = e^\phi \delta_{ij} \).

Note: If treated as an exact solution this can be shown to have *finite energy*.

**Cylindrical Solution:** Here

\[ h \propto \log(r_2) , \]

Back-reaction on geometry: \( \delta_{\alpha'} (e^\phi) \propto \alpha' / r_2^2 \) and \( g_{ij} = e^\phi \delta_{ij} \).

**Saddle Point Solution [Pantev-Wijnholt ’09, Braun etal ’18]:** Here e.g.

\[ h \propto x_1 x_2 + x_2 x_3 + x_3 x_1 . \]

Back-reaction: \( \delta_{\alpha'} (e^\phi) \propto \alpha' h \) and \( g_{ij} = e^\phi \delta_{ij} \).
To find non-Abelian solution, utilise that we have a complex flat connection (See also [Barbosa et al ’19]):

\[ A = -i G^{-1} dG , \]

where \( g \) lives in the complexified Lie algebra.

Consider an \( SU(2) \) gauge group \( \Rightarrow G \in \Gamma (SL(2, \mathbb{C})) \).

“Radial” example: Ansatz inspired by the t’Hooft-Polyakov monopole:

\[ G = \sinh(g(r))\hat{x}_i \sigma_i + \cosh(g(r))I_2 \ , \quad \hat{x}_i = \frac{x_i}{r} \ . \]

Stability equation \( d^\dagger \nabla A^\phi = 0 \) results in the equation for \( g(r) \):

\[ \sinh(4g(t)) = 2t^2 g''(t) , \quad t = \frac{1}{r} . \]
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- (Numerical) solutions have one or more divergencies at finite \( r \) (sources).
- Find solutions with localised matter. Investigate further using systematic methods of [Barbosa et al ’19]).
- Equations for \( SU(2) \) system very often of (Euclidean) Sinh-Gordon type. Classification in terms of \( n \)-solitons possible?
Reduced Moduli System
Six-dimensional heterotic moduli are governed by a holomorphic Chern-Simons action derived from the superpotential [Ashmore-delaOssa-Minasian-Strickland-Constable-ES '18]:

\[ \Delta W = \int_{X_6} \left( \langle y, \overline{D} y \rangle - \frac{2}{3} \langle y, [y, y] \rangle \right) \wedge \Omega, \]

where \( y \in \Omega^{(0,1)} \left( T^{*^{(1,0)}} \oplus \text{End}(V) \oplus T^{(1,0)} \right). \) EOM gives Maurer-Cartan equation for heterotic moduli problem (Should be supplemented with D-term conditions).

Reduces to an interesting generalisation of complex CS theory

\[ S = \int_{M_3} \left( \langle x, D x \rangle - \frac{2}{3} \langle x, [x, x] \rangle \right) \]

where now \( x \in \Omega^1_{\mathbb{C}}(T^* M \oplus \text{End}(V) \oplus TM). \)

- Study explicit local realisations of moduli structure, e.g. Atiyah mechanism?
- Supersymmetric version? Can we localise theory given the explicit nature of the 3d system?
- Get CS theory coupled to gravitational dof’s. Relation to gravitational corrections to open string?
Thank you for your attention!