The Calabi-Yau Hypersurface Landscape

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String Pheno 2019

Based on works with (various subsets of):
Cody Long, Liam McAllister, Mike Stillman, Andres Rios Tascon

What?

We study large ensembles of Calabi-Yau hypersurfaces in toric varieties.

- Develop algorithms and computational tools.

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- Study landscape statistics.
  - Black hole superradiance: See Viraf Mehta's talk!
    [MD, Cody Long, David J. E. Marsh, Liam McAllister, Viraf M. Mehta, Matthew J. Stott , work in progress]
  - Large $h^{1,1}$ + α'under control = Ultralight axions
    [MD, Cody Long, Liam McAllister, Mike Stillman, 1808.01282]

Plan

Part 1 - Tools and Algorithms

Part 2 - Machine Learning

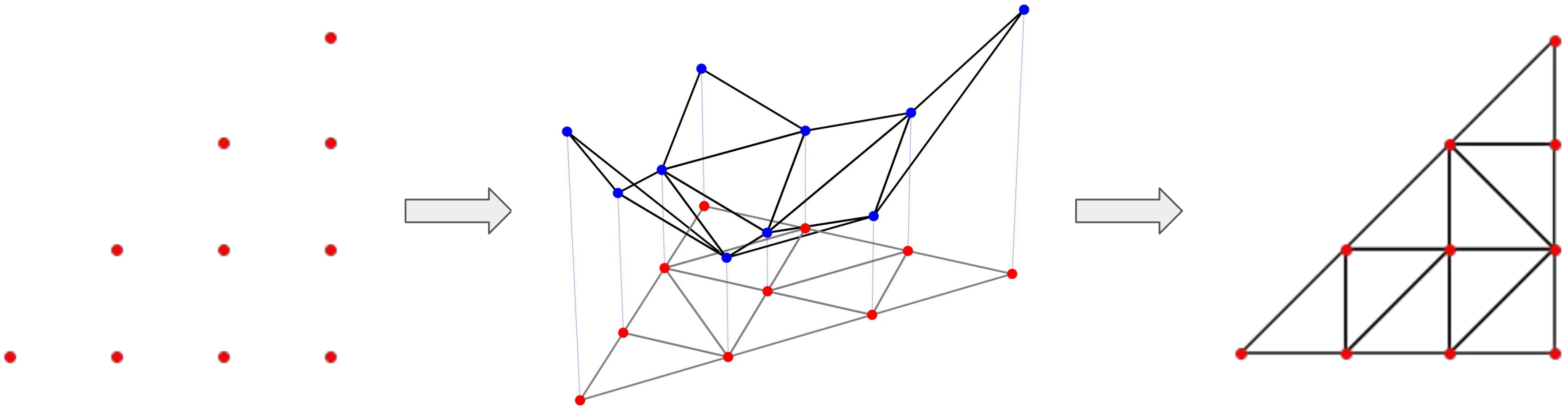
A Quick Review

- A fine, regular, star triangulation (FRST) of a reflexive polytope Δ° defines a toric variety V that has a CY hypersurface X. [Batyrev, alg-geom/9310003]
  - Fine: Use all points of Δ°.
  - Regular: See next slide.
  - Star: All simplices include the origin.
- Number of points of Δ°: $h^{1,1}+4$. (When Δ° is favorable)
- 473,800,776 reflexive polytopes in 4d. [Kreuzer, Skarke, hep-th/0002240]
  - $1 \leq h^{1,1} \leq 491$
- Number of FRSTs increases *exponentially* with $h^{1,1}$.
  - Can compute upper bounds on the number of FRSTs. See Andres Rios Tascon's talk!

[MD, Liam McAllister, Andres Rios Tascon, 1908.XXXXX]

Regular Triangulations

- 'Lift' each point: $\vec{p_i} = (x_i, y_i) \to \vec{p_i}\,' = (x_i, y_i, h_i)$
- Take the lower part of the convex hull.
- Explore the space of regular triangulations by changing $\vec{h}$!



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- Find genus-one fibrations
  - Need Mori Cone of X to find *all* fibrations. [MD, Liam McAllister, Andres Rios Tascon, 1908.XXXXX]

[Huang, Taylor, 1805.05907]
[Huang, Taylor, 1809.05160]
[Huang, Taylor, 1811.04947]
[Anderson, Gao, Gray, Lee, 1708.07907]
[Kollár, math.AG/1206.5721]

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2014	25	a few hours
2017	491	2s
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Compute κ^{ijk}

Year	$h^{1,1}$	CPU time
2017	100	30 mins
2018	491	30s
2019	491	3s

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- Main difficulty: How do you feed a triangulation to an ML algorithm?
    - Machine learning with structured data:

      [Battaglia et al., cs.LG/1806.01261]
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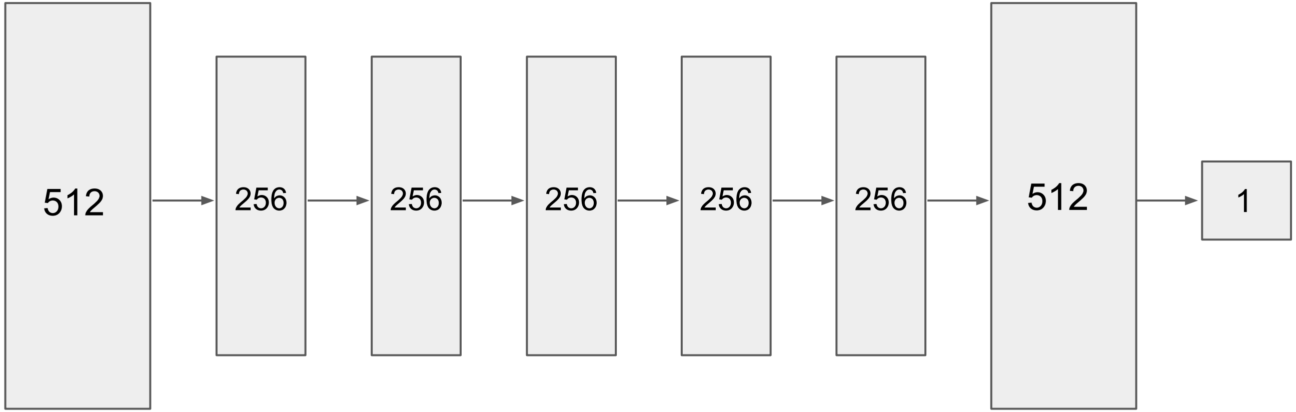
- Solution for regular triangulations: Use $\vec{h}$!

A demonstration at $h^{1,1} = 15$

- Single (randomly chosen) reflexive polytope.
- Training set: ~850,000 height vectors corresponding to ~ 255,000 FRSTs.
- Test set: ~150,000 height vectors corresponding to ~45,000 FRSTs.
- A simple neural network architecture:



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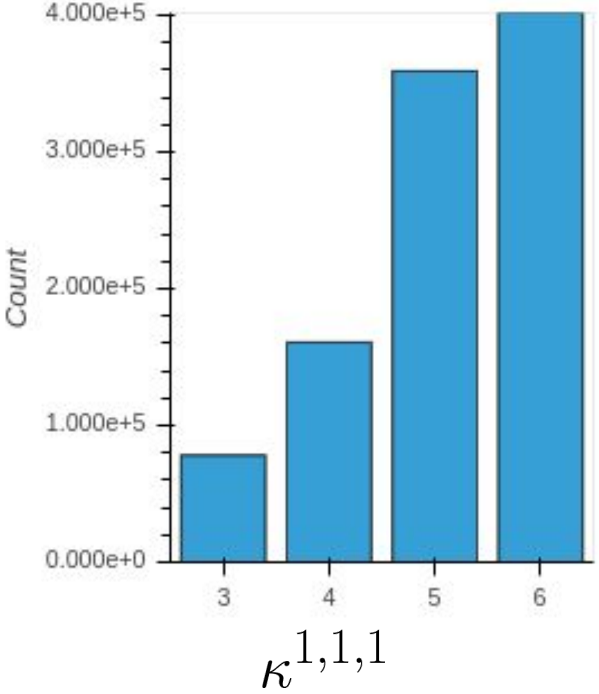

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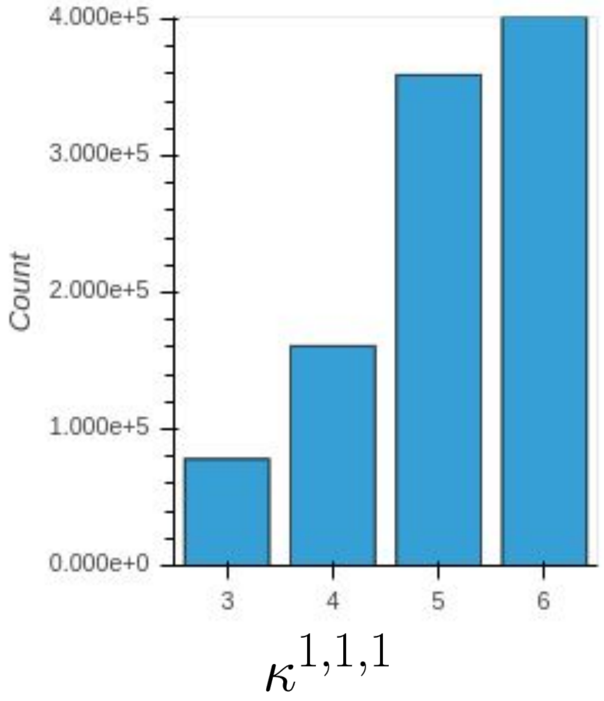

Performance:

Training Time	Accuracy
5 minutes	92%
1 hour	97.3%

★ We can predict intersection numbers with ML!

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1. Pick a basis $\{\omega_1, \ldots, \omega_{15}\}$ for $H^2(X, \mathbb{Z})$.
2. Expand the Kähler form $J = t^i \omega_i$.
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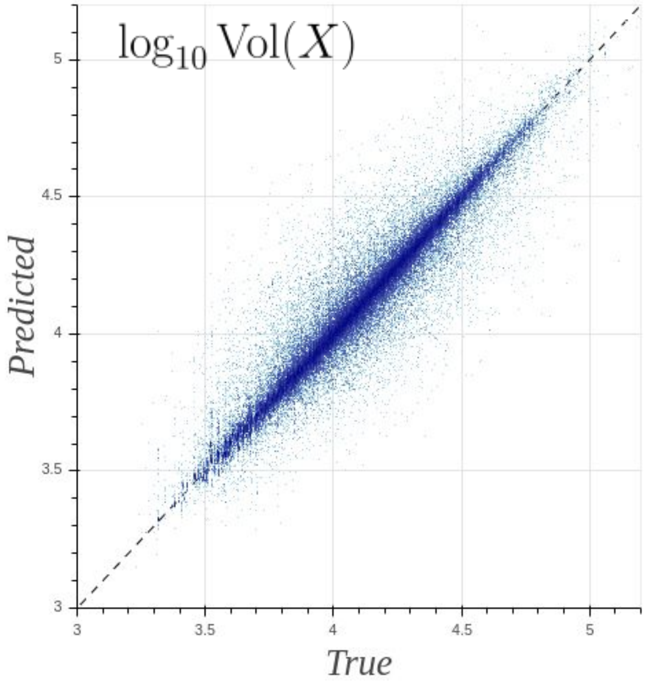

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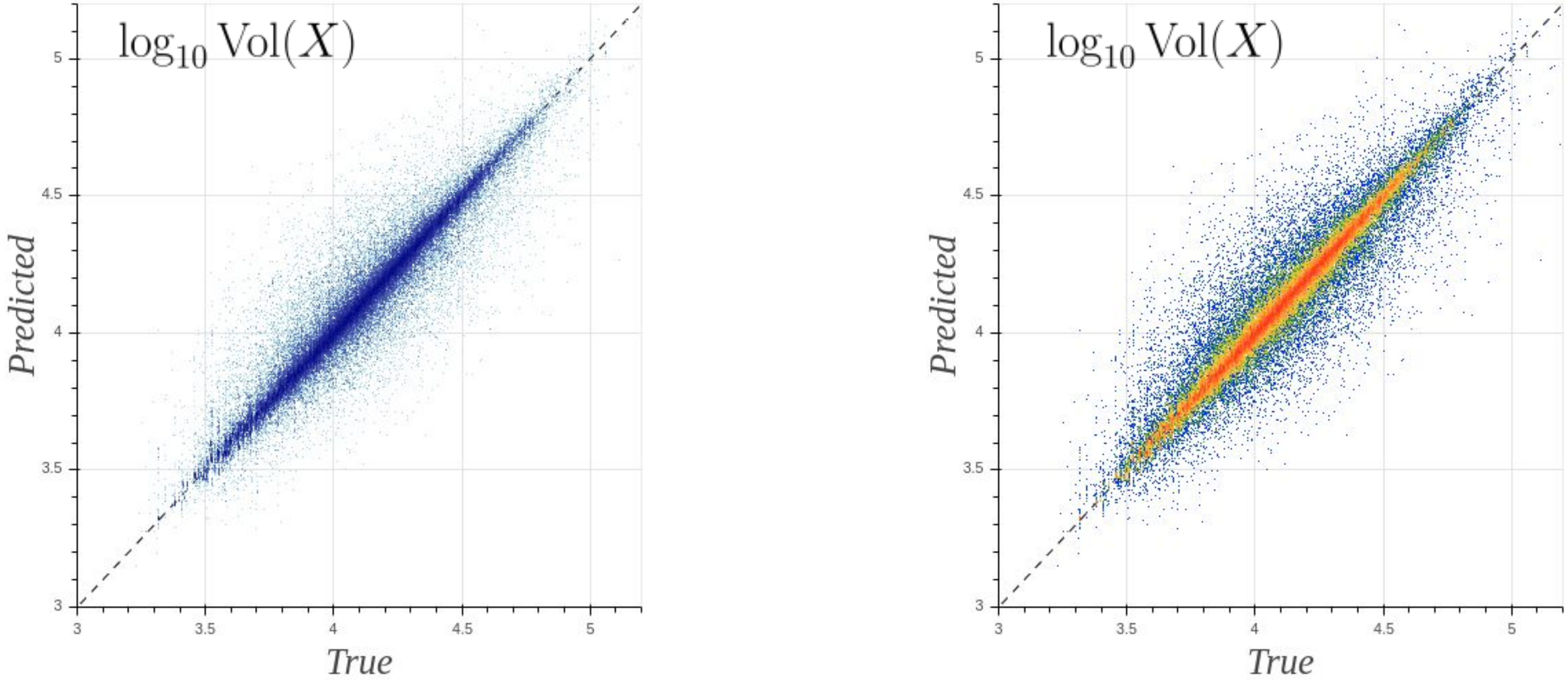

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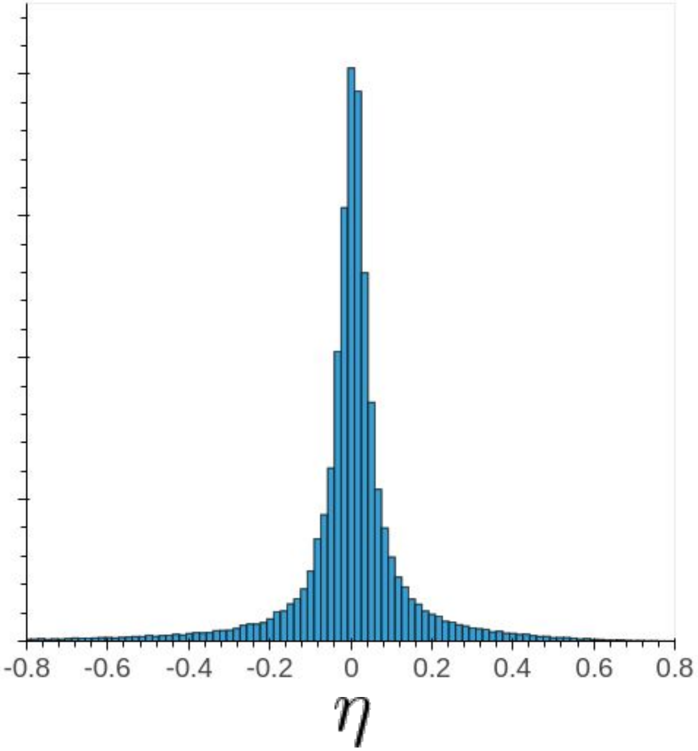
Performance:

$$\eta = \frac{\mathrm{Vol}(X)_{\mathrm{True}} - \mathrm{Vol}(X)_{\mathrm{Pred}}}{\mathrm{Vol}(X)_{\mathrm{True}}}$$

- $\eta < 0.25$: 90% of the time
- $\eta < 0.5$: 99% of the time



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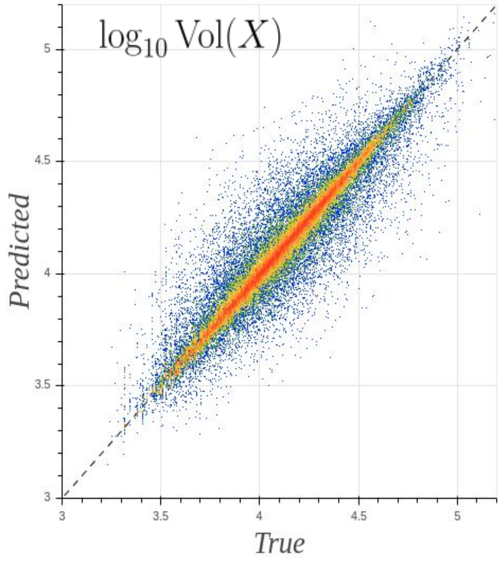

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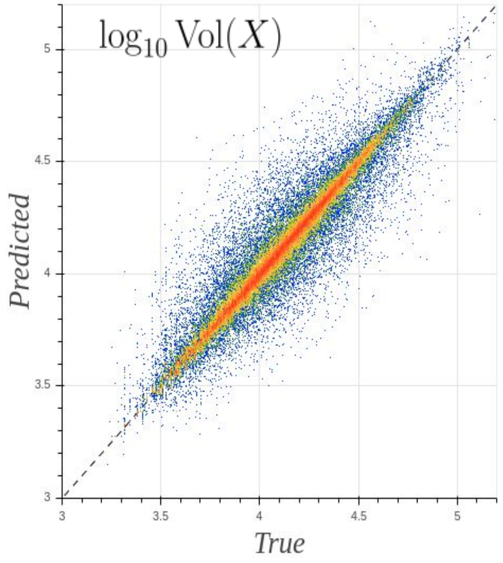

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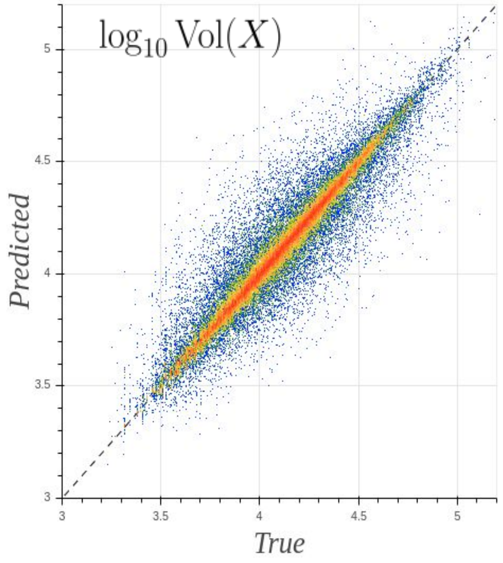

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t ~ 50μs

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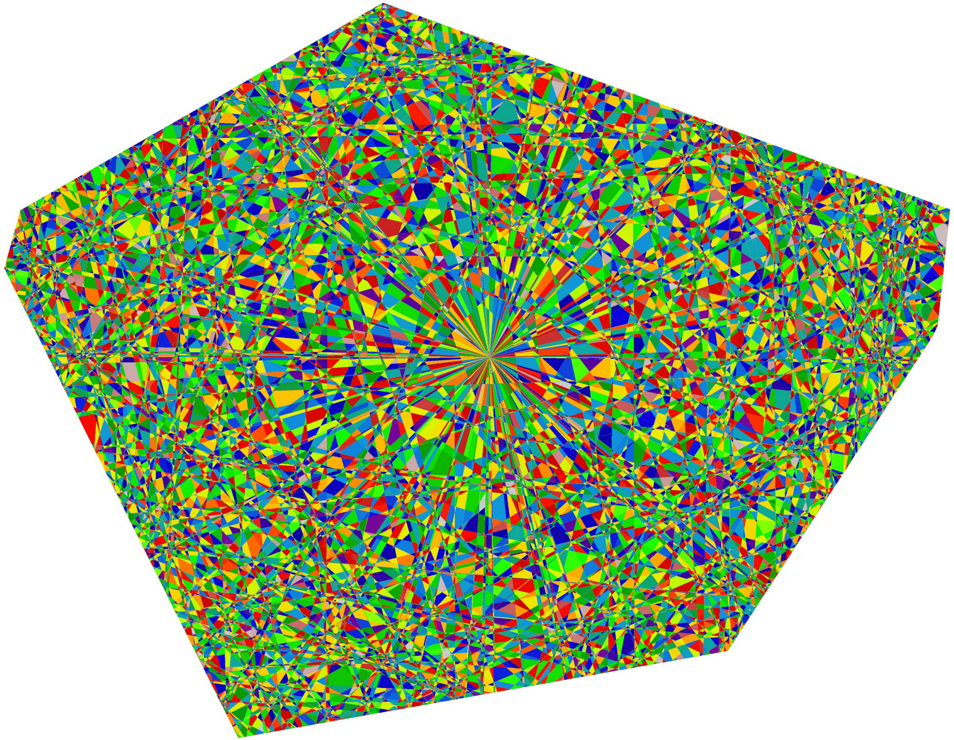
- A Calabi Yau database:
    - Study <u>*every polytope*</u> in the Kreuzer-Skarke database!

THANK YOU!

A 2-d cross section of the secondary fan of the reflexive polytope with $h^{1,1} = 491$.

Each colored region represents the Kähler Cone of a toric variety.

The landscape is **beautiful!**



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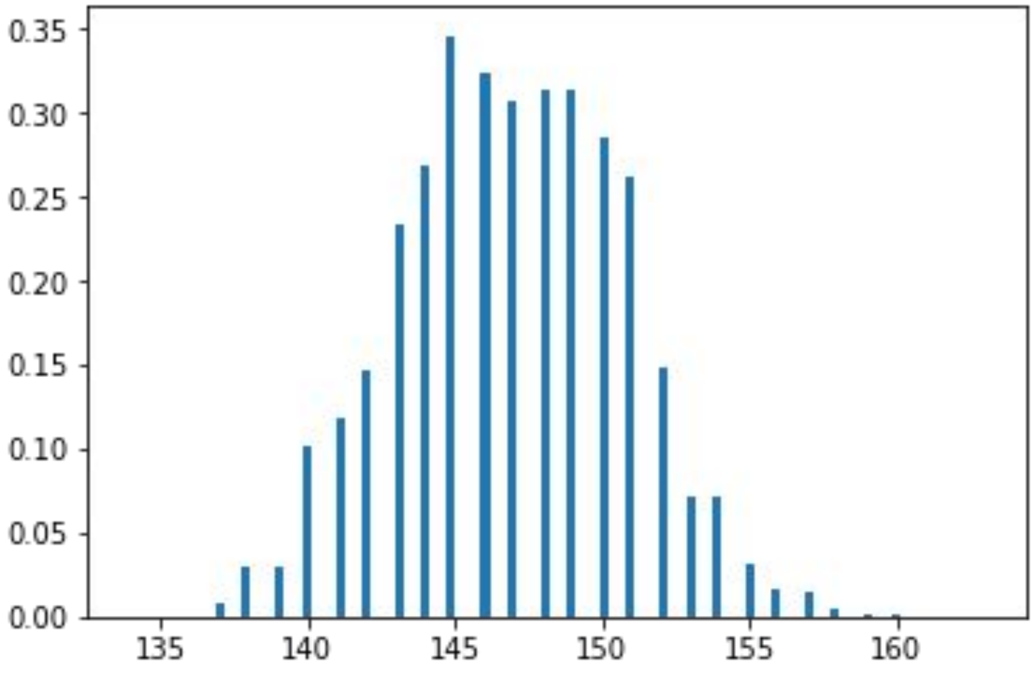

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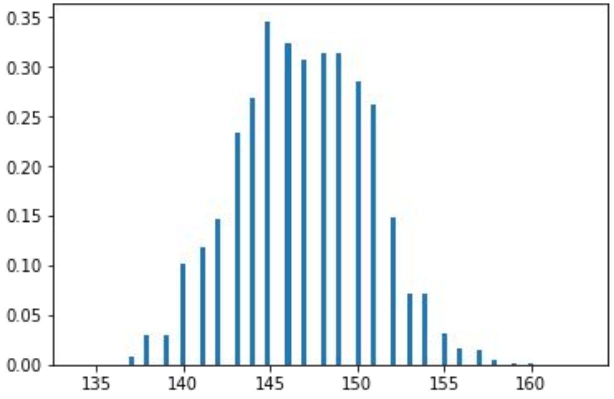

Performance:

Tolerance	Accuracy
Correct	91%
Within ± 1	94%
Within ± 3	98%