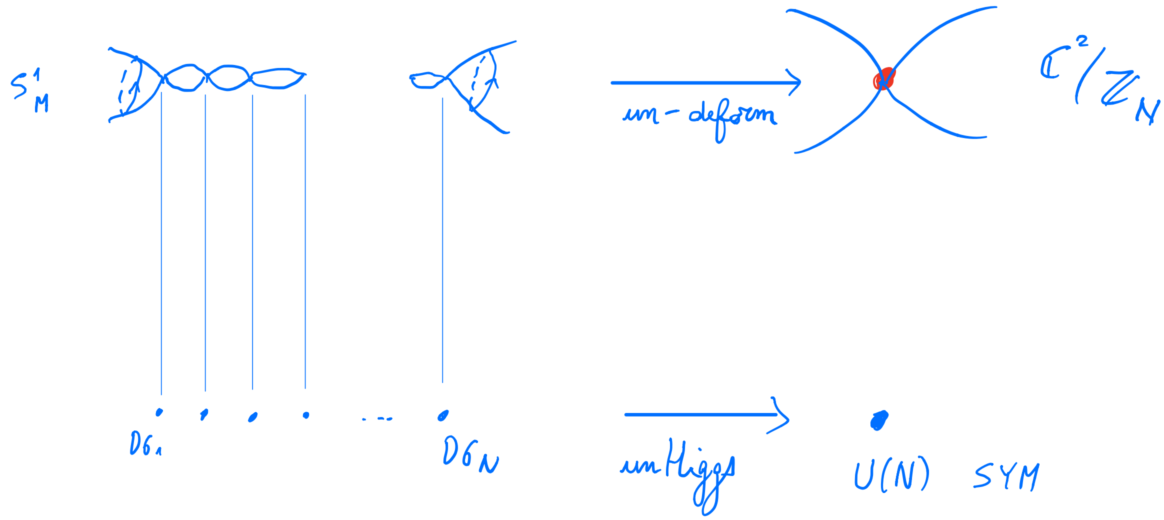
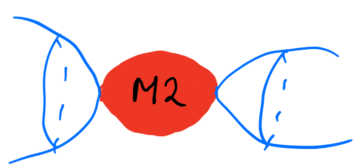
D6-branes and M-theory

$TN_N \quad uv = \det(\mathbb{1}_N z - \langle\Phi\rangle)$ $\qquad\qquad$ $uv = z^N$



11d SUGRA sees only $\underline{U(1)^N} \subset U(N)$

rest comes from M2-branes



The nilpotent puzzle

$$\langle \Phi_{D6} \rangle = \begin{pmatrix} \delta & 0 \\ 0 & -\delta \end{pmatrix} \longrightarrow \quad uv = (z+\delta)(z-\delta)$$

$SU(2) \longrightarrow U(1)_e$ $\qquad\qquad$ $U(1)$

$$\langle \Phi_{D6} \rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \longrightarrow \quad uv = z^2$$

$SU(2) \longrightarrow \emptyset$ $\qquad\qquad$! $\quad$ WTF?

answer: coherent state on M2's.

Goal: A systematic way to describe this in M-theory

Strategy: Probe with a D2-brane

3d mirror symmetry

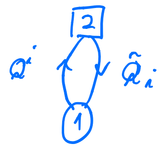
A-side

$d=3$, $\mathcal{N}=4$, SQED

$M^i{}_j = \tilde{Q}^i Q_j$, φ,

$W = \varphi \,\mathrm{tr}\, M \quad \Longrightarrow \quad \mathrm{tr}\, M = 0$

$\det M = 0$



v.m. (A_μ, σ) $\qquad dA = * d\gamma$

$V_\pm \equiv e^{\pm(i\gamma+\sigma)}$ $\qquad V_+ V_- = \varphi^2$

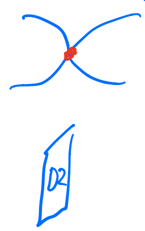
quantum

monopole operators : $\langle .. V_+(x) ... \rangle \Longrightarrow dF = \delta(x)$ in path integral

shift sym. $\gamma \to \gamma + c$ $\qquad U(1)_{Top}$.

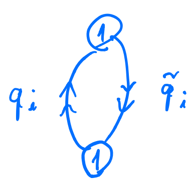
B - side

$\mathbb{C}^2/\mathbb{Z}_2 + 1 \times D6$



$d=3, \quad \mathcal{N}=4 \qquad U(1)^2/U(1)$

$m^i{}_j = \tilde{q}^i \tilde{q}_j \quad , \quad S \quad , \quad W_\pm$



$W = S \operatorname{tr} m$

mirror map : $\qquad M \longrightarrow \begin{pmatrix} S & W_+ \\ W_- & -S \end{pmatrix}$

$$\begin{pmatrix} \varphi & V_+ \\ V_- & -\varphi \end{pmatrix} \longrightarrow m$$

9-11 flip

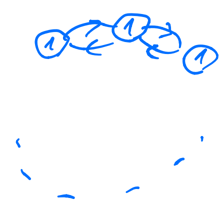
M-th on $\mathbb{C}^2 \times \mathbb{C}^2/\mathbb{Z}_N$

$$\begin{tikzcd}[column sep=small] & \text{M-th on } \mathbb{C}^2 \times \mathbb{C}^2/\mathbb{Z}_N \arrow[dl] \arrow[dr] & \\ \text{IIA} / \ \mathbb{C}^2 \times \mathbb{R}^3 & & \text{IIA} / \ \mathbb{R}^3 \times \mathbb{C}^2/\mathbb{Z}_N \end{tikzcd}$$

$+ N \times D6$ $\qquad\qquad$ $+ 1 \times D6$

in general:

$\longrightarrow$



$\hat{A}_{N-1}$ Dynkin

T-branes are monopole deformations

$$\langle \Phi_{D6} \rangle = \begin{pmatrix} 0 & m \\ 0 & \alpha \end{pmatrix}$$

$\downarrow$

$$\Delta W = \hat{Q} \cdot \langle \Phi_{D6} \rangle \cdot Q \longrightarrow \Delta W = m \, W_+$$

$$= m \, \hat{Q}^1 Q_2$$

$$\Delta W = m \, W_+ \xrightarrow{\text{IR}} \ ?$$

$$W = \begin{pmatrix}\tilde{Q}^1 & \tilde{Q}^2\end{pmatrix}\begin{pmatrix}\varphi & m \\ 0 & \varphi\end{pmatrix}\begin{pmatrix}Q_1 \\ Q_2\end{pmatrix}$$

$$\sim \tilde{Q}\cdot\begin{pmatrix}-\frac{\varphi^2}{m} & 0 \\ 0 & 1\end{pmatrix}\cdot Q$$

$$\sim -\frac{\varphi^2}{m}\tilde{Q}Q$$

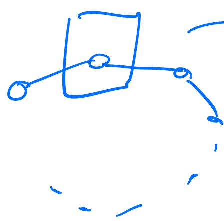
SQED $N_f = 1$
$+ \Delta W$

$$W = XYZ - S^2 Z$$

$X, Y = V_\pm$
$Z = Q\tilde{Q}$

$$W = Z\begin{vmatrix} S & -X \\ Y & -S\end{vmatrix}$$

"local" mirror :



$N_f = 2 \longrightarrow$ treat & reattach.

<u>Check : partition function on squashed S^3</u>

$S^3 \longrightarrow$ localization . Mirror symmetry becomes <u>Fourier transform</u>

<u>E.g.</u> : $\mathcal{N}=4$ SQED , $N_f = 1$
round S^3

$U(1) \longrightarrow \int_{-\infty}^{+\infty} d\sigma$

hyper $\longrightarrow \frac{1}{\mathrm{ch}(\sigma)}$

$\cosh(\sigma)$

FI $\longrightarrow$ $\exp(-2\pi i \sigma \xi)$

$$Z_{QED_1}(\xi) = \int d\sigma \frac{e^{-2\pi i\sigma\xi}}{\cosh(\sigma)} = \frac{1}{\cosh(\xi)} = Z_{1\text{-hyper}} \quad \checkmark$$

<u>Squashed $\hat{S}^3$</u> : $\quad b^2|z_1|^2 + b^{-2}|z_2|^2 = 1 \subset \mathbb{C}^2,$

$$s_b(x) = \prod_{m,n\in\mathbb{Z}_{\geq 0}} \frac{mb + nb^{-1} + \frac{Q}{2} - ix}{mb + nb^{-1} + \frac{Q}{2} + ix}.$$

$$s_1(x) = \prod \left(\frac{m+n+1-ix}{m+n+1+ix} \right)$$

χ-multiplet with $U(1)$-charges $\vec{q} = (q_1, \dots, q_m)$

$$U(1)_R = U(1)_0 + \sum_{i=1}^{m} c_i\, U(1)_i$$

complex twisted mass $\tilde{m}_\chi = \sum_i q_i (m^i + i c^i)$

vev of bckd scalar

$\Delta_\chi = \sum_i q_i c_i$

IR conf. dim.

$$\longrightarrow Z_\chi = s_b(i - \tilde{m}_\chi)$$

<u>E.g.</u> :

$$Z_{\text{SQED}_1\mathcal{N}=4}(\tilde{m}_A, \xi) = s_b(\tilde{m}_A) \int_{-\infty}^{\infty} e^{-2\pi i\xi} s_b\left(i\tfrac{Q}{4} - u - \tfrac{1}{2}\tilde{m}_A\right) s_b\left(i\tfrac{Q}{4} + u - \tfrac{1}{2}\tilde{m}_A\right)$$

φ $\qquad\qquad$ $Q, \tilde{Q}$

SQED , $N_f = 2$ + monopole

Jafferis: holomorphy of Z in $\hat{m}$, also for $U(1)_T$

$\longrightarrow \quad \Delta W = W_+ \implies U(1)_R \sim \Delta_T U(1)_T \implies \hat{m}_T = \xi + i\Delta_T$

$$Z_{SQED_2 + W_+}(m_F, \xi) = Z_{SQED_2}(m_F, \xi + i\Delta_T)$$

$$\int d\sigma\, e^{-2\pi i \sigma \xi} \longrightarrow \int d\sigma\, e^{-2\pi i \sigma(\xi + i\Delta_T)}$$

$$Z_{SQED_2}(m_F, \xi + i\Delta_T) \underset{\substack{\uparrow \\ \text{mirror sym.}}}{=} Z_{SQED_2}(\xi + i\Delta_T, m_F)$$

$$\dots = s_1\left(-i(\Delta_T - 1) - \xi\right) s_1\left(i(2\Delta_T - 1) - 2\xi\right)$$
$$s_1\left(-i(\Delta_T - 1) - m_F - \xi\right) s_1\left(-i(\Delta_T - 1) + m_F - \xi\right)$$

$$W = XYZ + S^2 X$$

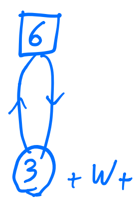
with

$$\begin{array}{ccccc} & X & Y & Z & S \\ \Delta = (& 2-2\Delta_T, & \Delta_T, & \Delta_T, & \Delta_T \;) \end{array}$$

Non-Lagrangian case

E-type quivers:

$$\hat{E}_8 \qquad \begin{array}{ccccccccccccccc} &&&&&&&&&& 3 &&&& \\ &&&&&&&&&& | &&&& \\ 1 & - & 2 & - & 3 & - & 4 & - & 5 & - & 6 & - & 4 & - & 2 \end{array}$$



use 4-d S-duality

$$4d,\ \mathcal{N}=2,\ N_f=6,\ G=SU(3) \xleftrightarrow{A.S.} \boxed{1} \leftrightarrow SU(2) — MN_{E_6}$$

$$F_{MN_{E_6}} = E_6 \supset SU(6) \times \underbrace{SU(2)}_{\text{gauge it}} \longrightarrow F = SU(6)$$

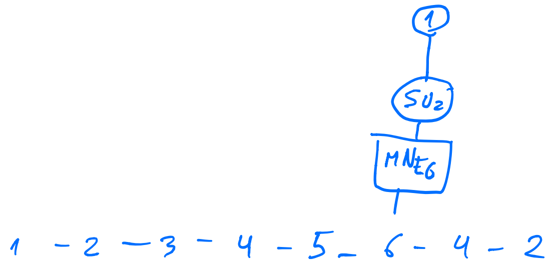
Now we gauge baryonic $U(1)$ $\longleftrightarrow$ gauge $SO(2)$ flavor

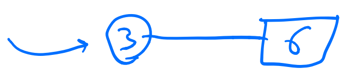
$$1 — SU(2) — MN_{E_6}$$

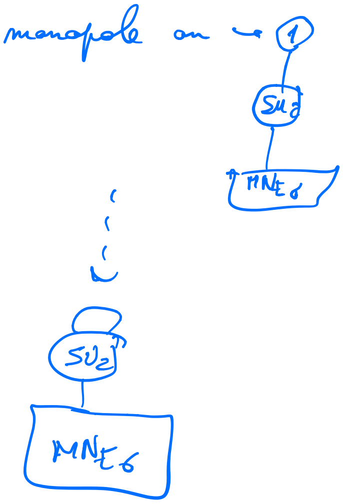
contains nilp moment map

$X \in \mathrm{adj}(SU(6))$, $X^2 = 0$

can recouple to quiver



monopole on 

= monopole on 

Can repeat story for other balanced nodes

$$R_{N,0}$$

$$T_N \qquad SU(N)^3 \longrightarrow SU(2N)\times U(1)$$

$$+\ X_{SU_{2N}} \quad \text{w/} \quad X^2=0.$$