SU(3) structures on Calabi-Yau manifolds

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Based on Larfors, Lukas, Ruehle 1805.08499, and work in progress
Why, what and how?

Motivation: 4D physics from string compactifications

- CY manifolds \(\rightarrow\) large set of semi-realistic string vacua
- Still lack fully realistic compactifications:
  moduli, physical couplings, stability, cosmological constant, ...

While CY geometry is *useful* it is not *necessary*.

This talk

- SU(3) structure \(\rightarrow\) 4D \(\mathcal{N} = 1\) SUSY
- SUSY, BI, EOM constrain torsion
- Can we get a large class of example manifolds?

Idea:

- Construct explicit SU(3) structures on CY manifolds
- Bonus: get explicit metric
- How far can we get at satisfying all constraints?
Why, what and how?

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1 Motivation: 4D Heterotic $\mathcal{N} = 1$ Minkowski solutions

2 SU(3) structure

3 Construction of SU(3) structures on CY

4 SU(3) structures on CY: Torsion classes

5 SU(3) structures on CY: Strominger–Hull system

6 Conclusions and outlook
Motivation: 4D Heterotic $\mathcal{N} = 1$ Minkowski solutions


Geometry

SUSY equations, $H = 0$ \implies \text{covariantly constant spinor $\eta$ on $X$: $\nabla \eta = 0$}
\iff $X$ is Calabi–Yau

SUSY equations, $H \neq 0$ \implies \text{globally defined spinor $\eta$ on $X$: $\nabla_T \eta = 0$}
\iff SU(3) structure on $X$ with torsion $T \sim H$

Gauge field & vector bundle

SUSY equations \implies \text{holomorphic vector bundle $V \to X$ with HYM connection}

Must also satisfy BI
\[ dH = \frac{\alpha'}{4} \left( \text{tr}(F \wedge F) - \text{tr}(R^- \wedge R^-) \right) \]
2. FROM \( G \)-STRUCTURES TO CALABI-YAU GEOMETRY

(a) Structure group \( O(d) \)

(b) A globally defined vector reduces the structure to \( O(d-1) \)

Figure 1: A set of non-degenerate tensors describes a \( G \)-structure. On the left: in the special case of the figure we assume that the structure group is reduced to \( O(d) \) (see example 2.1). On the right: an everywhere non-vanishing vector field \( v \) is introduced. Because of the existence of this vector field it is possible to construct a reduced frame bundle, where on the overlap between the patches only the rotations that leave the vector \( v \)-axis invariant are allowed, i.e. proper rotations in a plane orthogonal to the \( v \)-axis, making up \( O(d-1) \). The figure is inspired by a similar picture from a talk by Davide Cassani.

A convenient way to describe a \( G \)-structure, used a lot by physicists, is via one or more \( G \)-invariant tensors — or spinors as we will see later — that are globally defined on \( M \) and non-degenerate. Indeed, since these objects are globally defined it is possible to choose frames \( e_a \) in each patch so that they take exactly the same form in all patches. It follows that only those transition functions that leave these objects invariant are allowed and the structure group reduces to \( G \) or a subgroup thereof, see figure 1.

Note that, typically, such a set of \( G \)-invariant tensors is not unique, so that there are several descriptions of the same \( G \)-structure. Furthermore, it is possible that these tensors are actually invariant under a larger group \( G' \), in which case one can add more tensors to more accurately describe the \( G \)-structure. The \( G \)-invariant tensors can be found in a systematic way using representation theory. Indeed, one should decompose the different representations of \( GL(d, \mathbb{R}) \), in which a tensor on \( M \) transforms, into irreducible representations of \( G \) and scan for invariants. These invariants will then correspond to non-degenerate \( G \)-invariant tensors.

If the \( G \)-structure is already reduced to \( SO(d) \) (see example 2.1) and the manifold is spin, which means one can lift the \( SO(d) \) in the transition functions to its double cover \( Spin(d) \) in a globally consistent way, we can also consider spinor bundles. We will especially be interested in invariant spinors since they are needed to construct the generators of unbroken supersymmetry.
SU(3) structure

\[ M_6 \] orientable with metric: \( G = SO(6) \subset GL(6). \)

\[ M_6 \] spin: \( SO(6) \) lifts to \( Spin(6) \cong SU(4). \)

Let \( \eta \) Weyl, positive chirality: \( \eta \in 4 \) of \( SU(4) \). Choose basis:

\[
\eta = \begin{pmatrix}
0 \\
0 \\
0 \\
\eta_0
\end{pmatrix}
\]

invariant under

\[
\begin{pmatrix}
U & 0_{3 \times 1} \\
0_{1 \times 3} & 1
\end{pmatrix}, \quad U \in SU(3)
\]

Globally defined \( \eta \implies G = SU(3). \)

All orientable, spin \( M_6 \) admit a nowhere vanishing \( \eta \); torsion undetermined.

cf. Bryant:05
SU(3) structure

\( \mathcal{M}_6 \) orientable with metric: \( G = \text{SO}(6) \subset \text{GL}(6). \)

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SU(3) structure

\( \eta \leftrightarrow \) real two-form \( J \) and complex decomposable three-form \( \Omega \) s.t.

\[
\Omega \wedge J = 0, \quad \frac{3i}{4} \Omega \wedge \overline{\Omega} = J \wedge J \wedge J = 3! d\text{vol}
\]

where

\[
J_{mn} = -i \eta^+ \gamma_{mn} \eta^+, \quad \Omega_{mnp} = -i \eta^+ \gamma_{mnp} \eta^+
\]

- Almost complex structure: \( I_{mn} \sim \epsilon^{nk_1 \ldots k_5} \Re \Omega_{mk_1 k_2} \Re \Omega_{k_3 k_4 k_5} \)
- \( J, \Omega \Rightarrow \) metric \( g_{mn} = I_{mp} J_{pn} \) \quad Hitchin:00

\( J, \Omega \) closed \( \Leftrightarrow M_6 \) is Calabi–Yau.
Otherwise non-zero torsion \quad Chiossi–Salamon:02

\[
dJ = -\frac{3}{2} \Im(\mathcal{W}_1 \overline{\Omega}) + \mathcal{W}_4 \wedge J + \mathcal{W}_3
\]

\[
d\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}_5} \wedge \Omega
\]
4D $\mathcal{N} = 1$ solutions from $SU(3)$ structure manifolds

Remark:
many Calabi–Yau $\rightarrow$ many fluxless compactifications

Why so few explicit examples with flux?

$SU(3)$ structure not enough: SUSY, BI and EOM selects $W_i$

Complications:
- $W_1, W_2 \neq 0 \Rightarrow I_m^p$ not integrable (non-complex)
- $W_1, W_4, W_3 \neq 0$: not symplectic (non-Kähler)

Idea of this talk: Construct explicit $SU(3)$ structures on CY manifolds.

Alternative: Construct non-explicit $SU(3)$ structures as deformations of CY

Witten–Witten:87, Li–Yau:05, Andreas–Garcia-Fernandez:12
**4D Heterotic $\mathcal{N} = 1$ Minkowski solutions: Equations**

**No flux: Calabi–Yau**

$\mathcal{N} = 1$, $\text{Mkw}$, $H = 0$ $\iff$ $X$ is Calabi–Yau, dilaton constant.

$dJ = d\Omega = 0$, $H = 0$

**With flux: Strominger–Hull system**

$\mathcal{N} = 1$, $\text{Mkw}$, $H \neq 0$ $\iff$ $\text{SU}(3)$ structure on $X$ with torsion:

\[
d(e^{-2\phi} J \wedge J) = d(e^{-2\phi} \Omega) = 0, \quad H = i(\partial - \bar{\partial})J
\]

$W_0 = W_2 = 0$, $W_5 = 2W_4 = 2d\phi$.

**Heterotic vector bundle**

$\mathcal{N} = 1$ vector bundle $V \to X$ with connection $A$ and field strength $F$ must satisfy

\[
F \wedge \Omega = 0, \quad F \wedge J \wedge J = 0.
\]

Must also satisfy BI

\[
dH = \frac{\alpha'}{4} \left( \text{tr}(F \wedge F) - \text{tr}(R^- \wedge R^-) \right)
\]
Construction of SU(3) structures on CY

Motivational example: the quintic

- Hypersurface $X \subset \mathbb{P}^4$,

$$0 = P(x_0, \ldots, x_4) = x_0^5 p(z_1, \ldots, z_4) = 0 \text{, } z_a = \frac{x_a}{x_0} \text{ in } U_0 : x_0 \neq 0$$

- Inherit Kahler form: $J_0 = \mathcal{J}|_X$

- FS Kahler form $\mathcal{J} = \frac{i}{2\pi} \partial \bar{\partial} \ln \kappa, \kappa = 1 + \sum_{a=1}^4 |z_a|^2$

- Inherit hol. top form: $\Omega_0 = \frac{dz_1 \wedge dz_2 \wedge dz_3}{p,4}$

- Check SU(3) structure conditions:

$$J_0 \wedge \Omega_0 = 0 \text{ but } J_0 \wedge J_0 \wedge J_0 = \frac{3i}{4} \mathcal{F} \Omega_0 \wedge \bar{\Omega}_0$$
Construction of SU(3) structures on CY

Motivational example: the quintic

- Inherit Kahler form: $J_0 = \mathcal{J}|_X$ and holomorphic top form: $\Omega_0$

- $J_0 \wedge \Omega_0 = 0$ but $J_0 \wedge J_0 \wedge J_0 = \frac{3i}{4} \mathcal{F} \Omega_0 \wedge \bar{\Omega}_0$

- Rescale forms to get SU(3) structure $J = \mathcal{F}^k J_0$, $\Omega = \mathcal{F}^{\frac{3k+1}{2}} \Omega_0$

- Complex, non-Kahler manifold

  $W_1 = W_2 = W_3 = 0$, $W_4 = k \, d(\ln \mathcal{F})$, $W_5 = \frac{3k+1}{2} \, d(\ln \mathcal{F})$

- Strominger–Hull system if $k = 1$, with flux

  $H = i(\bar{\partial} - \partial) J = i(\bar{\partial} - \partial) \ln \mathcal{F} \wedge J$
Construction of SU(3) structures on CY

Method generalizes to any favourable CICY

\[ X \sim \left[ \begin{array}{ccc}
\mathbb{P}^{n_1} & q_1^1 & \cdots & q_K^1 \\
\vdots & \vdots & \ddots & \vdots \\
\mathbb{P}^{n_m} & q_1^m & \cdots & q_K^m 
\end{array} \right]^{h_1,1, h_2,1}, \]

E.g. for co-dim 1 CICY

- 1 Kahler form \( J_i = J_i|_X \) from each \( \mathbb{P}^{n_i} \subset A \): \( J = \sum_{i=1}^{m} a_i J_i \)

- Holomorphic top form: \( \Omega_0 = \hat{\Omega}|_X \)

\[ \hat{\Omega} \wedge dP_1 \wedge \cdots \wedge dP_K = \mu_1 \wedge \cdots \wedge \mu_m, \quad \mu_i = \frac{1}{n_i!} \epsilon_{A_0 A_1 \cdots A_{n_i}} x_i A_0 \cdots d x_i A_{n_i} \]

Check SU(3) structure:

- \( J \wedge \Omega_0 = 0 \checkmark \)

- \( J_i \wedge J_j \wedge J_k = \frac{3i}{4} \Lambda_{ijk} \Omega_0 \wedge \bar{\Omega}_0 \)

\[ \Lambda_{ijk} = \frac{c_{ijk}}{6 \pi^3} \left[ \prod_{l=1}^{m} \frac{|\nabla_l P|^{2n_l}}{\sigma_l} \right] (|\nabla_i P|^2 |\nabla_j P|^2 |\nabla_k P|^2 \sigma_i \sigma_j \sigma_k)^{-1}, \quad \sigma_i = \sum_{A=0}^{n_i} |x_i A|^2 \]
Construction of SU(3) structures on CY

Example: SU(3) structure on tetraquadric

\[
\begin{bmatrix}
\mathbb{P}^1 & 2 \\
\mathbb{P}^1 & 2 \\
\mathbb{P}^1 & 2 \\
\mathbb{P}^1 & 2 \\
\end{bmatrix}
\begin{pmatrix}
4,68 \\
-128 \\
\end{pmatrix}
\]

\[
X \sim \begin{pmatrix}
x_1 &= (x_{10}, x_{11}) \\
x_2 &= (x_{20}, x_{21}) \\
x_3 &= (x_{30}, x_{31}) \\
x_4 &= (x_{40}, x_{41}) \\
z_1 &= x_{11} \\
z_2 &= x_{21} \\
z_3 &= x_{31} \\
z_4 &= x_{41} \\
\end{pmatrix}
\]

- Hypersurface in \((\mathbb{P}^1)^4\) set by \(P(x_1, x_2, x_3, x_4) = 0\)
- 1 FS Kähler forms: \(\mathcal{J}_i = \frac{i}{2\pi} \frac{dz_i \wedge d\bar{z}_i}{\kappa_i^2}, \kappa_i = 1 + |z_i|^2\)

Restrict to tetra-quadric:
\[
J_\alpha = \frac{i}{2\pi} \frac{dz_\alpha \wedge d\bar{z}_\alpha}{\kappa_\alpha^2}, \quad J_4 = \frac{i}{2\pi \kappa_4} \sum_{\alpha, \beta=1}^3 v_\alpha \bar{v}_\beta dz_\alpha \wedge d\bar{z}_\beta \quad \text{with} \quad v_\alpha := \frac{p_{,\alpha}}{p_{,4}}, \text{ on } U_0
\]

- Holomorphic top form \(\Omega_0 = \frac{dz_1 \wedge dz_2 \wedge dz_3}{p_{,4}}\)
Check SU(3) structure \(J = \sum_{i=1}^4 a_i J_i, \quad \Omega = A \Omega_0:\)

- \(J \wedge \Omega_0 = 0 \checkmark\)
- \(\frac{3i}{4} \Omega \wedge \bar{\Omega} = J \wedge J \wedge J \iff |A|^2 = a_1 a_2 a_3 a_4 \sum_{i=1}^4 a_i^{-1} \Lambda_i\)

\[
\frac{1}{6} \Lambda_i := \Lambda_{ijk} = \frac{1}{6\pi^3} \frac{|p_l|^2 \kappa_1^2}{\kappa_1^2 \kappa_2^2 \kappa_3^2 \kappa_4^2}
\]
CICY SU(3) structure

\[ J = \sum_{i=1}^{m} a_i J_i \quad , \quad \Omega = A \Omega_0 \]

subject to \( |A|^2 = \sum_{i,j,k=1}^{m} \Lambda_{ijk} a_i a_j a_k \)

Torsion classes easily computed:

\[ dJ = \sum_{i=1}^{m} da_i \wedge J_i \quad , \quad d\Omega = d \ln(A) \wedge \Omega \]

\[ \Rightarrow W_1 = W_2 = 0 \quad , \quad W_3 = \sum_i (da_i - W_4) \wedge J_i \quad , \quad W_4 = \frac{1}{2} \sum_i J \cdot (da_i \wedge J_i) \quad , \quad W_5 = d \ln(A). \]

Integrable complex structure with exact \( W_5 \); rest set by \( a_i \).

Metric and torsion explicit, and slightly tuneable by choosing \( a_i \).
SU(3) structures on CY: Torsion classes

Universal CICY SU(3) structure

Choose $a_i = a \ t_i$ for $i = 1, \ldots, m$

$$J = a \ J_0 , \quad J_0 := \sum_{i=1}^{m} t_i \ J_i , \quad \Omega = A \ \Omega_0$$

With $g_{0,\alpha \bar{\beta}} = -2iJ_{0,\alpha \bar{\beta}}$ get

$$|A|^2 = a^3 \mathcal{F} , \quad \text{where} \quad \mathcal{F} := \sum_{i,j,k=1}^{m} \Lambda_{ijk} t_i t_j t_k = |\det B|^2 \det \left( g_{0,\alpha \bar{\beta}} \right) > 0 .$$

Torsion classes

$$W_1 = W_2 = W_3 = 0 , \quad W_4 = d \ln a , \quad W_5 = d \ln A = \frac{3}{2} d \ln a + \frac{1}{2} d \ln \mathcal{F} .$$
SU(3) structures on CY: Strominger–Hull system

In summary:
Any CICY allows a Universal SU(3) structure \((J, \Omega)\) with torsion

\[
W_1 = W_2 = W_3 = 0, \quad W_4 = d \ln a, \quad W_5 = \frac{3}{2} d \ln a + \frac{1}{2} d \ln \mathcal{F},
\]

where \(\mathcal{F} := \sum_{i,j,k=1}^m \Lambda_{ijk} t_i t_j t_k = |\det B|^2 \det (g_{0,\alpha \bar{\beta}}) > 0,\) and metric

\[
g_{\alpha \bar{\beta}} = a g_{0,\alpha \bar{\beta}},
\]

Choose \(a = \mathcal{F}\): reproduce torsion for Strominger–Hull system with

\[
H = i(\partial - \bar{\partial}) \mathcal{F} \wedge J_0
\]
SU(3) structures on CY: Strominger–Hull system

Any CICY allows Strominger–Hull type SU(3) structure \((J, \Omega)\) with torsion

\[
W_1 = W_2 = W_3 = 0, \quad W_4 = d \ln F, \quad W_5 = 2d \ln F,
\]

Right torsion is not enough: must construct suitable vector bundle and solve BI.

SUSY and bundle stability — Work in progress

SUSY \iff \text{holomorphic vector bundle } V \to X \text{ with HYM connection:}

\[
F \wedge \Omega = 0, \quad F \wedge J \wedge J = 0.
\]

Li–Yau theorem: \(V \to X\) allows HYM connection \iff \(V \to X\) is stable.

Donaldson’85, Uhlenbeck–Yau’86, Li–Yau’87, Kobayashi’87, Hitchin

Here: know \(J \leadsto\) solve for \(F\).
SU(3) structures on CY: Strominger–Hull system

Bianchi identity — Work in progress

\[ 2i \bar{\partial} \partial F \wedge J_0 = dH = \frac{\alpha'}{4} \left( \text{tr}(F \wedge F) - \text{tr}(R \wedge R) \right) + \ldots, \]

\[ \text{tr}(R \wedge R) \]

Invariant under conformal re-scaling: \( \text{tr}(R \wedge R) = \text{tr}(R_0 \wedge R_0) \)

Computable but lack manageable form for general CICY.

\[ \text{tr}(F \wedge F) \]

Construct stable vector bundle \( V \to X \) with suitable connection \( A \).
SU(3) structures on CY: Strominger–Hull system

Example: $\text{tr}(R \wedge R)$ on tetraquadric

\[ X \sim \left[ \begin{array}{c|c}
\mathbb{P}^1 & 2 \\
\mathbb{P}^1 & 2 \\
\mathbb{P}^1 & 2 \\
\mathbb{P}^1 & 2 \\
\end{array} \right]^{4,68}_{-128} \]

\[ x_1 = (x_{10}, x_{11}) \quad z_1 = (x_{11}, x_{10}) \]
\[ x_2 = (x_{20}, x_{21}) \quad z_2 = (x_{21}, x_{20}) \]
\[ x_3 = (x_{30}, x_{31}) \quad z_3 = (x_{31}, x_{30}) \]
\[ x_4 = (x_{40}, x_{41}) \quad z_4 = (x_{41}, x_{40}) \]

Set $z_4 = f(z_1, z_2, z_3)$, $v_\alpha = p_\alpha/p_4$, $\Omega_\alpha = t_1t_2t_3t_4 \bar{\partial} \left( \frac{\Lambda_4 \Lambda_\beta}{\mathcal{F} \Lambda_\alpha} \partial \left( \frac{\Lambda_\alpha}{\Lambda_4} \right) \right)$. 

Compute curvature 2-form:

\[ R_\alpha^\beta = -4\pi i j_\alpha \delta_\alpha^\beta - \frac{v_\alpha}{v_\beta} \Omega_\alpha^\beta \]

Finally

\[ \text{tr}(R \wedge R) = \sum_{\alpha, \beta=1}^{3} (R_\alpha^\beta \wedge R_\beta^\alpha) + \text{c.c.} = 3 \sum_{\alpha=1}^{3} j_\alpha \wedge \Omega_\alpha^\alpha + 3 \sum_{\alpha, \beta=1}^{3} \Omega_\alpha^\beta \wedge \Omega_\beta^\alpha + \text{c.c.} \]
Conclusions and outlook

Conclusions

All CY manifolds allow several SU(3) structures

CICY: ambient space provide building blocks for non-trivial SU(3) structures

- \( J = \sum_{i=1}^{m} a_i J_i \), \( \Omega = A \Omega_0 \), subject to \(|A|^2 = \sum_{i,j,k=1}^{m} \Lambda_{ijk} a_i a_j a_k\)
- metric computable \( g_{mn} = I_m^k J_{kn} \)
- torsion computable:

\[
W_3 = \sum_i (d a_i - a_i W_4) \wedge J_i , \quad W_4 = \frac{1}{2} \sum_i J_\downarrow (d a_i \wedge J_i) , \quad W_5 = d \ln(A) .
\]

- With \( a_i = a \ \forall i \implies \) Strominger–Hull system, with \( dH \neq 0 \)

Work on heterotic BI: compute \( \text{tr}(R \wedge R) \), construct vector bundles, ...
Conclusions and outlook

Conclusions

Use ambient space forms to build complex SU(3) structures on CICY
Necessary constraints for e.g. Strominger–Hull system fulfilled

What we should do next:

- Construct vector bundles with HYM connections satisfying heterotic BI
- Explore “non-universal” SU(3) structures
- Type IIB vacua: “smeared” sources required?
- Extend construction: other types of CY manifolds, other dimensions, ...
- Generalise method: non-complex SU(3) structures, ...
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Thank You
### $\mathcal{N} = 1$ solutions from $SU(3)$ structure manifolds

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