SU(3) structures on Calabi-Yau manifolds

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Based on *Larfors, Lukas, Ruehle* 1805.08499, and work in progress

Why, what and how?

Motivation: 4D physics from string compactifications

- CY manifolds $\longrightarrow$ large set of semi-realistic string vacua
- Still lack fully realistic compactifications:
  moduli, physical couplings, stability, cosmological constant,...

While CY geometry is *useful* it is not *necessary*.

This talk

- SU(3) structure $\rightarrow$ 4D $\mathcal{N} = 1$ SUSY
- SUSY, BI, EOM constrain torsion
- Can we get a large class of example manifolds?

Idea:

- Construct explicit SU(3) structures on CY manifolds
- Bonus: get explicit metric
- How far can we get at satisfying all constraints?

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Outline

Motivation: 4D Heterotic $\mathcal{N} = 1$ Minkowski solutions

Candelas–Horowitz–Strominger–Witten:85,Strominger:86, Hull:86

Geometry

SUSY equations, $H = 0$ $\Rightarrow$ covariantly constant spinor η on X: $\nabla\eta = 0$

$\iff$ X is Calabi–Yau

SUSY equations, $H \neq 0$ $\Rightarrow$ globally defined spinor η on X: $\nabla_T\eta = 0$

$\iff$ SU(3) structure on X with torsion $T \sim H$

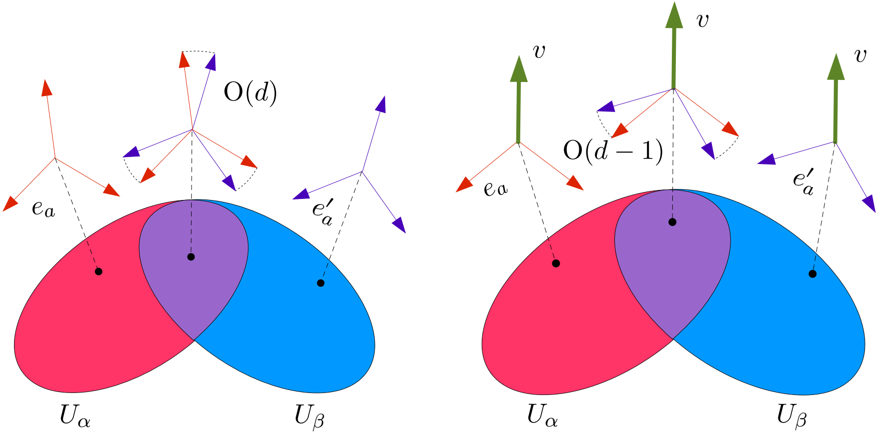
Gauge field & vector bundle

SUSY equations $\Rightarrow$ holomorphic vector bundle $V \to X$ with HYM connection

Must also satisfy BI $dH = \frac{\alpha'}{4}\left(\mathrm{tr}(F \wedge F) - \mathrm{tr}(R^- \wedge R^-)\right)$

SU(3) structure

G structure: restrict transition functions patching local frames of cotangent bundle



picture from *Koerber:10*

SU(3) structure

$\mathcal{M}_6$ orientable with metric: $G = \mathrm{SO}(6) \subset \mathrm{GL}(6)$.
$\mathcal{M}_6$ spin: $\mathrm{SO}(6)$ lifts to $\mathrm{Spin}(6) \cong \mathrm{SU}(4)$.

Let η Weyl, positive chirality: $\eta \in \mathbf{4}$ of SU(4). Choose basis:

$$\eta = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \eta_0 \end{pmatrix} \text{ invariant under } \begin{pmatrix} U & \mathbf{0_{3\times 1}} \\ \mathbf{0_{1\times 3}} & 1 \end{pmatrix}, \; U \in \mathrm{SU}(3)$$

Globally defined $\eta \implies \mathrm{G} = \mathrm{SU}(3)$.

All orientable, spin $\mathcal{M}_6$ admit a nowhere vanishing η; torsion undetermined.

cf. Bryant:05

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SU(3) structure

$\eta \Leftrightarrow$ real two-form J and complex decomposable three-form Ω s.t.

$$\boxed{\Omega \wedge J = 0, \qquad \tfrac{3i}{4}\Omega \wedge \overline{\Omega} = J \wedge J \wedge J = 3!\,d\mathrm{vol}}$$

where $\quad J_{mn} = -i\eta_+^\dagger \gamma_{mn}\eta_+, \qquad \Omega_{mnp} = -i\eta_-^\dagger \gamma_{mnp}\eta_+$

- Almost complex structure: $I_m{}^n \sim \epsilon^{nk_1..k_5}\mathrm{Re}\Omega_{mk1k2}\mathrm{Re}\Omega_{k3k4k5}$
- $J, \Omega \Rightarrow$ metric $g_{mn} = I_m{}^p J_{pn}$ Hitchin:00

J, Ω closed $\Leftrightarrow \mathcal{M}_6$ is Calabi–Yau.

Otherwise non-zero torsion Chiossi–Salamon:02

$$dJ = -\tfrac{3}{2}\mathrm{Im}(W_1\overline{\Omega}) + W_4 \wedge J + W_3$$
$$d\Omega = W_1 J \wedge J + W_2 \wedge J + \overline{W}_5 \wedge \Omega$$

4D $\mathcal{N} = 1$ solutions from $SU(3)$ structure manifolds

Remark:
many Calabi–Yau $\rightarrow$ many fluxless compactifications

Why so few explicit examples with flux?

SU(3) structure not enough: SUSY, BI and EOM selects W_i

Complications:

- $W_1, W_2 \neq 0 \Rightarrow {I_m}^p$ not integrable (non-complex)
- $W_1, W_4, W_3 \neq 0$: not symplectic (non-Kähler)

Idea of this talk: Construct explicit $SU(3)$ structures on CY manifolds.

Alternative: Construct non-explicit $SU(3)$ structures as deformations of CY
Witten–Witten:87, Li–Yau:05, Andreas–Garcia-Fernandez:12

4D Heterotic $\mathcal{N} = 1$ Minkowski solutions: Equations

No flux: Calabi–Yau

Candelas–Horowitz–Strominger–Witten:85

$\mathcal{N} = 1$, Mkw, $H = 0$ $\iff$ X is Calabi–Yau, dilaton constant.

$$\mathrm{d}J = \mathrm{d}\Omega = 0, H = 0$$

With flux: Strominger–Hull system

Strominger:86, Hull:86

$\mathcal{N} = 1$, Mkw, $H \neq 0$ $\iff$ SU(3) structure on X with torsion:

$$\mathrm{d}(e^{-2\phi} J \wedge J) = \mathrm{d}(e^{-2\phi}\Omega) = 0, H = i(\partial - \bar{\partial})J$$
$$W_0 = W_2 = 0,\ W_5 = 2W_4 = 2\mathrm{d}\phi\ .$$

Heterotic vector bundle

$\mathcal{N} = 1$ vector bundle $V \to X$ with connection A and field strength F must satisfy

$$F \wedge \Omega = 0\ , \quad F \wedge J \wedge J = 0\ .$$

Must also satisfy BI $\quad dH = \frac{\alpha'}{4}\left(\mathrm{tr}(F \wedge F) - \mathrm{tr}(R^- \wedge R^-)\right)$

Construction of SU(3) structures on CY

Motivational example: the quintic

- Hypersurface $X \subset \mathbb{P}^4$,

$$0 = P(x_0, .., x_4) = x_0^5 p(z_1, .., z_4) = 0 \ , \ z_a = \tfrac{x_a}{x_0} \text{ in } U_0 : x_0 \neq 0$$

- Inherit Kahler form: $\boxed{J_0 = \mathcal{J}|_X}$

FS Kahler form $\mathcal{J} = \frac{i}{2\pi}\partial\bar{\partial}\ln\kappa, \kappa = 1 + \sum_{a=1}^4 |z_a|^2$

- Inherit hol. top form: $\boxed{\Omega_0 = \frac{dz_1 \wedge dz_2 \wedge dz_3}{p_{,4}}}$
- Check SU(3) structure conditions:

$$J_0 \wedge \Omega_0 = 0 \text{ but } J_0 \wedge J_0 \wedge J_0 = \tfrac{3i}{4}\mathcal{F}\, \Omega_0 \wedge \bar{\Omega}_0$$

Construction of SU(3) structures on CY

Motivational example: the quintic

- Inherit Kahler form: $J_0 = \mathcal{J}|_X$ and holomorphic top form: Ω_0
- $J_0 \wedge \Omega_0 = 0$ but $J_0 \wedge J_0 \wedge J_0 = \frac{3i}{4}\mathcal{F}\,\Omega_0 \wedge \bar{\Omega}_0$

- Rescale forms to get SU(3) structure $\boxed{J = \mathcal{F}^k J_0 \,, \qquad \Omega = \mathcal{F}^{\frac{3k+1}{2}}\Omega_0}$
- Complex, non-Kahler manifold

$$W_1 = W_2 = W_3 = 0 \,, \quad W_4 = k\, d(\ln \mathcal{F}) \,, \quad W_5 = \frac{3k+1}{2}\, d(\ln \mathcal{F})$$

- Strominger–Hull system if $k = 1$, with flux

$$H = i(\partial - \bar{\partial})J = i(\partial - \bar{\partial}) \ln \mathcal{F} \wedge J$$

Construction of SU(3) structures on CY

Method generalizes to any favourable CICY

$$X \sim \left[\begin{array}{c|ccc} \mathbb{P}^{n_1} & q_1^1 & \cdots & q_K^1 \\ \vdots & \vdots & & \vdots \\ \mathbb{P}^{n_m} & q_1^m & \cdots & q_K^m \end{array}\right]^{h^{1,1},h^{2,1}}_{\eta} ,$$

E.g. for co-dim 1 CICY

- 1 Kahler form $J_i = \mathcal{J}_i|_X$ from each $\mathbb{P}^{n_i} \subset \mathcal{A}$: $\boxed{J = \sum_{i=1}^m a_i J_i}$
- Holomorphic top form: $\boxed{\Omega_0 = \hat{\Omega}|_X}$

$$\hat{\Omega} \wedge dP_1 \wedge \cdots \wedge dP_K = \mu_1 \wedge \cdots \wedge \mu_m \ , \quad \mu_i = \frac{1}{n_i!} \epsilon_{A_0 A_1 \cdots A_{n_i}} x_{iA_0} dx_{iA_1} \wedge \cdots \wedge dx_{iA_{n_i}}$$

Check SU(3) structure:

- $J \wedge \Omega_0 = 0$ ✓
- $J_i \wedge J_j \wedge J_k = \frac{3i}{4} \Lambda_{ijk} \, \Omega_0 \wedge \bar{\Omega}_0$

$$\Lambda_{ijk} = \frac{c_{ijk}}{6\pi^3} \left[\prod_{l=1}^m \frac{|\nabla_l P|^{2n_l}}{\sigma_l}\right] \left(|\nabla_i P|^2 |\nabla_j P|^2 |\nabla_k P|^2 \sigma_i \sigma_j \sigma_k\right)^{-1} \ , \ \sigma_i = \sum_{A=0}^{n_i} |x_{iA}|^2$$

Construction of SU(3) structures on CY

Example: $SU(3)$ structure on tetraquadric

$$X \sim \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \end{array} \right]^{4,68}_{-128} \qquad \begin{array}{l} \mathbf{x}_1 = (x_{1\,0}, x_{1\,1}) \\ \mathbf{x}_2 = (x_{2\,0}, x_{2\,1}) \\ \mathbf{x}_3 = (x_{3\,0}, x_{3\,1}) \\ \mathbf{x}_4 = (x_{4\,0}, x_{4\,1}) \end{array} \qquad \begin{array}{l} z_1 = \frac{x_{1\,1}}{x_{1\,0}} \\ z_2 = \frac{x_{2\,1}}{x_{2\,0}} \\ z_3 = \frac{x_{3\,1}}{x_{3\,0}} \\ z_4 = \frac{x_{4\,1}}{x_{4\,0}} \end{array}$$

- Hypersurface in $(\mathbb{P}^1)^4$ set by $P(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) = 0$
- 1 FS Kähler forms: $\mathcal{J}_i = \frac{i}{2\pi} \frac{dz_i \wedge d\bar{z}_i}{\kappa_i^2} \quad , \kappa_i = 1 + |z_i|^2$

  Restrict to tetra-quadric:
  $J_\alpha = \frac{i\, dz_\alpha \wedge d\bar{z}_\alpha}{2\pi \kappa_\alpha^2}$, $\quad J_4 = \frac{i}{2\pi \kappa_4^2} \sum_{\alpha,\beta=1}^3 v_\alpha \bar{v}_\beta \, dz_\alpha \wedge d\bar{z}_\beta$ with $v_\alpha := \frac{p_{,\alpha}}{p_{,4}}$, on U_0
- Holomorphic top form $\Omega_0 = \frac{dz_1 \wedge dz_2 \wedge dz_3}{p_{,4}}$

Check SU(3) structure $J = \sum_{i=1}^4 a_i J_i$, $\qquad \Omega = A\,\Omega_0$:

- $J \wedge \Omega_0 = 0$ ✓
- $\frac{3i}{4} \Omega \wedge \overline{\Omega} = J \wedge J \wedge J \iff |A|^2 = a_1 a_2 a_3 a_4 \sum_{i=1}^4 a_i^{-1} \Lambda_i$

$$\frac{1}{6}\Lambda_l := \Lambda_{ijk} = \frac{1}{6\pi^3} \frac{|p_l|^2 \kappa_l^2}{\kappa_1^2 \kappa_2^2 \kappa_3^2 \kappa_4^2}$$

SU(3) structures on CY: Torsion classes

CICY SU(3) structure

$\boxed{J = \sum_{i=1}^{m} a_i J_i \quad , \quad \Omega = A\,\Omega_0}$ subject to $|A|^2 = \sum_{i,j,k=1}^{m} \Lambda_{ijk} a_i a_j a_k$

Torsion classes easily computed:

$$dJ = \sum_{i=1}^{m} da_i \wedge J_i \ , \qquad d\Omega = d\ln(A) \wedge \Omega$$

$$\implies W_1 = W_2 = 0 \ ,$$

$$W_3 = \textstyle\sum_i (da_i - W_4) \wedge J_i \ , \quad W_4 = \frac{1}{2} \sum_i J \lrcorner (da_i \wedge J_i) \ , \quad W_5 = d\ln(A).$$

Integrable complex structure with exact W_5; rest set by a_i

Metric and torsion explicit, and slightly tuneable by choosing a_i.

SU(3) structures on CY: Torsion classes

Universal CICY SU(3) structure

Choose $a_i = a\, t_i$ for $i = 1, \ldots, m$

$$J = a\, J_0 \ , \qquad J_0 := \sum_{i=1}^{m} t_i J_i \ , \qquad \Omega = A\, \Omega_0$$

With $g_{0,\alpha\bar{\beta}} = -2iJ_{0,\alpha\bar{\beta}}$ get

$$|A|^2 = a^3 \mathcal{F} \ , \quad \text{where} \quad \mathcal{F} := \sum_{i,j,k=1}^{m} \Lambda_{ijk} t_i t_j t_k = |\det B|^2 \det \left(g_{0,\alpha\bar{\beta}} \right) > 0 \ .$$

Torsion classes

$$W_1 = W_2 = W_3 = 0 \ , \quad W_4 = d \ln a \ , \quad W_5 = d \ln A = \frac{3}{2} d \ln a + \frac{1}{2} d \ln \mathcal{F} \ .$$

SU(3) structures on CY: Strominger–Hull system

In summary:
Any CICY allows a Universal SU(3) structure (J, Ω) with torsion

$$W_1 = W_2 = W_3 = 0 \ , \quad W_4 = d \ln a \ , \quad W_5 = \frac{3}{2} d \ln a + \frac{1}{2} d \ln \mathcal{F} \ ,$$

where $\mathcal{F} := \sum_{i,j,k=1}^{m} \Lambda_{ijk} t_i t_j t_k = |\mathrm{det} B|^2 \mathrm{det}\left(g_{0,\alpha\bar{\beta}}\right) > 0$, and metric

$$g_{\alpha\bar{\beta}} = a g_{0,\alpha\bar{\beta}},$$

Choose $a = \mathcal{F}$: reproduce torsion for Strominger–Hull system with

$$H = i(\partial - \bar{\partial})\mathcal{F} \wedge J_0$$

SU(3) structures on CY: Strominger–Hull system

Any CICY allows Strominger–Hull type SU(3) structure (J, Ω) with torsion

$$W_1 = W_2 = W_3 = 0\ , \quad W_4 = d \ln \mathcal{F}\ , \quad W_5 = 2d \ln \mathcal{F}\ ,$$

Right torsion is not enough: must construct suitable vector bundle and solve BI.

SUSY and bundle stability — Work in progress

SUSY $\iff$ holomorphic vector bundle $V \to X$ with HYM connection:

$$F \wedge \Omega = 0\ , \quad F \wedge J \wedge J = 0\ .$$

Li–Yau theorem: $V \to X$ allows HYM connection $\iff$ $V \to X$ is stable.

Donaldson'85, Uhlenbeck–Yau'86,Li–Yau'87, Kobayashi'87, Hitchin

Here: know $J \rightsquigarrow$ solve for F.

SU(3) structures on CY: Strominger–Hull system

Bianchi identity — Work in progress

$$2i\,\bar{\partial}\partial\mathcal{F}\wedge J_0 = dH = \frac{\alpha'}{4}\left(\mathrm{tr}(F\wedge F) - \mathrm{tr}(R\wedge R)\right) + \ldots\,,$$

$\mathrm{tr}(R\wedge R)$

Invariant under conformal re-scaling: $\mathrm{tr}(R\wedge R) = \mathrm{tr}(R_0\wedge R_0)$
Computable but lack manageable form for general CICY.

$\mathrm{tr}(F\wedge F)$

Construct stable vector bundle $V\to X$ with suitable connection A.

SU(3) structures on CY: Strominger–Hull system

Example: $\mathrm{tr}(R \wedge R)$ on tetraquadric

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Set $z_4 = f(z_1, z_2, z_3)$, $v_\alpha = p_{,\alpha}/p_{,4}$, $\Omega_\alpha^\beta = \frac{t_1 t_2 t_3 t_4}{t_\beta} \bar{\partial} \left(\frac{\Lambda_4 \Lambda_\beta}{\mathcal{F} \Lambda_\alpha} \partial \left(\frac{\Lambda_\alpha}{\Lambda_4} \right) \right)$.

Compute curvature 2-form:

$$R_\alpha^\beta = -4\pi i J_\alpha \delta_\alpha^\beta - \frac{v_\alpha}{v_\beta} \Omega_\alpha^\beta$$

$$\mathrm{tr} R = \partial \bar{\partial} \ln \det g_0 \quad \checkmark$$

Finally

$$\mathrm{tr}(R \wedge R) = \sum_{\alpha,\beta=1}^{3} (R_\alpha^\beta \wedge R_\beta^\alpha) + \text{c.c.} = 8\pi i \sum_{\alpha=1}^{3} J_\alpha \wedge \Omega_\alpha^\alpha + \sum_{\alpha,\beta=1}^{3} \Omega_\alpha^\beta \wedge \Omega_\beta^\alpha + \text{c.c.} \,.$$

Conclusions and outlook

Conclusions

All CY manifolds allow several SU(3) structures

CICY: ambient space provide building blocks for non-trivial SU(3) structures

- $J = \sum_{i=1}^{m} a_i J_i$, $\Omega = A\,\Omega_0$, subject to $|A|^2 = \sum_{i,j,k=1}^{m} \Lambda_{ijk} a_i a_j a_k$
- metric computable $g_{mn} = I_m{}^k J_{kn}$
- torsion computable:

$$W_3 = \sum_i (da_i - a_i W_4) \wedge J_i \ , \quad W_4 = \frac{1}{2} \sum_i J \lrcorner (da_i \wedge J_i) \ , \quad W_5 = d \ln(A) \ .$$

- With $a_i = a \ \forall i \implies$ Strominger–Hull system, with $dH \neq 0$

  Work on heterotic BI: compute $\mathrm{tr}(R \wedge R)$, construct vector bundles, ...

Conclusions and outlook

Conclusions

Use ambient space forms to build complex SU(3) structures on CICY
Necessary constraints for e.g. Strominger–Hull system fulfilled

What we should do next:

- Construct vector bundles with HYM connections satisfying heterotic BI
- Explore "non-universal" SU(3) structures
- Type IIB vacua: "smeared" sources required?
- Extend construction: other types of CY manifolds, other dimensions, ...
- Generalise method: non-complex SU(3) structures, ...

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Thank You

4D $\mathcal{N} = 1$ solutions from $SU(3)$ structure manifolds

4D geometry	**String vacuum**	**Non-vanishing torsion**	**$SU(3)$ type**
$\mathcal{N} = 1$ Mkw	Heterotic, Type II (H_3) Heterotic ($H_3 = 0$)	$W_3, W_4 = d\phi, W_5 = 2W_4$ $W_i = 0 \ , \forall i$	Complex CY
$\mathcal{N} = 1$ Mkw	Type IIB ($H_3, F_3, F_5, O_3/O_7$) Type IIB ($F_3, O_5/O_9$) Type IIB/F-theory ($H_3, F_3, F_5, O_3/O_7$)	$3W_4 = 2W_5$ $W_3, W_4 = d\phi, W_5 = 2W_4$ $W_4 = W_5 = d\phi$	Conf. CY Complex Complex
$\mathcal{N} = 1$ Mkw	Type IIA (F_2, F_4, O_6)	$W_2, 3W_5 = d\phi$	Symplectic
$\mathcal{N} = 1$ AdS	Type IIA ($H_3, F_{\rm even}$)	$W_1^+, W_2^+, dW_2^+ \propto \Omega^+$	Half-flat