CIC with AHEAD

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A few trivialities first

If the flux $\phi$ of (primary) cosmic rays is isotropic, and ignoring for the moment the attenuation in the atmosphere, the rate for an horizontal detector with a sensitive $A$ area is:

$$\frac{dN}{dt} = 2\pi \phi A \cos \theta \sin \theta \, d\theta$$

But since $\cos \theta \sin \theta \, d\theta = -d(\cos^2 \theta)$, one should have:

$$\frac{dN}{dt \, d(\cos^2 \theta)} = \text{constant}$$

Hence attenuation effects should appear as departing from a constant "intensity" when studying it as function of $\cos \theta$. 
Atmospheric attenuation

\[(R + z)^2 = (R + h)^2 + (h \tan \theta)^2\]
\[\Rightarrow z + \frac{z^2}{2R} = h + \frac{h^2}{2R \cos^2 \theta}\]

Using \(\frac{z}{2R} \ll 1\):

\[z \approx h \left(1 + \frac{h}{2R \cos^2 \theta}\right)\]

With \(h < 20\) km and \(R = 6378\) km, the approximation \(z \approx h\) can safely be used down to \(\theta < 80^\circ\).

Hence

\[X_{\text{slent}} = \int_0^\infty \rho(z)dr \approx \int_0^\infty \rho(h) \frac{dh}{\cos \theta} = \frac{X_{\text{vert}}}{\cos \theta}\]

So the attenuation effect is expected to be to first order a function of \(\sec \theta = 1/\cos \theta\).
Early MIT work was done at Agassiz, near MIT

First use of scintillators to measure time of arrival and directions

Bassi, Clark and Rossi: Phys Rev 92 441 1953
$N_v = N \exp\left[\frac{(x-x_0)}{\lambda}\right]$. 

$N_v = \text{Vertical Equivalent Size}$

Extensive discussion about relation of this size to Geiger Counter Size

Fig. 14. Variation of shower intensity with atmospheric depth. The straight lines represent exponential variation. Their slopes are calculated by the method of maximum likelihood.

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Array operated at El Alto at 4200 m: forerunner of Chacaltaya project at 5000m

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Schematic of Constant Intensity Cut Method

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Whereas, at sea level, $S$ for fixed $N$ can be expressed as an exponential function of $\sec \theta$ (which is proportional to the slant thickness of the atmosphere), at 4200 m we find that $S$ (for $N$ near $30 \times 10^6$) is approximately stationary with respect to changes in $\sec \theta$ near $\sec \theta = 1.0$. This is the behavior expected if showers with $N \approx 30 \times 10^6$ are near their maximum longitudinal development.
FIG. 2. Experimental results on the longitudinal development of extensive air showers. The plot is derived from the observed integral size spectra at various zenith angles and represents the dependence of $N$ on $x$ at constant $S$. The upper and lower sets of points are for $S = 10^{-14}$ cm$^{-2}$ sec$^{-1}$ sr$^{-1}$ and $S = 10^{-13}$ cm$^{-2}$ sec$^{-1}$ sr$^{-1}$, respectively. The straight line with an error indicator represents the analogous results derived from sea level data for $S = 10^{-13}$ cm$^{-2}$ sec$^{-1}$ sr$^{-1}$. 
3.2. Determination of attenuation length

Assuming exponential attenuation of showers in the atmosphere over the depth corresponding to the extremes of inclination, the values of \( \rho(600) \) observed for showers derived from primaries of the same energy but incident at various zenith angles \( \theta \), will be related by

\[
(\rho(600))_\theta \exp\left(\frac{1018}{\lambda} \text{sec } \theta\right) = \text{constant},
\]

where \( \lambda \) (in g cm\(^{-2}\)) is the attenuation length (the atmospheric depth of Haverah Park being taken to be 1018 g cm\(^{-2}\)).

To derive \( \lambda \), cuts at constant intensity, and so corresponding to constant primary energy, are taken across the eight spectra of the zenith angle bands. Since the experimental exponents of these spectra are not identical, the value of \( \lambda \) derived will depend upon the intensity at which the cut is made. At \( I = 10^{-12} \text{ m}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \), the value is \( 760 \pm 40 \text{ g cm}^{-2} \). In order to determine \( \lambda \) also from a cut at \( I = 10^{-13} \text{ m}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \), the spectral slopes were redetermined in the range \( 2 \times 10^{-12} > I > 5 \times 10^{-14} \text{ m}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \), to ensure that the spectra were in correct fit to the experimental data over this particular range. The redetermined values of \( k \) and \( \Gamma \) differed only slightly from those initially determined (table 4), and do not support the existence of any systematic changes. The value of \( \lambda \) for a cut at \( 10^{-13} \text{ m}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \) is \( 780 \pm 35 \text{ g cm}^{-2} \).
Fig. 5. Attenuation of $\rho(600)$ with zenith from the constant intensity cut method, as obtained previously [20], and in the present work.

Fig. 6. Attenuation of $\rho(600)$ with zenith angle for different primaries and energies from Monte Carlo calculations. The errors bars correspond to the spread of the value of $\rho(600)$, to illustrate the shower fluctuations.
Auger Collaboration has adopted the same approach except that reference angle is not vertical but 38 degrees.

This is about the median zenith angle of the events on the 1500 m array.

\[ S_{38} = S(1000 \text{ at } 38^\circ) \]
Figure 3: Zenith angle distribution for events in the fiducial region. The energies are $\geq 10^{19}$ eV and zenith angles $\leq 60$ deg. The solid curve is the expected distribution.

Figure 4: Relative flux vs $\text{VEM}_{1000}$ for bins of $\sec(\theta)$ of width 0.3 centered at 1.1, 1.3, 1.5, 1.7, and 1.9. The dashed line is a guess for $\sec(\theta) = 1.0$. The energy lines were chosen assuming a vertical shower at $10^{19}$ eV corresponds to $\text{VEM}_{1000} = 70$. The lines for lower energy are selected assuming that the energy is proportional to $(\text{VEM}_{1000})^{1.05}$.

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A way to derive the attenuation length from the constant intensity cut method

I am setting down what I think is a reliable way to get $\lambda$ from the constant intensity cut method (CIC).

1. It is essential to use integral spectra to make the cut. This is because the working hypothesis is that the showers that are recorded at the same rate ABOVE some size/ground parameter value are produced by primary particles of the same energy.

2. The integral spectrum should be derived from the differential spectrum (with uncertainties in slope and the constant) using the maximum likelihood technique. Then with an integral spectrum one can find the value of $S(1000)$ for each zenith angle band for some chosen intensities.

3. It is not correct to use the differential spectrum. The point is that $dS(1000)$ in the denominator of differential intensity ($N/dS(1000)$) does not have the same meaning at different angles. This may be why the derived $\lambda$ values are so strange.

I think that you now have the tools: let’s see what this method gives us.

aaw/230405
The CIC method applied to AHEAD

- With AHEAD, we are far enough from shower maximum and at "low" enough energy to consider that \( N_e \) the shower size at the shower foot as measured from the LDF fit, is a reasonable estimator of the shower energy.
- The LDF is the lateral distribution of the particle density in each station, where the distance to the shower axis is used (projected in the shower plane).
- The LDF is fitted with a NKG type function (Nishimura, Kamata, Greisen):
  \[
  \rho(r) = \frac{N_e C_s}{r_m^2} \left( \frac{r}{r_m} \right)^{s-2} \left( 1 + \frac{r}{r_m} \right)^{s-4.5}
  \]
  where \( s = 1.2 \) is set to this fixed value, \( r_m = 80 \text{m} \) and the constant
  \[
  C_s = 0.366 \times s^2 \times (2.07 - s)^{1.25} = 0.443.
  \]
  The value of \( N_e \) is extracted from the fit.
- The next step is to correct for the attenuation using the CIC method...
The CIC method applied to AHEAD

- The next step is to correct for the attenuation using the CIC method:
  - Plot the number of events with shower sizes $< N_e$ as a function of $N_e$ for events in bins of $\sec \theta$ of width 0.2 centered at 1.1, 1.3, 1.5, 1.7 and 1.9.
  - The CIC methods assume that for a given true energy, the intensity (the number of events) is independent of the arrival direction i.e. a given energy corresponds to a horizontal line across the plot.
  - Using that plot, it is then possible to extrapolate from the measured $N_e$ to a standardize rescaled value corresponding to a vertical shower $N_e^{0^\circ}$ or to a 38° shower $N_e^{38^\circ}$ (38° is used because it is corresponds to a median value).
  - The correction formula that I propose to use is
    \[ N_e^{0^\circ} = N_e(\theta) e^{\frac{X_{vert}}{\Lambda_{att}} (\sec \theta - 1)} \]
  This corresponds to an exponential attenuation in the atmosphere.
  - The last step relies on models and Monte-Carlo simulation to convert this median value to an energy in eV. We will use, for example:
    \[ E[eV] = 2.183 \times 10^{10} \times (N_e^{0^\circ})^{0.9} \]
CIC curves from AHEAD

278500 events since July 2017

Intensity i.e. event rate

1 \leq \sec \theta < 1.2
1.2 \leq \sec \theta < 1.4
1.4 \leq \sec \theta < 1.6
1.6 \leq \sec \theta < 1.8
1.8 \leq \sec \theta < 2.0
CIC curves from AHEAD

278500 events since July 2017

Intensity i.e. event rate

1 ≤ sec θ < 1.2
1.2 ≤ sec θ < 1.4
1.4 ≤ sec θ < 1.6
1.6 ≤ sec θ < 1.8
1.8 ≤ sec θ < 2.0

Corrected with

\[ N_e^{0°} = N_e(\theta) e^{\frac{X_{vert}}{\Lambda_{att}} (\sec \theta - 1)} \]