Interaction of particles with matter (lecture 1)
A brief review of a few typical situations is going to greatly simplify the subject.

Mean free path of a particle, i.e. average distance travelled between two consecutive collisions in matter:

\[ \lambda = \frac{1}{\sigma n} \]

where:

\( \sigma \) total interaction cross-section of the particle
\( n \) number of scattering centers per unit volume

example: \( n = \frac{\rho N_A}{M} \) for a monoatomic element of molar mass \( M \) and specific mass \( \rho \).

\( N_A \) Avogadro number

Electromagnetic interaction: \( \lambda \leq 1 \mu m \) (charged particles)

Strong interaction: \( \lambda \geq 1 \text{ cm} \) (neutrons ....)

Weak interaction: \( \lambda \geq 10^{15} \text{ m} \approx 0.1 \text{ light year} \) (neutrinos)

A practical signal (>100 interactions or hits) can only come from electromagnetic interaction.

Particle detection proceeds in two steps: 1) primary interaction 2) charged particle interaction producing the signals.
typical examples: photon detection

Signal is induced by electrons

Compton scattering

Pair production
neutral pion detection:

A $\pi^0$ decays into two photons with a mean lifetime of $8.5 \times 10^{-17}$ s.
neutrino detection:

A 2800 MW nuclear power station produces 130 MW of neutrinos!

A detector of 1 m³ located 20 m away from the reactor core can detect 100 neutrinos/h.

\[ \bar{\nu}_e + p \rightarrow n + e^+ \]

\[ n_{th} + L_i^6 \rightarrow \alpha + t + 4.8 \text{ MeV} \]

Charged particles produce light in the target scintillator
Interaction of charged particles with matter

For heavy particles ionization and excitation are the dominant processes producing energy loss.

Particle $P$ of $Z$ charge state

Excitation: $P^{(Z)} + \text{atom} \rightarrow \text{atom}^* + P^{(Z)}$ followed by: $\text{atom}^* \rightarrow \text{atom} + \gamma$

Ionization: $P^{(Z)} + \text{atom} \rightarrow \text{atom}^+ + e^- + P^{(Z)}$

Ionization + excitation: $P^{(Z)} + \text{atom} \rightarrow \text{atom}^{*+} + e^- + P^{(Z)}$
$T_e^{\text{max}}$, Maximal kinetic energy transferred to an ionized electron:

$$m_0 \quad \vec{v}$$

$$\vec{p} = \gamma m_0 \vec{v}$$

$$E = (p^2 + m_0^2)^{-1/2}$$

Hypothesis: $V > \langle v_e \rangle = Z \alpha c$, speed of deepest atomic orbit electrons where $\alpha$ is the fine structure constant: $\alpha = 1/137$.

One may show (exercise) that: $T_e^{\text{max}} = E_e^{\text{max}} - m_e = \frac{2 m_e \beta^2 \gamma^2}{(E_{CM}/m_0)^2}$ (In natural units, $c=\hbar=1$)

where: $E_{CM} = (m_0^2 + m_e^2 + 2 m_e E)^{1/2}$ total energy in center-of-mass frame
Two cases:

\[ m_0 \gg m_e, \text{ i.e. the incoming particle is not an electron and if its energy is not too big} \]

\[ \left( \frac{E_{\text{CM}}}{m_0} \right)^2 = \left( \frac{m_0^2}{m_0^2} + \frac{m_e^2}{m_0^2} + \frac{2m_e E}{m_0^2} \right) \approx 1 \quad \text{with} \quad E = \gamma m_0 \]

\[ \frac{2 \gamma m_e}{m_0} \ll 1 \quad \text{proton } E_p < 50 \text{ GeV, muon } E_\mu < 500 \text{ MeV (medium energy range)} \]

then:

\[ T_e^{\text{max}} = E_e^{\text{max}} - m_e = 2m_e \beta^2 \gamma^2 \]

\[ m_0 = m_e \quad \text{the incoming particle is an electron} \]

\[ T_e^{\text{max}} = (E - m_e) \quad \text{due to undistinguishibility of electrons, max transferable energy} = T^{\text{max}} / 2 \]

If the incoming particle is not an electron then in practice \( m_0 \gg m_e \).
Stopping power of heavy particles by excitation and ionization in matter.
Average energy loss by a charged particle (other than an electron) in matter.

Bethe and Bloch formula
(see Nuclei and particles, Émilio Segré, W.A. Benjamin ; Principles of Radiation Interaction in Matter and Detection, C. Leroy and P.G. Rancoita, World Scientific ; Introduction to experimental particle physics, R. Fernow)

\[-\frac{dE}{dx}[\text{MeV g/cm}^2] = \frac{0.3071}{A(\text{g mol}^{-1})} \frac{z^2 Z}{\beta^2} \left( \frac{1}{2} \ln \left( \frac{2 m_e \beta^2 \gamma^2 T_e^{\text{max}}}{I^2} \right) - \beta^2 - \frac{\delta (\gamma \beta)}{2} - \frac{C_e}{Z} \right)\]

Stopping power or mean specific energy loss

charge of incoming particle
Z of medium
density effect correction at high energy
Atomic shell correction at low energy
(not covered in this lecture, see Leroy & Rancoita)

mean excitation energy
Atomic mass of medium

Surface mass density of medium \( dx = \rho \, dl \)
(or mass thickness of medium)
show that:

\[ \beta^2 = \frac{(\beta \gamma)^2}{1 + (\beta \gamma)^2} \]
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Particle identification in Alice TPC

negative particles,
PbPb, 2011 run,
$\sqrt{s_{NN}} = 2.76\text{TeV}$
few remarks:

- for $\beta \gamma < 1$: $\frac{-dE}{dx} \sim \beta^{-5/3}$ non relativistic particles

- for $\beta \gamma \sim 3-4$: $\frac{-dE}{dx}$ is minimal over a large energy plateau. A particle in this state is called a minimum ionizing particle (MIP)

In media composed of light elements: $\frac{-dE}{dx} \approx 2 \frac{\text{MeV}}{\text{g cm}^{-2}}$

- for $\beta \gamma > 4$: relativistic increase of $\frac{-dE}{dx}$ as $\ln(\gamma)$ which is tempered by $-\delta/2$ correction.

- $I$: mean excitation and ionization energy, $I = 15$ eV for atomic H and 19.2 eV for $H_2$
  $I = 41.8$ for He
  $I = 15 Z^{0.9}$ eV for $Z > 2$

At medium energy: $\frac{2y m_e}{m_a} \ll 1$     $T_{e \text{max}} = 2m_e \beta^2 y^2$

$-(\frac{dE}{dx})[\frac{\text{MeV}}{\text{g/cm}^2}] = 0.3071 \cdot \frac{A(g)}{Z^2} \cdot \frac{Z^2 Z}{\beta^2 \ln(\frac{2m_e \beta^2 y^2}{I}) - \beta^2 - \frac{\delta}{2} - \frac{C_e}{Z}}$
Density effect correction

When energy increases, stopping power decreases to a minimum ($1/\beta^2$ dependance) and then starts rising again due to logarithmic term. In fact, the max. transverse electric field increases as $\gamma$ but its influence is screened by nearby atoms beyond a distance of $70 \text{ Å} (\text{shown by Bohr})$. This density effect tempers the relativistic rise.

Studies have been carried-out by Sernheimer, Peierls, Berger & Seltzer (see Leroy & Rancoita).

The density correction effect term, $\delta$ is given by:

\[
\begin{align*}
\text{for } \beta \gamma < 10^{S_0} &: \quad \delta = \delta_0 \left( \frac{\beta \gamma}{10^{S_0}} \right)^2 \\
\text{for } 10^{S_0} < \beta \gamma < 10^{S_1} &: \quad \delta = 2 \ln(\beta \gamma) + C + a \left[ \frac{1}{\ln(10)} \ln \left( \frac{10^{S_1}}{\beta \gamma} \right) \right]^{md} \\
\text{for } \beta \gamma > 10^{S_1} &: \quad \delta = 2 \ln(\beta \gamma) + C
\end{align*}
\]

where: $C = -2 \ln \left( \frac{I}{h \nu_p} \right) - 1$

with $\nu_p = \sqrt{\frac{nr_e c^2}{\pi}}$ in which $n$ is the density of electrons and $r_e$ the classical radius of $e^{-}$: $r_e = 2.82 \text{ fm}

$h \nu_p \approx 28.7 \sqrt{\frac{\rho (g/cm^3)}{A(g)}} Z \text{ eV}$

show that at very high energy: $\beta \gamma > 10^{S_1}$

\[
-(\frac{dE}{dx})[\text{MeV}] = 0.3071 \frac{z^2 Z}{2A(g)} \left[ \ln \left( \frac{2m_e T_{e,max}}{(h \nu_p)^2} \right) - 1 \right]
\]
Table 2.1 Values of $Z$, $Z/A$, $I$, $\rho$ in units of g/cm$^3$, $h\nu_p$ and density-effect parameters $S_0$, $S_1$, $a$, $md$, and $\delta_0$ for elemental substances.

<table>
<thead>
<tr>
<th>El.</th>
<th>$Z$</th>
<th>$Z/A$</th>
<th>$I$ eV</th>
<th>$\rho$</th>
<th>$h\nu_p$ eV</th>
<th>$S_0$</th>
<th>$S_1$</th>
<th>$a$</th>
<th>$md$</th>
<th>$\delta_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>2</td>
<td>0.500</td>
<td>41.8</td>
<td>$1.66 \times 10^{-4}$</td>
<td>0.26</td>
<td>2.202</td>
<td>3.612</td>
<td>0.134</td>
<td>5.835</td>
<td>0.00</td>
</tr>
<tr>
<td>Li</td>
<td>3</td>
<td>0.432</td>
<td>40.0</td>
<td>0.53</td>
<td>13.84</td>
<td>0.130</td>
<td>1.640</td>
<td>0.951</td>
<td>2.500</td>
<td>0.14</td>
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<tr>
<td>O</td>
<td>8</td>
<td>0.500</td>
<td>95.0</td>
<td>$1.33 \times 10^{-3}$</td>
<td>0.74</td>
<td>1.754</td>
<td>4.321</td>
<td>0.118</td>
<td>3.291</td>
<td>0.00</td>
</tr>
<tr>
<td>Ne</td>
<td>10</td>
<td>0.496</td>
<td>137.0</td>
<td>$8.36 \times 10^{-4}$</td>
<td>0.59</td>
<td>2.074</td>
<td>4.642</td>
<td>0.081</td>
<td>3.577</td>
<td>0.00</td>
</tr>
<tr>
<td>Al</td>
<td>13</td>
<td>0.482</td>
<td>166.0</td>
<td>2.70</td>
<td>32.86</td>
<td>0.171</td>
<td>3.013</td>
<td>0.080</td>
<td>3.635</td>
<td>0.12</td>
</tr>
<tr>
<td>Si</td>
<td>14</td>
<td>0.498</td>
<td>173.0</td>
<td>2.33</td>
<td>31.06</td>
<td>0.201</td>
<td>2.872</td>
<td>0.149</td>
<td>3.255</td>
<td>0.14</td>
</tr>
<tr>
<td>Ar</td>
<td>18</td>
<td>0.451</td>
<td>188.0</td>
<td>$1.66 \times 10^{-3}$</td>
<td>0.79</td>
<td>1.764</td>
<td>4.486</td>
<td>0.197</td>
<td>2.962</td>
<td>0.00</td>
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<tr>
<td>Fe</td>
<td>26</td>
<td>0.466</td>
<td>286.0</td>
<td>7.87</td>
<td>55.17</td>
<td>-0.001</td>
<td>3.153</td>
<td>0.147</td>
<td>2.963</td>
<td>0.12</td>
</tr>
<tr>
<td>Cu</td>
<td>29</td>
<td>0.456</td>
<td>322.0</td>
<td>8.96</td>
<td>58.27</td>
<td>-0.025</td>
<td>3.279</td>
<td>0.143</td>
<td>2.904</td>
<td>0.08</td>
</tr>
<tr>
<td>Ge</td>
<td>32</td>
<td>0.441</td>
<td>350.0</td>
<td>5.32</td>
<td>44.14</td>
<td>0.338</td>
<td>3.610</td>
<td>0.072</td>
<td>3.331</td>
<td>0.14</td>
</tr>
<tr>
<td>Kr</td>
<td>36</td>
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<td>352.0</td>
<td>$3.48 \times 10^{-3}$</td>
<td>1.11</td>
<td>1.716</td>
<td>5.075</td>
<td>0.074</td>
<td>3.405</td>
<td>0.00</td>
</tr>
<tr>
<td>Ag</td>
<td>47</td>
<td>0.436</td>
<td>470.0</td>
<td>10.50</td>
<td>61.64</td>
<td>0.066</td>
<td>3.107</td>
<td>0.246</td>
<td>2.690</td>
<td>0.14</td>
</tr>
<tr>
<td>Xe</td>
<td>54</td>
<td>0.411</td>
<td>482.0</td>
<td>$5.49 \times 10^{-3}$</td>
<td>1.37</td>
<td>1.563</td>
<td>4.737</td>
<td>0.233</td>
<td>2.741</td>
<td>0.0</td>
</tr>
<tr>
<td>Ta</td>
<td>73</td>
<td>0.403</td>
<td>718.0</td>
<td>16.65</td>
<td>74.69</td>
<td>0.212</td>
<td>3.481</td>
<td>0.178</td>
<td>2.762</td>
<td>0.14</td>
</tr>
<tr>
<td>W</td>
<td>74</td>
<td>0.403</td>
<td>727.0</td>
<td>19.30</td>
<td>80.32</td>
<td>0.217</td>
<td>3.496</td>
<td>0.155</td>
<td>2.845</td>
<td>0.14</td>
</tr>
<tr>
<td>Au</td>
<td>79</td>
<td>0.401</td>
<td>790.0</td>
<td>19.32</td>
<td>80.22</td>
<td>0.202</td>
<td>3.698</td>
<td>0.098</td>
<td>3.110</td>
<td>0.14</td>
</tr>
<tr>
<td>Pb</td>
<td>82</td>
<td>0.396</td>
<td>823.0</td>
<td>11.35</td>
<td>61.07</td>
<td>0.378</td>
<td>3.807</td>
<td>0.094</td>
<td>3.161</td>
<td>0.14</td>
</tr>
<tr>
<td>U</td>
<td>92</td>
<td>0.387</td>
<td>890.0</td>
<td>18.95</td>
<td>77.99</td>
<td>0.226</td>
<td>3.372</td>
<td>0.197</td>
<td>2.817</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Data are from [Sternheimer, Berger and Seltzer (1984)]
Restricted energy loss

knock-on electron (delta ray) generated by a 180 GeV muon as observed by the experiment GridPix at CERN SPS.

High energy transfers generate delta rays that may escape the detector if it is too thin. So average energy deposits are very often much smaller than predicted by B&B.

If $T_0$ is the average maximal delta ray energy that can be absorbed in the detecting medium, a better estimate of the average deposited energy is given by:

$$-(\frac{dE}{dx})[\frac{\text{MeV}}{\text{g/cm}^2}] = \frac{0.3071}{A(\text{g mol}^{-1})} \frac{Z^2 Z}{\beta^2} \left( \frac{1}{2} \ln \left( \frac{2m_e \beta^2 \gamma^2 T_0}{I^2} \right) - \frac{\beta^2}{2} \left( 1 + \frac{T_0}{2m_e \beta^2 \gamma^2} \right) - \frac{\delta(\gamma \beta)}{2} - \frac{C_e}{Z} \right)$$

At extremely high energies, when $\beta \gamma > 10^5$, stopping power reaches a constant called Fermi plateau.

$$-(\frac{dE}{dx})[\frac{\text{MeV}}{\text{g/cm}^2}] = 0.3071 \frac{z^2 Z}{2A(g)} \ln \left( \frac{2m_e T_0}{(h \nu_p)^2} \right)$$
Fermi plateau measured in silicon

Fig. 2.5 Energy loss in silicon (in units of eV/μm) versus $\beta \gamma (= p/M_0c$, where $M_0$ is the rest mass of the incoming particle) from [Rancoita (1984)]. From the top the first two curves are: the $-\Delta E/\Delta x$ without (broken curve) and with (full curve) the density-effect correction. The following second two curves are compared to experimental data for detector thicknesses of 300 (x from [Hancock, James, Movchet, Rancoita and Van Rossum (1983)]) and 900 μm (o and ● from [Esbensen et al. (1978)]): the restricted energy loss with the density-effect taken into account and the prediction of the most probable energy loss.
Delta rays (secondary electrons)

The differential probability to generate a delta ray of kinetic energy $T$ is given by:

$$\frac{dw(T, E)}{dTdx} = 0.3071 \frac{z^2 Z}{2. A(g) \beta^2} \frac{F(T)}{T^2} \text{MeV}^{-1} \text{cm}^2 \text{g}^{-1}$$

$F(T)$ is a spin-dependent factor.

For spin-0 particles:

$$F(T) = F_0(T) = (1 - \beta^2 \frac{T}{T_{\text{max}}})$$

For spin-1/2 particles:

$$F(T) = F_{1/2}(T) = F_0(T) + \frac{1}{2} \left( \frac{T}{E} \right)^2$$

For spin-1 particles:

$$F(T) = F_1(T) = F_0(T) \left( 1 + \frac{1}{3} \frac{T m_e}{m_0^2} \right) + \frac{1}{3} \left( \frac{T}{E} \right)^2 \left( 1 + \frac{1}{2} \frac{T m_e}{m_0^2} \right)$$
Delta rays (secondary electrons)

For $T \ll T_{\text{max}}$ and $T \ll m_0^2 / m_e$,

$$\frac{dw(T, E)}{dT dx} = 0.3071 \frac{z^2 Z}{2.A (g) \beta^2} \frac{1}{T^2} \text{MeV}^{-1} \text{cm}^2 \text{g}^{-1}$$

This allows to compute an approximate probability to generate a delta ray of kinetic energy greater than $T_s$ in a thin absorber of mass thickness $x$:

$$w(T_s, E, x) \approx 0.3071 x \frac{z^2 Z}{2.A (g) \beta^2} \frac{1}{T_s}$$

show this expression
Energy straggling distribution

So far, only the average energy loss has been considered. But energy loss is subjected to large fluctuations that in thin absorbers results in asymmetric distributions. The subject is quite complex and has no general exact solutions, but a few approximate formulas help to estimate it.

For thin absorbers in which $\frac{\epsilon}{T_{\text{max}}}$ $\ll$ 1, where:

$$\epsilon = 0.3071 x \frac{z^2 Z}{2. A(g) \beta^2} \text{ MeV}$$

where $x$ is mass thickness of the absorber in g cm$^{-2}$.

The problem was first studied by Landau and then Vavilov. Their distribution functions are not analitic. A useful approximation of the Landau distribution is:

$$L(\lambda) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(\lambda + e^{-\lambda})\right)$$

where:

$$\lambda = \frac{\Delta E - \Delta E_{\text{MP}}}{\epsilon}$$

and $\Delta E$ is the energy loss $\Delta E_{\text{MP}}$ is the most probable energy loss.

$$\Delta E_{\text{MP}} = \Delta E_{\text{Bethe}} + \epsilon \left(\beta^2 + \ln\left(\frac{\epsilon}{T_{\text{max}}}\right) + 0.194\right) \text{ MeV}$$
Energy straggling

Still for thin absorbers, an improved generalized energy loss distribution that takes into account the distant collisions that are neglected in Landau’s approach, can be obtained by convoluting a Landau distribution with a Gaussian distribution:

\[ f(\Delta E, x)_I = \frac{1}{\sqrt{2\pi\sigma_I^2}} \int_{-\infty}^{\infty} L(\Delta E - \Delta E', x) \exp\left( -\frac{\Delta E'}{2\sigma_I^2} \right) d(\Delta E') \]

Energy deposited by protons in silicon

736 MeV/c protons

115 GeV/c protons

Fig. 2.10 Curves (a) and (b) (adapted and republished with permission from Hancock, S., James, F., Mowchett, J., Rancoita, P.G. and Van Rossum, L., Phys. Rev. A 28, 615 (1983); Copyright (1983) by the American Physical Society) show the energy loss spectra at 0.736 and 115 GeV/c of incoming particle momentum. Continuous curves are the complete fit to experimental data, i.e., the Landau straggling function folded over the Gaussian distribution taking into account distant collisions.
Energy straggling

In thick absorbers in which \( \varepsilon / T_{\text{max}} \gg 1 \), both the Landau and the Vavilov distributions tend to a Gaussian:

\[
f(\Delta E, x) \approx \frac{1}{\sqrt{2\pi T_{\text{max}} \varepsilon (1 - \frac{\beta^2}{2})}} \exp \left( -\frac{(\Delta E - \Delta E_{\text{Bethe}})^2}{2 T_{\text{max}} \varepsilon (1 - \frac{\beta^2}{2})} \right)
\]

with:

\[
\sigma \approx \sqrt{\varepsilon T_{\text{max}} (1 - \frac{\beta^2}{2})}
\]
Stopping power of electrons by ionization and excitation in matter.

Incoming and outgoing particles are identical.
Energy transfer is bigger.

\[-(\frac{dE}{dx})[\text{MeV}\frac{g}{cm^2}] = \frac{0.3071}{A(g)} \cdot \frac{Z}{\beta^2} \left[ \frac{1}{2} \ln \left( \frac{T m_e \beta^2 \gamma^2}{2 I^2} \right) + \frac{1}{2 \gamma^2} \left( 1 - (2 \gamma - 1) \ln(2) \right) + \frac{1}{16} \left( \frac{\gamma - 1}{\gamma} \right)^2 \right] \]

Kinetic energy of incoming electron: \(T = (\gamma - 1) m_e = E - m_e\)

Stopping power of positrons by ionization and excitation in matter.

\[-(\frac{dE}{dx})[\text{MeV}\frac{g}{cm^2}] = \frac{0.3071}{A(g)} \cdot \frac{Z}{\beta^2} \left[ \frac{1}{2} \ln \left( \frac{T m_e \beta^2 \gamma^2}{2 I^2} \right) - \frac{\beta^2}{24} \left( 23 + \frac{14}{\gamma + 1} + \frac{10}{(\gamma + 1)^2} + \frac{4}{(\gamma + 1)^3} \right) \right] \]

When a positron comes to a rest it annihilates: \(e^+ + e^- \rightarrow \gamma \gamma\) of 511 KeV each.

A positron may also undergo an annihilation in flight according to the following cross section:

\[\sigma(Z, E) = \frac{Z \pi r_e^2}{\gamma + 1} \left[ \frac{\gamma^2 + 4 \gamma + 1}{\gamma^2 - 1} \ln \left( \gamma + \sqrt{\gamma^2 - 1} \right) - \frac{\gamma + 3}{\sqrt{\gamma^2 - 1}} \right] \]

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Stopping power of a compound medium

\[ \frac{dE}{dx} \approx \sum_i f_i \frac{dE}{dx} \bigg|_i \]

\[ f_i = \frac{m_i}{m}, \quad \sum_i m_i = m \quad \text{where } f_i \text{ is the massic ratio of element } i \]

\[ \frac{dE}{dx} \bigg|_i \quad \text{is the stopping power of element } i \]
Interaction of particles with matter
(lecture 2)
Bremsstrahlung: electromagnetic radiative energy loss

A decelerated or accelerated charged particle radiates photons. The mean radiative energy loss is given by:

\[- \frac{dE_{\text{rad}}}{dx} (\text{MeV} g/cm^2) = 0.3071 \frac{\alpha Z^2 z^2 (m_e/m)^2}{\pi A(g)} E \ln \left( \frac{183}{Z^{1/3}} \right)\]

The mean radiative energy loss of a particle of charge \(z\) and mass \(m\) is a function of the mean radiative energy loss of an electron:

\[\frac{dE_{\text{rad}}}{dx} (z, m) = \left( \frac{m_e}{m} \right)^2 z^2 \frac{dE_{\text{rad}}}{dx} (e^-)\]

Electrons are much more sensitive to this effect.
For an electron and taking into account the Bremsstrahlung radiation induced by atomic electrons:

\[- \frac{dE_{rad}}{dx} (e^-) = 4\alpha N_A \frac{Z(Z+1)}{A} r_e^2 E \ln \left( \frac{183}{Z^{1/3}} \right)\]

where \(r_e = \alpha/m_e\)

which can be rewritten as:

\[- \frac{dE_{rad}}{dx} (e^-) = \frac{E}{X_0}\]

where \(X_0\) is the medium radiation length

then over a path \(x\) in the medium, the mean radiated energy of an electron reads:

\[E_{rad}(e^-) = E(1 - e^{-x/X_0})\]

where \(x\) is expressed in cm or g/cm^2

and:

\[X_0(g/cm^2) = \frac{716.4 \ A(g)}{Z(Z+1) \ln \left( \frac{287}{Z^{1/2}} \right)}\]

In a compound medium:

\[X_o = \left[ \sum_i f_i X_{0i} \right]^{-1}\]

where \(f_i\) and \(X_{0i}\) are the mass ratio and the radiation length of element \(i\) respectively.
Critical energy: energy at which the ionization stopping power is equal to the mean radiative energy loss of electrons

\[ \frac{dE^{\text{rad}}}{dx}(E_c) = \frac{dE^{\text{ionization}}}{dx}(E_c) \]

\[ E_c = \frac{610}{Z + 1.24} \text{ MeV} \quad \text{for liquids and solids} \quad E_c = \frac{710}{Z + 0.92} \text{ MeV} \quad \text{for gas} \]

A better formula that can be used for liquids and solids is:

\[ E_c = 2.66 \left( \frac{Z}{A(g)} X_0(g/cm^2) \right)^{1.11} \text{ MeV} \]

In literature, both the radiation length and the critical energy are tabulated for electrons. For other particles they would scale according to the square of their masses with respect to the electron mass.
<table>
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<tr>
<th>medium</th>
<th>Z</th>
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<td>36</td>
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Electron-positron pair production

At very high energy, direct electron-positron pair production may play an important role.

\[-\frac{dE}{dx}^{\text{pair}} = b_{\text{pair}}(Z, A, E) \ E\]

Energy loss by photo-nuclear interaction:

example: electro-dissociation of deuteron \[e^- + d \rightarrow n + p + e^-\]

\[-\frac{dE}{dx}^{\text{nucl.}} = b_{\text{nucl.}}(Z, A, E) \ E\]

Total stopping power:

\[\frac{dE}{dx}^{\text{tot}} = \frac{dE}{dx}^{\text{ionization}} + \frac{dE}{dx}^{\text{rad}} + \frac{dE}{dx}^{\text{pair}} + \frac{dE}{dx}^{\text{nucl.}}\]

which could also be written as:

\[-\frac{dE}{dx}^{\text{tot}} = a(Z, A, E) + b(Z, A, E) \ E\]

where \(a(Z, A, E)\) is the ionization term and \(b(Z, A, E)\) the sum of the Bremsstrahlung, the pair production and the photo-nuclear terms.
Multiple scattering through small angles

A charged particle traversing a medium is deflected many times by small-angles essentially due to Coulomb scattering in the electromagnetic field of nuclei.

This effect is well reproduced by the Molière theory. On both $x$ and $y$, the angular deflection $\theta_{\text{proj}}$ of a particle almost follows a Gaussian which is centered around 0:

$$\left(\theta_{\text{space}}\right)^2 = \left(\theta_{\text{proj}}^x\right)^2 + \left(\theta_{\text{proj}}^y\right)^2$$

$$\theta_{\text{rms}}^{\text{proj}} = \frac{1}{\sqrt{2}} \theta_{\text{rms}}^{\text{space}}$$

$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta_p} z \frac{x}{X_0} \left(1 + 0.038 \ln \left(\frac{x}{X_0}\right)\right)$$

$P(\theta_{\text{proj}}) d\theta_{\text{proj}} = \frac{1}{\sqrt{2\pi \theta_0}} e^{-\frac{1}{2} \left(\frac{\theta_{\text{proj}}}{\theta_0}\right)^2} d\theta_{\text{proj}}$

$$P(\theta_{\text{space}}) d\Omega = \frac{1}{2\pi \theta_0^2} e^{-\frac{1}{2} \left(\frac{\theta_{\text{space}}}{\theta_0}\right)^2} d\Omega$$

Large deflection angles are more probable than what the Gaussian predicts. This results from Rutherford scattering of heavy particles off nuclei. Particles emerging from the medium are also laterally shifted:

$$X_{\text{rms}} = \frac{1}{\sqrt{3}} \theta_0 x$$

$p$ particle momentum

$x$ medium thickness

$z$ charge state of incoming particle

$X_0$ radiation length
Particle Range in matter

If the medium is thick enough, a particle will progressively decelerate while increasing its stopping power ($\beta^{-5/3}$) until it reaches a maximum (called the Bragg peak).

This stopping power profile is used in protontherapy for treating cancerous tumors.
Wanjie Proton Therapy Center in Zibo (Shandong Province)

A cyclotron delivering 230 MeV protons to treat cancerous tumors

30 centers of that sort around the world.
As the result of the stochastic behavior of particles interacting in matter, it is not possible to enunciate a perfect definition of a reproducible particle range.

Continuous slowing down approximation range:

\[
R(T_0) = \int_0^{T_0} \frac{dT}{dE} (T)
\]

where \( T_0 \) is the incident kinetic energy of the particle.

In practice, the integration is carried out down to 10 eV.

We also use the mean range \(<R>\) which corresponds to the distance at which half of the initial particles have been stopped.

If \( T > 1 \text{ MeV} \), \( R \approx <R> \).
If only ionization and excitation are used to calculate $R(T)$ (valid for heavy particles with energies $< 1$ GeV), the following relationship can be used:

\[
R_b(M_b, z_b, T_b) = \frac{M_b}{M_a} \frac{z_a^2}{z_b^2} R_a(M_a, z_a, T_b) \frac{M_a}{M_b}
\]

Particle $a$ with $z_a$, $M_a$

Particle $b$ with $z_b$, $M_b$ and kinetic energy $T_b$

One may also write for a particle of mass $M$ and charge state $z$ carrying a kinetic energy $T_0$:

\[
R(M, z, T_0) = \frac{M}{z^2} h\left(\frac{T_0}{M}\right)
\]

where $h$ is a universal function of the medium (Z, A and I fixed).
universal h function = R 

\[ R/M \text{ (g cm}^{-2}\text{ GeV}^{-1}) \]

\[ \beta_\gamma = p/Mc \]

- Muon momentum (GeV/c)
- Pion momentum (GeV/c)
- Proton momentum (GeV/c)
Alpha particle range in Si

http://physics.nist.gov/PhysRefData/Star/Text
Interactions of photons with matter:

Photons are indirectly detected: they first create electrons (and in some cases positrons when interacting by pair production) which subsequently interact with matter.

In their interactions with matter, photons may be absorbed (photoelectric effect or $e^+e^-$ pair creation) or scattered (Compton scattering) through large deflection angles.

As photon trajectories are particularly chaotic, it is impossible to define a mean range. We then proceed with an attenuation law:

\[
I(x) = I_0 e^{-\mu x}
\]

where:

- $I_0$ is the initial photon beam flux
- $I(x)$ is the photon beam flux exiting the layer of thickness $x$
- $\mu$ is the mass attenuation coefficient (cm$^2$/g)
- $x$ surface mass density of the layer (g/cm$^2$)
- $\sigma_{tot}$ is the total photon cross-section per atom

$\sigma_{tot}$

\[
\mu = \frac{N_A \sigma_{tot}}{A}
\]
Photon mean free paths in different media:

\[ \lambda = \frac{1}{\mu} \]
Photoelectric effect:

![Photoelectric effect diagram](image)

Because of their proximity to the nucleus, electrons of the deepest shells (K,L,M...) are favored. Following the emission of a photoelectron, the atom reorganises leading to the production of X rays or Auger electrons.

\[ E_e = E_K - 2E_L \]
\[ E_x = E_K - E_L \text{ if } E_x > E_L \]

Production scheme of an Auger electron
Photoelectron energy :

\[ E_e = E_Y - E_{\text{binding}} \]

where :

\[ E_{\text{binding}} = E_K \text{ or } E_L \text{ or } E_M \ldots \]

At low energy \((E_Y/m_e \ll 1)\), but if \(E \gg E_K\) :

\[ \sigma^K_{\text{photo}} = \left(\frac{32}{\epsilon^7}\right)^{\frac{1}{2}} \alpha^4 Z^5 \sigma^e_{\text{Th}} \text{ (per atom)} \]

\[ \alpha = 1/137 \text{ Fine structure constant} \]

Thomson scattering cross section

\[ \epsilon = E_Y/m_e \]

\[ \sigma^e_{\text{Th}} = \frac{8}{3} \pi r_e^2 \text{ with } r_e = \frac{\alpha}{m_e} \text{ classical electron radius} \]

photoelectric cross section strongly increases as \(Z^5\) and decreases as \(1/E_Y^{3.5}\)

At high energy \((E_Y/m_e \gg 1)\) :

\[ \sigma^K_{\text{photo}} = 4 \pi r_e^2 Z^5 \frac{\alpha^4}{\epsilon} \]

At low energy \((E_Y < 100 \text{ keV})\), the photoelectric effect dominates the total photon cross section
Compton scattering: Elastic scattering of a photon off an atomic electron considered as being free (if $E_\gamma > E_{binding}$)

$$\frac{E'_\gamma}{E_\gamma} = \frac{1}{1 + \epsilon (1 - \cos \theta'_\gamma)} \quad \text{with} \quad \epsilon = \frac{E_\gamma}{m_e}$$

$$\cot \theta_e = (1 + \epsilon) \tan \left(\frac{\theta'_\gamma}{2}\right)$$

$$\frac{E'_{\gamma min}}{E_\gamma} = \frac{1}{1 + 2 \epsilon}$$

(exercise, show these equations)

Klein-Nishina cross section:

$$\sigma^e_c = 2 \pi r_e^2 \left( \frac{1 + \epsilon}{\epsilon^2} \right) \left\{ \frac{2(1 + \epsilon)}{1 + 2 \epsilon} - \frac{1}{\epsilon} \ln(1 + 2 \epsilon) \right\} + \frac{1}{2} \ln(1 + 2 \epsilon) - \frac{1 + 3 \epsilon}{(1 + 2 \epsilon)^2} \right) \quad \text{(per electron)}$$

$$\frac{d \sigma^e_c}{d\Omega} = \frac{r_e^2}{2} \frac{1 + \cos^2 \theta'_\gamma}{\left(1 + \epsilon (1 - \cos \theta'_\gamma)\right)^2} \left(1 + \frac{\epsilon^2 (1 - \cos \theta'_\gamma)^2}{(1 + \cos^2 \theta'_\gamma)(1 + \epsilon (1 - \cos \theta'_\gamma))}\right) \quad \text{(per electron)}$$

Cross-section per atom: $$\sigma^e_{c \text{atom}} = Z \sigma^e_c$$
Pair production is the leading effect at high energy.

\[ e^+e^- \text{ pair production :} \]

\[ E_\gamma \geq 2m_e \]

\[ E_\gamma \geq 4m_e \]

\[
1 \ll \epsilon < \frac{1}{\alpha Z^{1/3}} \quad \Rightarrow \quad \sigma_{\text{atom}} = 4 \alpha r_e^2 Z^2 \left( \frac{7}{9} \ln(2\epsilon) - \frac{109}{54} \right)
\]

\[
\epsilon \gg \frac{1}{\alpha Z^{1/3}} \quad \Rightarrow \quad \sigma_{\text{pair}} = 4 \alpha r_e^2 Z^2 \left( \frac{7}{9} \ln \left( \frac{183}{Z^{1/3}} \right) - \frac{1}{54} \right)
\]

Where \( X_0 (\text{g/cm}^2) \) is the radiation length.

In this high energy regime:

\[
\sigma_{\text{pair}} \approx \frac{7}{9} \frac{A}{N_A} \frac{1}{X_0}
\]

Pair production is the leading effect at high energy.
Total absorption cross section:

In Compton scattering, photons are not totally absorbed

Let us define a Compton energy scattering cross section:

$$\sigma_{\text{cs}}^{\text{atom.}} = \frac{E'}{E_y} \sigma_{\text{c}}^{\text{atom.}}$$

And a Compton absorption cross section:

$$\sigma_{\text{ca}}^{\text{atom.}} = \sigma_{\text{c}}^{\text{atom.}} - \sigma_{\text{cs}}^{\text{atom.}}$$

Massic coefficients in cm$^2$/g:

$$\mu_{\text{cs}} = \frac{N}{A} \sigma_{\text{cs}}^{\text{atom.}}; \quad \mu_{\text{ca}} = \frac{N}{A} \sigma_{\text{ca}}^{\text{atom.}}; \quad \mu_{\text{c}} = \mu_{\text{cs}} + \mu_{\text{ca}}$$

$$\mu_{\text{p}} = \frac{N}{A} \sigma_{\text{pair}}^{\text{atom.}}; \quad \mu_{\text{ph}} = \frac{N}{A} \sigma_{\text{photo}}$$

$$\mu_{\text{a}} = \mu_{\text{ph}} + \mu_{\text{p}} + \mu_{\text{ca}}$$ total massic absorption coefficient

$$\mu = \mu_{\text{ph}} + \mu_{\text{p}} + \mu_{\text{c}}$$ total massic attenuation coefficient
Cerenkov light emission

When a particle moves faster than the phase velocity of light in the medium, an asymmetric polarization of the medium builds up along the longitudinal axis in the vicinity of the particle, that leads to the production of light, which in turn creates a coherent wave front as shown on the picture. It may be understood as the photonic shock wave of a particle that moves faster than $c/n$ where $n$ is the refractive index of the medium.

$\cos \theta = \frac{c/nt}{\beta ct} = \frac{1}{\beta n}$

$\beta > \frac{1}{n}$  Cerenkov emission has a velocity threshold
Cerenkov light emission

The number of photons emitted per unit path length and unit wave length reads:

\[
\frac{dN}{dx \, d\lambda} = 2\pi \alpha \frac{1}{\lambda^2} \left(1 - \frac{1}{\beta^2 n^2}\right)
\]

It is strongly peaked at short wave lengths.

The total number of photons per unit path length is then:

\[
\frac{dN}{dx} = 2\pi \alpha \int_{\beta n > 1} \left(1 - \frac{1}{\beta^2 n^2}\right) \frac{d\lambda}{\lambda^2}
\]

If the variation of \( n \) is small over the wavelength region detected then:

\[
\frac{dN}{dx} = 2\pi \alpha \sin^2 \theta \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)
\]

E.g. in a wavelength interval 350-500 nm (photomultiplier tube),

\[
\frac{dN}{dx} = 390 \sin^2 \theta \text{ photons/cm}
\]

d\(E/dx\) due to Cerenkov light is small compared to ionization loss and much weaker than scintillating output. It can be neglected in energy loss balance of a particle.
Transition radiation

This radiation is emitted mostly in the X-ray domain when a particle crosses a boundary between media of different dielectric properties.

The radiation is emitted in a cone at an angle: \[ \cos \theta = \frac{1}{\gamma} \]

The probability of radiation per transition surface is low ~ 1/2 \( \alpha \) (fine structure constant).

The energy of radiated photons increases as a function of \( \gamma \).

![Diagram of transition radiation detector (TRD)]

Fig. 6.21. Typical dependence of the average energy of transition radiation photons on the electron momentum for standard radiator arrangements [450].
Deposited energy:

Generally speaking, the energy loss is never equal to the deposited energy as the radiated photons or the secondary particles may escape the medium.

Deposited energy is what generates the signal in a particle detector.

Deposited energy is subjected to large stochastic fluctuations. Remember: Stopping power is the mean energy loss.
If the medium is thin and the number of interactions is small, the deposited energy distribution is asymmetric: it is sometimes called a Landau distribution.
If the medium is thick or the number of interactions is large, the deposited energy distribution tends to a Gaussian.

There are no simple and exact analytical formulae to compute deposited energy.

Nowadays, to estimate the energy deposited in a detector or more generally in a medium we use a Monte-Carlo program which simulates the propagation of the particle through matter: e.g. Geant4
creation of electron-ion pairs:

When the measured signal is a current or a charge liberated through ionizing interactions, it is useful to compute the mean number of created electron-ion pairs:

\[ n_{\text{e-ion}} = \frac{\Delta E_{\text{deposited}}}{W} \]

where: \( W \) is the required mean energy to produce an e-ion pair

\( W > I \) (mean excitation and ionization potential)

In most gases, \( W \sim 30 \text{ eV} \).

In semiconductor detectors (Ge, Si), \( W \) is much lower: e.g. \( W=3.6 \text{ eV} \) for Si and \( W=2.85 \text{ eV} \) for Ge.
Hadron collision and interaction lengths:
When dealing with very high energy hadrons, it is somewhat useful to express the total cross-section as:

\[ \sigma_T = \sigma_{\text{elastic}} + \sigma_{\text{inelastic}} \]

Only the inelastic part of the total cross-section is susceptible to induce a hadron shower. It is then useful to introduce two mean lengths:

\[ \lambda_T = \frac{A}{N_A \sigma_T} \text{g cm}^{-2} \] called the nuclear collision length

\[ \lambda_I = \frac{A}{N_A \sigma_{\text{inelastic}}} \text{g cm}^{-2} \] called the interaction length

95% containment of a hadronic shower can be obtained for a thickness of:

\[ L_{95\%} \text{ (in units of } \lambda_I \text{)} \approx 1 + 1.35 \ln(E \text{ (GeV)}) \]

Then approximately 10 interaction lengths are needed to contain a 1 TeV hadronic shower.

In high A materials : \( \lambda_I > X_0 \) which explains why hadron calorimeters are deeper than electromagnetic calorimeters.
Table 6.1. Abridged from pdg.lbl.gov/AtomicNuclearProperties by D. E. Groom (2007). Quantities in parentheses are for NTP (20°C and 1 atm), and square brackets indicate quantities evaluated at STP. Boiling points are at 1 atm. Refractive indices $n$ are evaluated at the sodium D line blend (589.2 nm); values $\gg 1$ in brackets are for $(n - 1) \times 10^6$ (gases).

<p>| Material   | $Z$ | $A$      | $\langle Z/A \rangle$ | Nucl.coll. length $\lambda_I$ | Nucl.inter. length $\lambda_I$ | Rad.len. $X_0$ | $dE/dx|_{\text{min}}$ | Density ${ \text{g cm}^{-3} }$ | Melting point (K) | Boiling point (K) | Refract. index (@ Na D) |
|------------|----|----------|------------------------|-------------------------------|-------------------------------|----------------|----------------------|--------------------------|----------------------|----------------------|------------------------|
| H$_2$      | 1  | 1.00794(7) | 0.99212                | 42.8                          | 52.0                          | 63.04          | (4.103)               | 0.071(0.084)           | 13.81                | 20.28                | 1.11[132.]            |
| D$_2$      | 1  | 2.04[1017750(8) | 0.49650                | 51.3                          | 71.8                          | 125.97         | (2.053)               | 0.169(0.168)           | 18.7                 | 23.65                | 1.11[138.]            |
| He         | 2  | 4.002602(2) | 0.49967                | 51.8                          | 71.0                          | 94.32          | (1.937)               | 0.125(0.166)           | 4.220                | 4.220                | 1.02[35.0]            |
| Li         | 3  | 6.941(2)    | 0.43221                | 52.2                          | 71.3                          | 82.78          | 1.639                | 0.534                   | 453.6                | 1615                 |                        |
| Be         | 4  | 9.012182(3) | 0.44384                | 55.3                          | 77.8                          | 65.19          | 1.595                | 1.848                   | 1560.0               | 2744.0               |                        |
| C diamond  | 6  | 12.0107(8)  | 0.49555                | 59.2                          | 85.8                          | 42.70          | 1.725                | 3.520                   |                      |                      | 2.42                  |
| C graphite | 6  | 12.0107(8)  | 0.49555                | 59.2                          | 85.8                          | 42.70          | 1.725                | 3.520                   |                      |                      | 2.42                  |
| N$_2$      | 7  | 11.0067(2)  | 0.49976                | 61.1                          | 89.7                          | 37.99          | (1.825)               | 0.807(1.165)           | 63.15                | 77.29                | 1.20[298.]            |
| O$_2$      | 8  | 15.9994(3)  | 0.50002                | 61.3                          | 90.2                          | 34.24          | (1.801)               | 1.141(1.332)           | 54.36                | 90.0                   | 1.22[271.]           |
| P$_2$      | 9  | 18.9984(32) | 0.47372                | 65.0                          | 97.4                          | 32.93          | (1.676)               | 1.507(1.580)           | 53.53                | 85.03                | [195.]                |
| Ne         | 10 | 20.1797(6)  | 0.49555                | 65.7                          | 99.0                          | 28.93          | (1.724)               | 1.204(0.839)           | 24.56                | 27.07                | 1.09[67.1]            |
| Al         | 13 | 26.9815386(8) | 0.48181              | 69.7                          | 107.2                         | 24.01          | 1.615                | 2.699                   | 933.5                | 2792.0               |                      |
| Si         | 14 | 28.0855(3)  | 0.49848                | 70.2                          | 108.4                         | 21.82          | 1.664                | 2.329                   | 1687.0               | 3538.0               | 3.95                  |
| Cl$_2$     | 17 | 35.453(2)   | 0.47951                | 73.8                          | 115.7                         | 19.28          | (1.630)               | 1.574(2.980)           | 171.6                | 239.1                | [773.]                |
| Ar         | 18 | 39.948(1)   | 0.45059                | 75.7                          | 119.7                         | 19.55          | (1.519)               | 1.396(1.662)           | 83.81                | 87.26                | 1.20[281.]            |
| Ti         | 22 | 47.867(1)   | 0.45961                | 78.8                          | 126.2                         | 16.16          | 1.477                | 4.540                   | 1941.0               | 3560.0               |                      |
| Fe         | 26 | 55.845(2)   | 0.46557                | 81.7                          | 132.1                         | 13.84          | 1.451                | 7.874                   | 1811.0               | 3134.0               |                      |
| Cu         | 29 | 63.546(3)   | 0.45636                | 84.2                          | 137.3                         | 12.86          | 1.403                | 8.960                   | 1358.0               | 2835.0               |                      |
| Ge         | 32 | 72.64(1)    | 0.44053                | 86.9                          | 143.0                         | 12.25          | 1.370                | 5.323                   | 1211.0               | 3106.0               |                      |
| Sn         | 50 | 118.710(7)  | 0.42119                | 98.2                          | 166.7                         | 8.82           | 1.263                | 7.310                   | 505.1                | 2875.0               |                      |
| Xe         | 54 | 131.393(6)  | 0.41129                | 100.8                         | 172.1                         | 8.48           | (1.255)              | 2.953(5.483)           | 161.4                | 165.1                | 1.39[701.]           |
| W          | 74 | 183.84(1)   | 0.40252                | 110.4                         | 191.9                         | 6.76           | 1.145                | 19.300                  | 3695.0               | 5828.0               |                      |
| Pt         | 78 | 195.084(9)  | 0.39983                | 112.2                         | 195.7                         | 6.54           | 1.128                | 21.450                  | 2042.0               | 4098.0               |                      |
| Au         | 79 | 196.960509(4) | 0.40108            | 112.5                         | 196.3                         | 6.46           | 1.134                | 19.320                  | 1337.0               | 3129.0               |                      |
| Pb         | 82 | 207.92(1)   | 0.39575                | 114.1                         | 199.6                         | 6.37           | 1.122                | 11.350                  | 600.6                | 2022.0               |                      |
| U          | 92 | [238.02891(3)] | 0.38651            | 118.6                         | 209.0                         | 6.00           | 1.081                | 18.950                  | 1408.0               | 4404.0               |                      |</p>
<table>
<thead>
<tr>
<th>Material</th>
<th>Z</th>
<th>A</th>
<th>$\langle Z/A \rangle$</th>
<th>Nucl. coll. length $\lambda_f$ (g cm$^{-2}$)</th>
<th>Nucl. inter. length $\lambda_i$ (g cm$^{-2}$)</th>
<th>Rad. len. $X_0$ (g cm$^{-2}$)</th>
<th>$dE/dx_{\text{min}}$ (MeV g$^{-1}$ cm$^2$)</th>
<th>Density (g cm$^{-3}$)</th>
<th>Melting point (K)</th>
<th>Boiling point (K)</th>
<th>Refractive index (@ Na D)</th>
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<tbody>
<tr>
<td>Air (dry, 1 atm)</td>
<td>0.49919</td>
<td>61.3</td>
<td>90.1</td>
<td>36.62</td>
<td>1.815</td>
<td>1.205</td>
<td></td>
<td>78.80</td>
<td></td>
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<tr>
<td>Shielding concrete</td>
<td>0.50274</td>
<td>65.1</td>
<td>97.5</td>
<td>26.57</td>
<td>1.711</td>
<td>2.300</td>
<td></td>
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<tr>
<td>Borosilicate glass (Pyrex)</td>
<td>0.49707</td>
<td>64.6</td>
<td>96.5</td>
<td>28.17</td>
<td>1.696</td>
<td>2.230</td>
<td></td>
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<tr>
<td>Lead glass</td>
<td>0.42101</td>
<td>95.9</td>
<td>158.0</td>
<td>7.87</td>
<td>1.255</td>
<td>6.220</td>
<td></td>
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<tr>
<td>Standard rock</td>
<td>0.50000</td>
<td>66.8</td>
<td>101.3</td>
<td>26.54</td>
<td>1.688</td>
<td>2.650</td>
<td></td>
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<tr>
<td>Methane (CH$_4$)</td>
<td>0.62334</td>
<td>54.0</td>
<td>73.8</td>
<td>46.47</td>
<td>2.417</td>
<td>0.667</td>
<td>90.68</td>
<td>111.7</td>
<td>[444.]</td>
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<tr>
<td>Ethane (C$_2$H$_6$)</td>
<td>0.59851</td>
<td>55.0</td>
<td>75.9</td>
<td>45.66</td>
<td>2.304</td>
<td>1.203</td>
<td>90.36</td>
<td>184.5</td>
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<tr>
<td>Butane (C$<em>4$H$</em>{10}$)</td>
<td>0.59497</td>
<td>55.5</td>
<td>77.1</td>
<td>45.23</td>
<td>2.278</td>
<td>2.489</td>
<td>134.9</td>
<td>272.6</td>
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<tr>
<td>Octane (C$<em>8$H$</em>{18}$)</td>
<td>0.57778</td>
<td>55.8</td>
<td>77.8</td>
<td>45.00</td>
<td>2.123</td>
<td>0.703</td>
<td>214.4</td>
<td>398.8</td>
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<tr>
<td>Paraffin (CH$_3$(C$_2$H$<em>4$)$</em>{n\approx 23}$CH$_3$)</td>
<td>0.57275</td>
<td>56.0</td>
<td>78.3</td>
<td>44.85</td>
<td>2.088</td>
<td>0.930</td>
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<tr>
<td>Nylon (type 6, 6/6)</td>
<td>0.54790</td>
<td>57.5</td>
<td>81.6</td>
<td>41.92</td>
<td>1.973</td>
<td>1.18</td>
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<td>Polycarbonate (Lexan)</td>
<td>0.52697</td>
<td>58.3</td>
<td>83.6</td>
<td>41.50</td>
<td>1.886</td>
<td>1.20</td>
<td></td>
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<tr>
<td>Polyethylene (CH$_2$CH$_2$n)</td>
<td>0.57034</td>
<td>56.1</td>
<td>78.5</td>
<td>44.77</td>
<td>2.079</td>
<td>0.89</td>
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<tr>
<td>Polyethylene terephthalate (Mylar)</td>
<td>0.52037</td>
<td>58.9</td>
<td>84.9</td>
<td>39.95</td>
<td>1.848</td>
<td>1.40</td>
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<tr>
<td>Poly(methylmethacrylate) (acrylic)</td>
<td>0.53937</td>
<td>58.1</td>
<td>82.8</td>
<td>40.55</td>
<td>1.929</td>
<td>1.19</td>
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<td>Polypropylene</td>
<td>0.55998</td>
<td>56.1</td>
<td>78.5</td>
<td>44.77</td>
<td>2.041</td>
<td>0.90</td>
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<td>Polystyrene ([C$_6$H$_5$CHCH$_2$]n)</td>
<td>0.53768</td>
<td>57.5</td>
<td>81.7</td>
<td>43.79</td>
<td>1.936</td>
<td>1.06</td>
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<tr>
<td>Polytetrafluoroethylene (Teflon)</td>
<td>0.47992</td>
<td>63.5</td>
<td>94.4</td>
<td>34.84</td>
<td>1.671</td>
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<td>Polyvinyltoluene</td>
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<td>57.3</td>
<td>81.3</td>
<td>43.90</td>
<td>1.956</td>
<td>1.03</td>
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<tr>
<td>Aluminum oxide (sapphire)</td>
<td>0.49038</td>
<td>65.5</td>
<td>98.4</td>
<td>27.94</td>
<td>1.647</td>
<td>3.970</td>
<td>2327.</td>
<td>3273.</td>
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<td>Barium fluoride (BaF$_2$)</td>
<td>0.42207</td>
<td>90.8</td>
<td>149.0</td>
<td>9.91</td>
<td>1.303</td>
<td>4.893</td>
<td>1641.</td>
<td>2533.</td>
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<tr>
<td>Carbon dioxide gas (CO$_2$)</td>
<td>0.49989</td>
<td>60.7</td>
<td>88.9</td>
<td>36.20</td>
<td>1.819</td>
<td>(1.842)</td>
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<td>[449.]</td>
</tr>
<tr>
<td>Solid carbon dioxide (dry ice)</td>
<td>0.49989</td>
<td>60.7</td>
<td>88.9</td>
<td>36.20</td>
<td>1.787</td>
<td>1.563</td>
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<td>Cerium iodide (CeI)</td>
<td>0.41559</td>
<td>100.6</td>
<td>171.5</td>
<td>8.39</td>
<td>1.124</td>
<td>4.510</td>
<td>894.2</td>
<td>1553.</td>
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<td>Lithium fluoride (LiF)</td>
<td>0.46262</td>
<td>61.0</td>
<td>88.7</td>
<td>39.26</td>
<td>1.614</td>
<td>2.635</td>
<td>1121.</td>
<td>1946.</td>
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<tr>
<td>Lithium hydride (LiH)</td>
<td>0.50321</td>
<td>50.8</td>
<td>81.7</td>
<td>79.62</td>
<td>1.897</td>
<td>0.820</td>
<td>965.</td>
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<tr>
<td>Lead tungstate (PbWO$_4$)</td>
<td>0.41315</td>
<td>100.6</td>
<td>168.3</td>
<td>7.39</td>
<td>1.229</td>
<td>8.300</td>
<td>1403.</td>
<td>2.20</td>
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<tr>
<td>Silicon dioxide (SiO$_2$, fused quartz)</td>
<td>0.49930</td>
<td>65.2</td>
<td>97.8</td>
<td>27.05</td>
<td>1.699</td>
<td>2.200</td>
<td>1986.</td>
<td>3223.</td>
<td></td>
<td></td>
<td>1.46</td>
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<tr>
<td>Sodium chloride (NaCl)</td>
<td>0.55569</td>
<td>71.2</td>
<td>110.1</td>
<td>21.91</td>
<td>1.847</td>
<td>2.170</td>
<td>1075.</td>
<td>1738.</td>
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<tr>
<td>Sodium iodide (NaI)</td>
<td>0.42897</td>
<td>93.1</td>
<td>154.6</td>
<td>9.49</td>
<td>1.305</td>
<td>3.687</td>
<td>933.2</td>
<td>1577.</td>
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<tr>
<td>Water (H$_2$O)</td>
<td>0.55509</td>
<td>58.5</td>
<td>83.3</td>
<td>36.08</td>
<td>1.992</td>
<td>1.000(0.756)</td>
<td>273.1</td>
<td>373.1</td>
<td></td>
<td></td>
<td>1.33</td>
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<tr>
<td>Silica aerogel</td>
<td>0.50093</td>
<td>65.0</td>
<td>97.3</td>
<td>27.25</td>
<td>1.740</td>
<td>0.200</td>
<td>(0.03 H$_2$O, 0.97 SiO$_2$)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
To learn more:


- Introduction to experimental particle physics, Richard Fernow, Cambridge University Press

- Particle penetration and Radiation effects, P. Sigmund, Springer

- Nuclei and particles, Émilio Segré, W.A. Benjamin

- Stopping powers and ranges for protons and alpha particles (ICRU Report 49,1993)
  Library of congress US-Cataloging-in-Publication Data

- Particle detectors, Claus Grupen, Cambridge monographs on particle physics

- Detectors for Particle radiation, Konrad Kleinknecht, Cambridge University Press

- Radiation detection and measurement, G.F. Knoll, J. Wiley & Sons

- Single Particle Detection and Measurement, R. Gilmore, Taylor & Francis

- Radiation detectors, C.F.G. Delaney and E.C. Finch, Oxford Science Publications

- High-Energy Particles, Bruno Rossi, Prentice-Hall
Interactions of neutrons with matter:

Neutrons are neutral particles which only interact with nuclei.

Neutrons can be absorbed or scattered by nuclei.

With respect to their energy, neutrons can be categorized as follow:

- thermal neutrons: in thermal equilibrium with matter, $<E_n> = 3/2 kT$
  
  $<E_n> = 0.038 \text{ eV for } T= 300 \text{ K}$  \quad $k$ being the Boltzmann constant
  
  $E_0$ (most probable energy) = $kT = 0.025 \text{ eV for } T = 300 \text{ K}$

- ultra-cold neutrons $E < 2 \times 10^{-7} \text{ eV}$
- very cold neutrons $2 \times 10^{-7} \text{ eV} < E < 50 \times 10^{-6} \text{ eV}$

- cold neutrons $50 \times 10^{-6} \text{ eV} < E < E_0 = 0.025 \text{ eV}$

- slow neutrons $0.025 \text{ eV} < E < 0.5 \text{ eV}$

- epithermal neutrons: $0.5 \text{ eV} < E < 1 \text{ keV}$

- intermediate energy neutrons: $1 \text{ keV} < E < 0.5 \text{ MeV}$

- fast neutrons: $0.5 \text{ MeV} < E < 50 \text{ MeV}$

- relativistic neutrons: $50 \text{ MeV} < E$
All neutrons may undergo elastic scattering and radiative capture (emission of photons).

Elastic scattering:

It is mostly used to slow down neutrons, e.g., in a nuclear reactor.

¹H, ²H and ¹²C are the best and preferred moderator nuclei.

Up to 10 MeV and for some target nuclei (like H), the energy spectrum of elastically scattered neutrons is approximately flat:

The probability of a neutron of mass mₙ and incident energy E₀ to be found - after elastic scattering off a nucleus of mass m - in an energy interval dE is:

\[ P(E₀)dE = \frac{dE}{(1-\alpha)E₀} \]

where \[ \alpha = \frac{(A-1)^2}{(A+1)^2} \leq 1 \]

The scattered neutron energy follows: \[ \alpha E₀ \leq E \leq E₀ \]

A is the mass number of the target nucleus.
Elastic scattering off protons is also used to detect fast neutrons: detection of recoiling protons.

Energy spectrum of elastically-scattered neutrons (if the incident neutron energy is less than 10 MeV and the scattering process is approximately isotropic in the center-of-mass-coordinate system.)
$(n, \gamma)$ radiative capture: As neutrons are neutral particles, radiative capture may happen at very low energies (no Coulomb interaction effects).

Example: Production of $^{239}$Pu in a reactor

$$n + ^{238}_{\text{92}} \text{U} \rightarrow ^{239}_{\text{92}} \text{U}^* \rightarrow ^{239}_{\text{92}} \text{U} + \gamma$$

$$^{239}_{\text{92}} \text{U} \rightarrow ^{239}_{\text{93}} \text{Np} + e^- + \bar{\nu}_e \rightarrow ^{239}_{\text{94}} \text{Pu} + e^- + \bar{\nu}_e$$

$(n, \gamma)$ radiative capture is used to produce artificial radioisotopes in nuclear reactors.

It may also be used to detect neutrons and measure neutron fluences (time integrated fluxes).

In general, the capture cross section increases as the inverse of the neutron velocity: the slower the neutrons, the bigger the cross section (Gamow Law). This general behavior may be affected by capture resonances.
\((n, \gamma n')\) neutron inelastic scattering: neutron energy less than a few tens of MeV

\[
n + ^A_Z X \rightarrow ^{A+1}_Z X^* \rightarrow ^A_Z X^* + n' \rightarrow ^A_Z X + \gamma + n'
\]

gamma decay of nucleus

decay into neutron + excited nucleus

neutron capture

formation of compound nucleus
(n, p) and (n, α) reactions: A few of these reactions are exoenergetic (produce energy).

The neutron is first captured to produce a compound nucleus which then decays into several products.

\[
\begin{align*}
  n + ^3\text{He} & \rightarrow p + ^3\text{H} + 760 \text{ keV} & \sigma &= 5400 \text{ barns} \\
  n + ^{14}\text{N} & \rightarrow p + ^{14}\text{C} + 630 \text{ keV} & \sigma &= 1.75 \text{ barns} \\
  n + ^{35}\text{Cl} & \rightarrow p + ^{35}\text{S} + 620 \text{ keV} & \sigma &= 0.8 \text{ barn} \\
  n + ^6\text{Li} & \rightarrow \alpha + ^3\text{H} + 4782 \text{ keV} & \sigma &= 940 \text{ barns} \\
  n + ^{10}\text{B} & \rightarrow \alpha + ^7\text{Li} + 2791 \text{ keV} & \sigma &= 4000 \text{ barns}
\end{align*}
\]

These reactions (in particular on $^3\text{He}$, $^6\text{Li}$, $^{10}\text{B}$) are used to detect low energy neutrons.
In general, the capture cross section increases as the inverse of the neutron velocity: the slower the neutrons, the bigger the cross section (Gamow Law). This general behavior may be affected by capture resonances.

Cross section of $^{10}\text{B}(n,\alpha)^7\text{Li}$
For some odd neutron number nuclei (odd N), fission may occur at all neutron energies and in particular at very low energy. This is the case of: $^{233}\text{U}$, $^{235}\text{U}$, $^{239}\text{Pu}$, $^{241}\text{Pu}$

For other nuclei, fission only takes place above a neutron energy threshold, e.g. $^{238}\text{U}$: 0.9 MeV, $^{232}\text{Th}$: 1.3 MeV

Apart from its well-known application for massive energy production, fission may be used to detect neutrons.
Neutron cross sections:

Macroscopic cross section is defined as:

\[ \Sigma = n \sigma \]

number of nuclei per unit volume

\[ \Sigma_{tot} = \Sigma_{\text{elastic scattering}} + \Sigma_{\text{absorption}} + \Sigma_{\text{inelastic scattering}} \]

As in the case of gammas, we can use an attenuation law:

\[ I(x) = I_0 e^{-\Sigma_{tot}x} \]

initial neutron flux
distance traversed by neutrons

\[ \lambda = \frac{1}{\Sigma_{tot}} \] being the neutron mean free path in the considered medium