Calorimetry: Exercises

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Useful Formulas (EM showers) [1]

Radiation Length:

$$X_0 \approx \frac{180A}{Z^2}$$ (g.cm^{-2})

Radiation Length for composite material:

$$\frac{1}{X_0} = \sum \frac{w_j}{X_j}$$

w_j: fraction of material j
X_j: radiation length of material j (in g.cm-2)

Moliere Radius:

$$R_M = \frac{21MeV}{E_C} X_0$$

Moliere Radius for composite material:

$$\frac{1}{R_M} = \sum \frac{w_j}{R_{M\,j}}$$

w_j: fraction of material j
R_{Mj}: Moliere Radius of material j (in g.cm-2)

Energy Resolution:

$$\frac{\sigma}{E} = \frac{S}{\sqrt{E}} \oplus \frac{N}{E} \oplus C$$

$\oplus$: quadratic sum
S: Stochastic
N: noise
C: constant

Useful Formulas (EM showers) [2]

$$E_C(solid) = \frac{610\,\mathrm{MeV}}{\mathrm{Z}+1.24}$$

E_C: critical energy

$$E_C(liquid) = \frac{710\,\mathrm{MeV}}{\mathrm{Z}+0.92}$$

Shower maximum

$$t_{\max} = \frac{\ln E_0 / E_C}{\ln 2}$$

$$N(t_{\max}) \approx \frac{E_0}{E_C}$$

Longitudinal containment:

$$t_{95\%} = t_{\max} + 0.08Z + 9.6$$

$$\frac{\sigma_E}{E} = 3.2\% \sqrt{\frac{E_c\,[\mathrm{MeV}] \cdot t_{\mathrm{abs}}}{F \cdot E\,[\mathrm{GeV}]}}$$

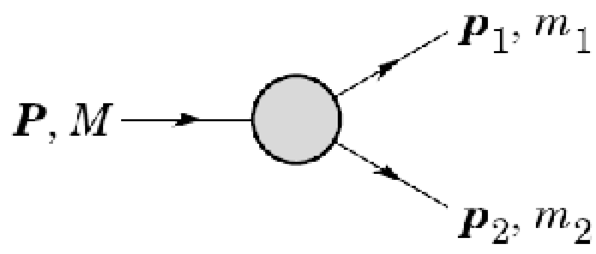
(stochastic contribution)

t_{abs}: thickness of absorber (in units of X_0)

F: factor (~0.2 for liquid noble gaz, 0.06 for Si, ~1 for scintillators)

Resolution

- Two-body decay. Ex: H→γγ



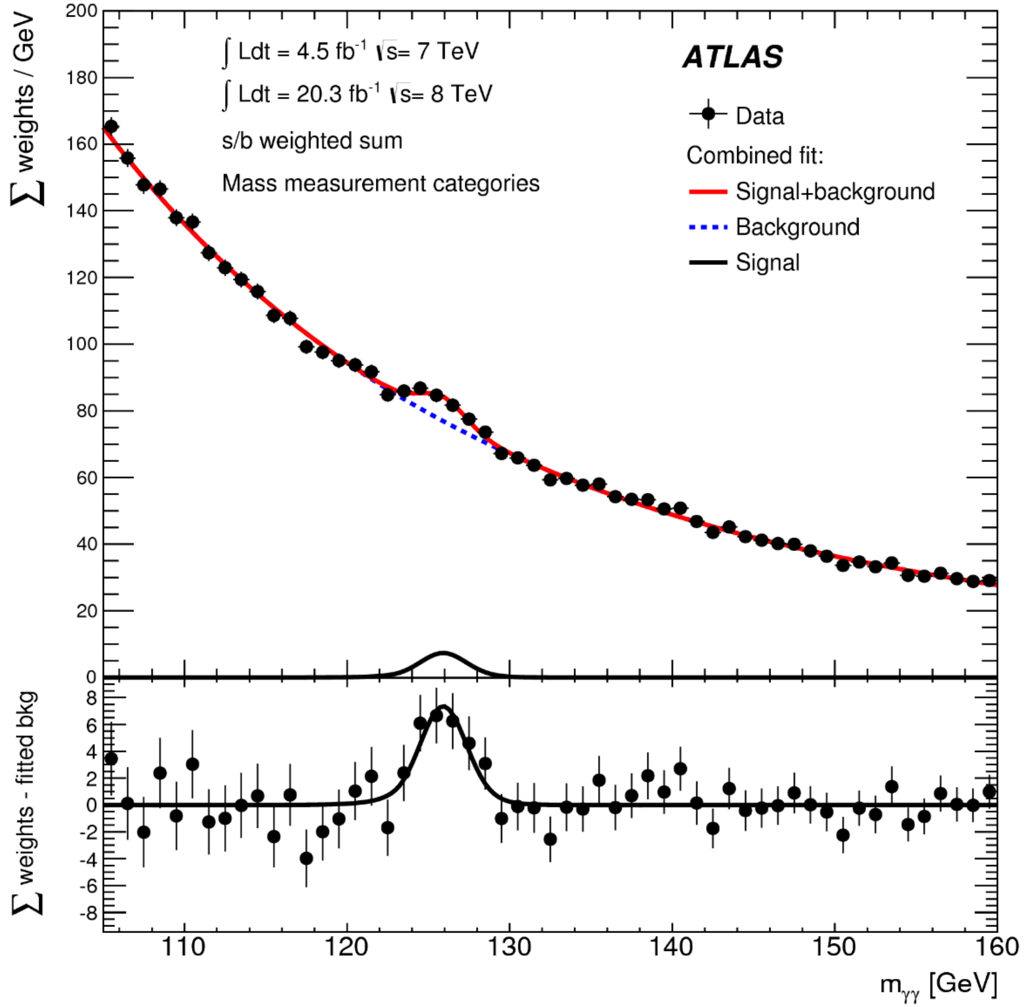
$$m_{\gamma\gamma} = 2E_1E_2(1-\cos\theta_{\gamma\gamma})$$

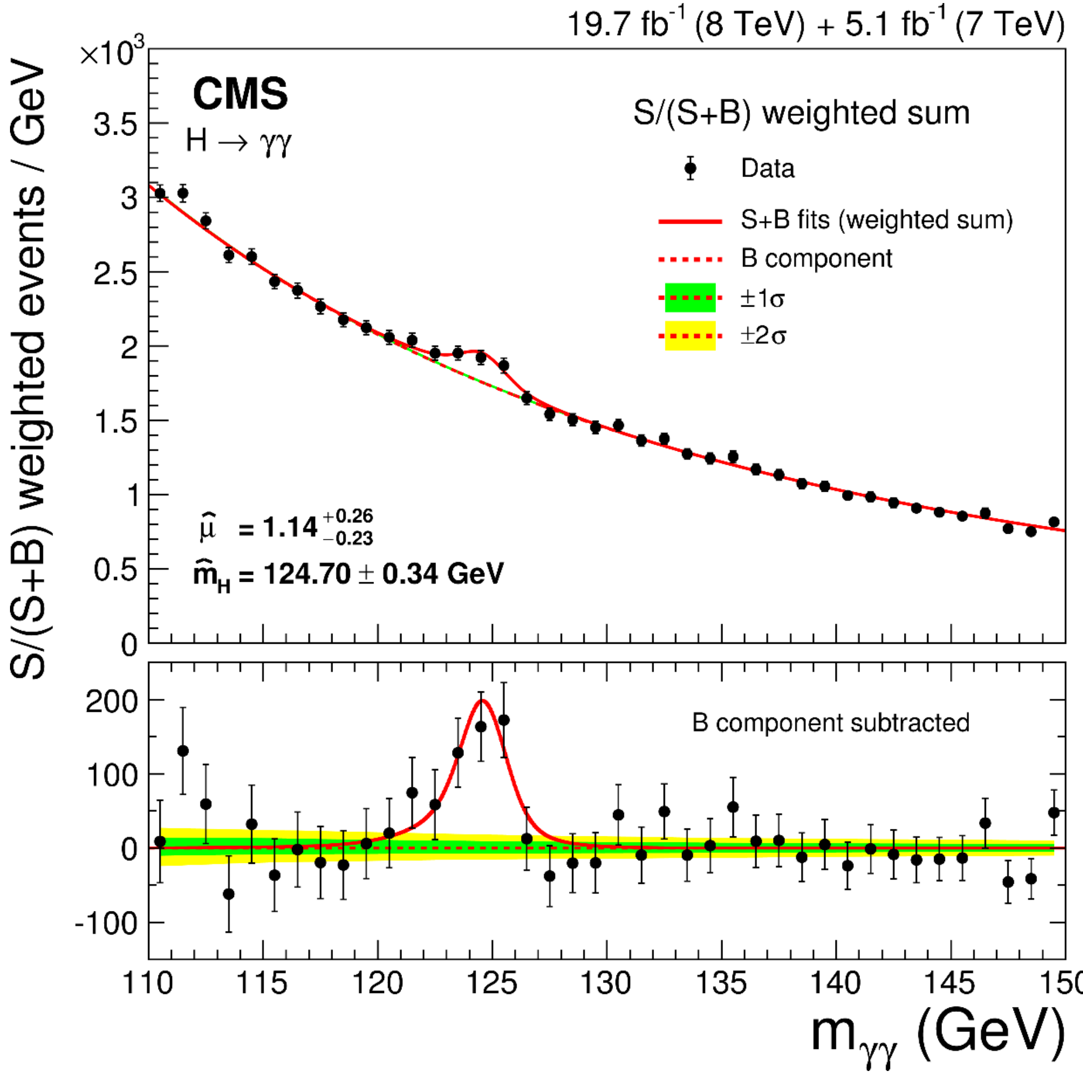
$$\frac{\sigma_m}{m_{\gamma\gamma}} = \frac{1}{2}\sqrt{\left(\frac{\sigma_{E1}}{E_1}\right)^2 + \left(\frac{\sigma_{E2}}{E_2}\right)^2 + \left(\frac{\sigma_\theta}{tg\theta/2}\right)^2}$$

- Resolution on E comes from calorimeters

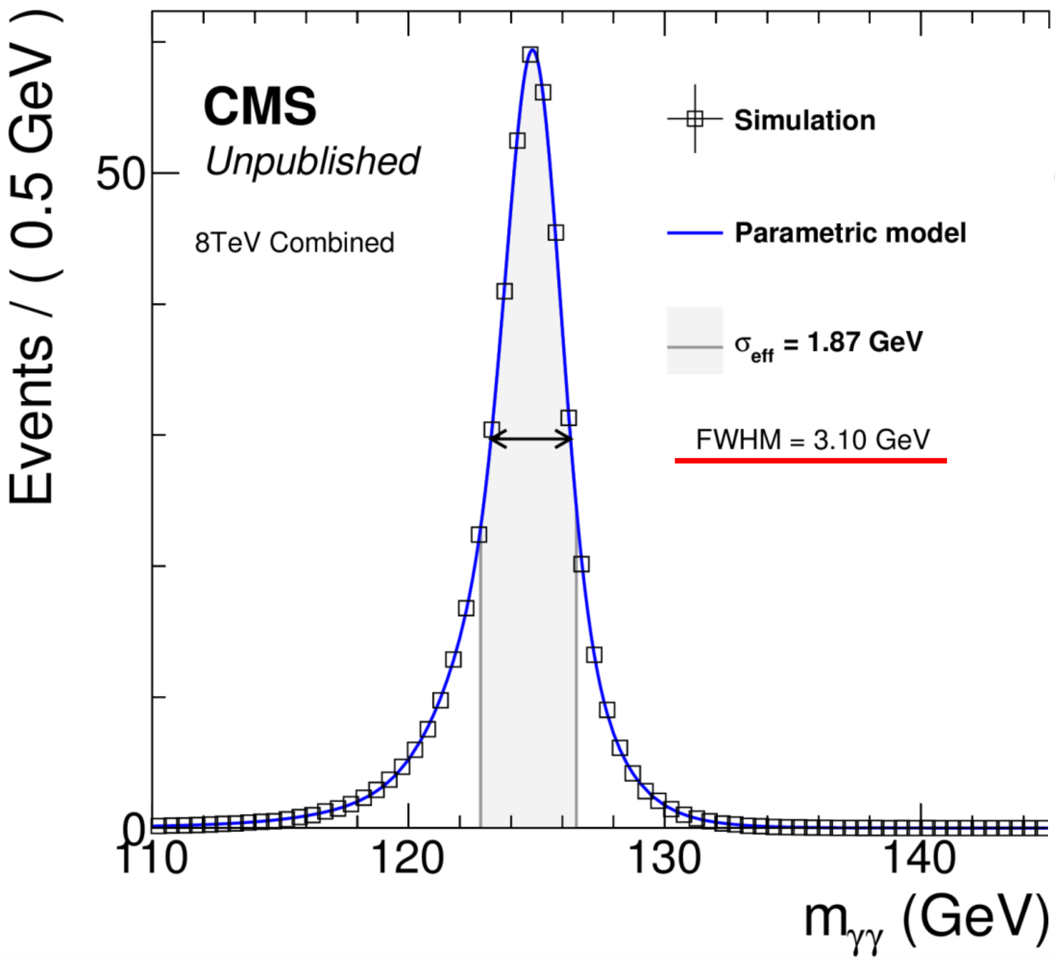
- How do we measure position of photons ? (in CMS and ATLAS)

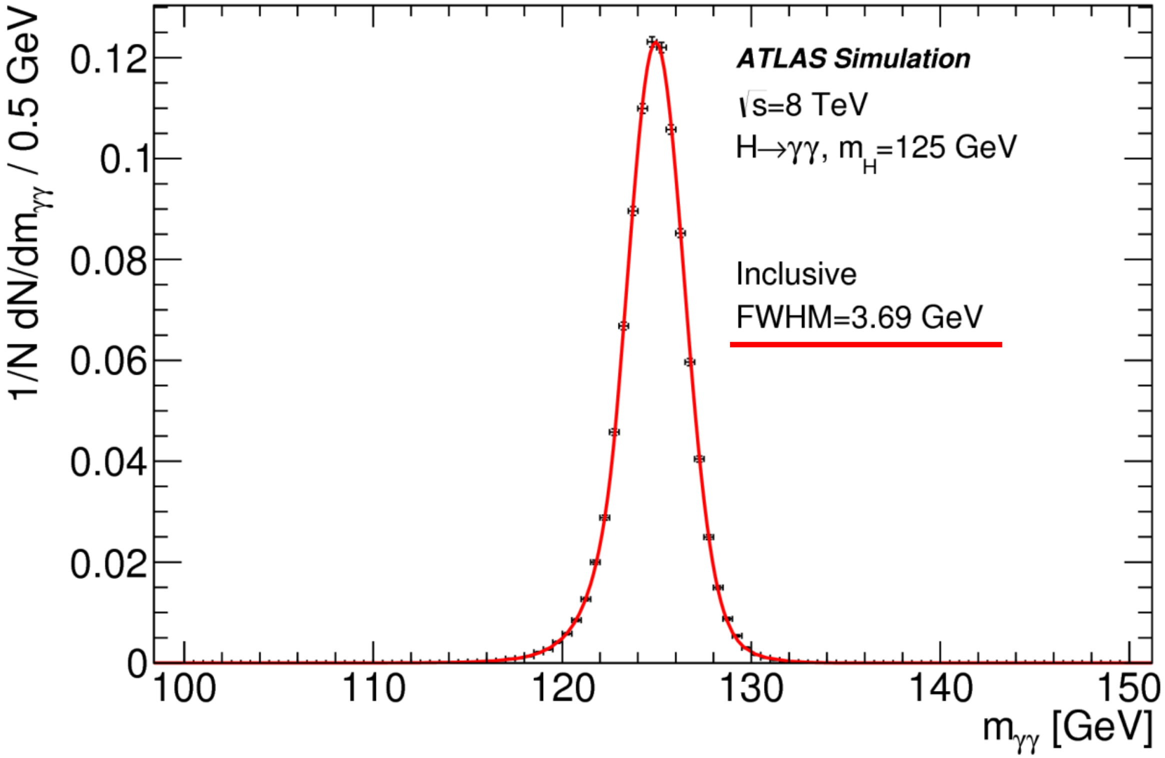
ATLAS/CMS Results (1)





ATLAS/CMS Results (2)





Resolution (again)

CMS

$$\frac{\sigma(E)}{E} = \frac{0.03}{\sqrt{E(GeV)}} \oplus \frac{0.3}{E(GeV)} \oplus 0.005$$

(test beam)

ATLAS

$$\frac{\sigma(E)}{E} = \frac{0.1}{\sqrt{E(GeV)}} \oplus \frac{0.3}{E(GeV)} \oplus 0.007$$

(test beam)

- Fill the table for both calorimeters
- Comment ?

	10 GeV	1 TeV
Stochastic (GeV)		
Noise (GeV)		
Constant (GeV)		
σ(E) (GeV)		
σ(E) / E (%)		

Resolution (again and again)

The ATLAS LAr calorimeter has Pb absorber plates of 1.53mm.

a) What will be the expected contribution to the stochastic term ?

b) Comparison with test beam ?

Ec(Pb) = 7,43 MeV
X0(Pb) = 5,6 mm

Exercise: Crystal Calorimeter

	Atomic Mass	X0 (g.cm-2)	R_M(g.cm-2)
Cs	132.9	8.31	15.53
I	126.9	8.48	15.75

1) Compute the radiation length of a CsI crystal (g.cm-2)
2) Given its density (4.51 g.cm-3), give X0 in cm
3) Given the critical Energy E_C=11.17 MeV, deduce the Moliere Radius (g.cm-2 and cm)
4) Compute the Moliere Radius with the formula for composite material. Compare to 3).

BACK UP SLIDES