Tracking

Jérôme Baudot (baudot@in2p3.fr)

30 January 2019, Archamps
Hypothesis:

- Two sensors
  - perfect positions
  - Infinitely thin
- 1 straight tracks
  - 2 parameters (a, b)

Estimation of track parameters

- Assuming track model is straight

No uncertainty!
What are we talking about?

Hypothesis:

- Two sensors
  - Positions with UNCERTAINTY $\sigma_{\text{det}}$
  - Infinitely thin
- 1 straight tracks
  - 2 parameters ($a, b$)

Estimation of track parameters

- Assuming track model is straight
- Uncertainties from error propagation

\[
a = \frac{x_1}{z_1} \frac{x_0}{z_0}, \quad b = \frac{x_0 z_1}{z_1} \frac{x_1 z_0}{z_0}
\]

\[
a = \frac{\sqrt{2}}{z_1} \frac{z_0}{z_0} \quad \text{det}, \quad b = \frac{\sqrt{z_1^2 + z_0^2}}{z_1} \frac{z_0}{z_0} \quad \text{det}
\]

\[
\text{cov}_{a,b} = \frac{\sqrt{z_1 + z_0}}{z_1} \frac{z_0}{z_0} \quad \text{det}
\]
Hypothesis:

- More than two sensors
  - Positions with uncertainty $\sigma_{\text{det}}$
  - Infinitely thin
- 1 straight tracks
  - 2 parameters ($a, b$)

Estimation of track parameters

- Assuming track model is straight
  - Need FITTING PROCEDURE least square
  - Need covariance matrix of measurements (here diagonal)
- Uncertainties from error propagation
  - Detail depends on geometry
  - Both estimation & uncertainties improve

A = \frac{S_1 S_{xz}}{S_1 S_{z}^2} \frac{S_x S_z}{(S_z)^2} \quad b = \frac{S_x S_z^2}{S_1 S_{z}^2} \frac{S_z S_{xz}}{(S_z)^2}

\Sigma_a = \frac{S_1}{S_1 S_{z}^2} \frac{S_z}{(S_z)^2} \quad \Sigma_b = \frac{S_z^2}{S_1 S_{z}^2} \frac{S_z}{(S_z)^2}

\text{COV}_{a,b} = \frac{S_z}{S_1 S_{z}^2} \frac{S_z}{(S_z)^2}
Hypothesis:
- More than two sensors
  - Positions with uncertainty $\sigma_{\text{det}}$
  - With some THICKNESS ➔ physics effect
- 1 straight tracks
  - 2 parameters $(a,b)$

Estimation of track parameters
- Assuming track model is straight
  - Need fitting procedure least square
  - Need covariance matrix of measurements physics effect ➔ NON DIAGONAL terms
- Uncertainties from error propagation ➔ same estimators but increased uncertainties

What are we talking about?

$$a = \frac{S_1 S_{xz}}{S_1 S_{z^2}} \frac{S_x S_z}{(S_z)^2}, \quad b = \frac{S_x S_{z^2}}{S_1 S_{z^2}} \frac{S_z S_{xz}}{(S_z)^2}$$

Complex covariant matrix expression
- correlation between sensors
- Various implementations possible
What are we talking about?

Hypothesis:
- More than two sensors
  - Positions with uncertainty $\sigma_{\text{det}}$
  - With some thickness
- MANY straight tracks
  - Still 2 parameters ($a,b$)...per track!
  - But may change along track path

New step = FINDING
- Which hits to which tracks?
  - Strongly depends on geometry

Estimation of track parameters
- Happens after finder
- Uncertainties involve correlation
Lecture outline

1. Basic concepts
2. Position sensitive detectors
3. Standard algorithms
4. Advanced algorithms
5. Optimizing a tracking system
6. References

first lecture
second lecture
third lecture
practice
1. Motivations & basic concepts

- Motivations
- Types of measurements
- The 2 main tasks
- Environmental considerations
- Figures of merit
1. Motivations & Basic Concepts

Understanding an event

- Individualize tracks $\simeq$ particles
- Measure their properties
- LHC: $\sim$1000 particles per 25 ns “event”

Track properties

- **Momentum** $\Leftrightarrow$ curvature in B field
  - Reconstruct invariant masses
  - Contribute to jet energy estimation
- **Energy** $\Leftrightarrow$ range measurement
  - Limited to low penetrating particle
- **Mass** $\Leftrightarrow$ dE/dx measurement
- **Origin** $\Leftrightarrow$ vertexing (connecting track)
  - Identify decays
  - Measure flight distance
- **Extension** $\Leftrightarrow$ particle flow algorithm (pfa)
  - Association with calorimetric shower

8 jets event (tt-bar h) @ 1 TeV ILC
1. Motivations & Basic Concepts

- Magnetic field curves trajectories
  \[ \frac{dp}{dt} = qv \cdot \vec{B} \]
  - Rewritten with position \((x)\) and path length \((l)\) → basic equation:
  \[ \frac{d^2 \vec{r}}{dl^2} = \frac{\mu}{\|\vec{p}\|} \cdot q \vec{B}(x) \cdot \frac{dr}{dl} \]
  - In \(B=4T\) a 10 GeV/c particle will get a sagitta of 1.5 cm @ 1m

- Fixed-target experiments
  - Dipole magnet on a restricted path segment
  - Measurement of deflection (angle variation)
    \[ \frac{p_T}{q} = \frac{0.3 \cdot B(T) \cdot L}{\Delta \alpha} \]

- Collider experiment
  - Barrel-type with axial \(B\) over the whole path
  - Measurement of curvature (sagitta)
    \[ \frac{p_T(\text{GeV/c})}{q} = 0.3 \times B(T) \times R(m) \]

- Other arrangements
  - Toroidal \(B\)... not covered

- Two consequences
  - Position sensitive detectors needed
  - Perturbation effects on trajectories limit precision on track parameters
1. Motivations & Basic Concepts

Identifying through topology

- Short-lived weakly decaying particles
  - Charm $c\tau \sim 120 \mu m$
  - Beauty $c\tau \sim 470 \mu m$
  - tau, strange/charmed/beauty particle

Exclusive reconstruction

- Decay topology with secondary vertex
- Exclusive = all particles associated

Inclusive “kink” reconstruction

- Some particles are invisible ($\nu$)
1. Motivations & Basic Concepts

Inclusive reconstruction
- Selecting parts of the daughter particles = flavor tagging
- based on impact parameter (IP)
- \( \sigma_{\text{IP}} \sim 20\text{-}100 \mu m \) requested

Definition of impact parameter (IP)
- Also \( \text{DCA} = \text{distance of closest approach} \) from the trajectory to the primary vertex
- Full 3D or 2D (transverse plane \( d_{\rho} \)) +1D (beam axis)
- Sign extremely useful for flavor-tagging

\( \vec{B} \)
\[
\begin{align*}
\text{Sign defined by charge + traj. Position /VP} \\
\text{Sign defined by angle dca / jet momentum}
\end{align*}
\]
Finding the event origin

- Where did the collision did occur?
  - Primary vertex
- (life)Time dependent measurements
  - CP-asymmetries @ B factories ($\Delta z \approx 60-120 \mu m$)
- Case of multiple collisions / event
  - $>> 10$ vertex @ LHC

Remarks for collider

- Usually no measurement below 1-2 cm / primary vertex
  - Due to beam-pipe maintaining vacuum
- Requires extrapolation $\rightarrow$ expect “unreducible” uncertainties
1. Motivations & Basic Concepts

- **Usually not a tracker task**
  - CALORIMETERs (see lecture by Isabelle)
  - Indeed calorimeters gather material to stop particles while trackers try to avoid material (multiple scattering)
  - however...calorimetry tries to improve granularity

- **Particle flow algorithm**
  - Colliders (pp and ee)

- **Energy evaluation by counting particles**
  - Clearly heretic for calorimetry experts
  - Requires to separate $E_{\text{deposit}}$ in dense environment

- **Range measurement for low energy particles**
  - Stack of tracking layers
  - Modern version of nuclear emulsion

Energy measurement
Reminder on the physics (see other courses)

- Coulomb scattering mostly on nuclei
- Molière theory description as a **centered** gaussian process
  - the thinner the material, the less true → large tails

**In-plane** description (defined by vectors $p_{in}, p_{out}$)

- Corresponds to $(\varphi, \theta)$ with $p_{in} = p_z$ and

\[
p_{out}^2 = p_{out,z}^2 + p_{out,x}^2
\]

\[
p_{out} \cos \theta \approx p_{out,z}
\]

\[
p_{out,x} = p_{out} \sin \theta \approx p_{out} \theta
\]

(\text{note: } [0,2] \text{ uniform})

\[
\frac{\text{sq}}{p} = 13.6 \ (\text{MeV/c}) \cdot z \cdot \sqrt{\frac{\text{thickness}}{X_0}} \cdot \left[1 + 0.038 \ln\left(\frac{\text{thickness}}{X_0}\right)\right]
\]

$X_0 = \text{radiation length}$

Same definition as in calorimetry... though this is accidental.
1. Motivations & Basic Concepts

In-space description (defined by fixed x/y axes)

- Corresponds to \((\theta_x, \theta_y)\) with

\[
p_{\text{out},T}^2 = p_{\text{out},x}^2 + p_{\text{out},y}^2
\]

\[
\begin{align*}
\sin \theta_x & \approx p_{\text{out}} \theta_x \\
\sin \theta_y & \approx p_{\text{out}} \theta_y
\end{align*}
\]

- \(\theta_x\) and \(\theta_y\) are independent gaussian processes

\[
\theta \in [p_{\text{out}}, p_{\text{out},T}] \text{ plane}
\]

\[
\Phi \in [p_{\text{out},x}, p_{\text{out},T}] \text{ plane}
\]

\[
\theta_x \in [p_{\text{out}}, p_{\text{out},x}] \text{ plane}
\]

\[
\theta_y \in [p_{\text{out}}, p_{\text{out},y}] \text{ plane}
\]

\[
\begin{align*}
\theta &= \sqrt{\frac{2}{x} + \frac{2}{y}} \\
x &= y = \frac{1}{\sqrt{2}}
\end{align*}
\]
Important remark when combining materials

- Total thickness $T = \Sigma T_i$, each material $(i)$ with $X_0(i)$

- Definition of effective radiation length $X_{0,\text{eff}} = \frac{T_i X_0(i)}{T}$

- Consider single gaussian process

and never do variance addition (which minimize deviation)
Impact on tracking algorithm

- The track parameters evolves along the track!
- May drive choice of reconstruction method

Photon conversion

- Alternative definition of radiation length probability for a high-energy photon to generate a pair over a path $dx$:

$$\gamma \rightarrow e^+e^- = \text{conversion vertex}$$

- Generate troubles:
  - Additional unwanted tracks
  - Decrease statistics for electromagnetic calorimeter

Remember this simple case

$$\text{Prob} = \frac{dx}{\frac{9}{7}X_0}$$

CMS “picture” of material budget through photon conversion vertices (silicon tracker only)
1. Motivations & Basic Concepts

The collider paradigm

- Basic inputs from detectors
  - Succession of 2D or 3D points (or track segments)
    - Who's who?

- 2 steps process
  - Step 1: track identification = finding = pattern recognition
    - Associating a set of points to a track
  - Step 2: track fitting
    - Estimating trajectory parameters ➔ momentum

- Both steps require
  - Track model (signal, background)
  - Knowledge of measurement uncertainties
  - Knowledge of materials traversed (Eloss, mult. scattering)

- Vertexing needs same 2 steps
  - Identifying tracks belonging to same vertex
  - Estimating vertex properties (position + 4-vector)
1. Motivations & Basic Concepts

Telescope mode

- Single particle at a time
  - Sole nuisance = noise
- Trigger from beam
  - Often synchronous
- Goal = get the incoming direction

The astroparticle way

- Similar to telescope mode
- No synchronous timing
- Ex: deep-water $\nu$ telescopes

=> For 2 last cases: mostly a fitting problem
- Usually with straight track model
Life in a real experiment is tough (for detectors of course)

- Chasing small cross-sections ➔ large luminosity and/or energy
- Short interval between beam crossing
  - LHC: 25 ns (and >10 collisions / crossing)
  - CLIC: 5 ns (but not continuous)
- Large amount of particles (could be > $10^7$ part/cm$^2$/s) ➔ background, radiation
  - makes the finding more complicated
- Vacuum could be required (space, very low momentum particles (CBM, LHCb))

Radiation tolerance

- Two types of energy loss
  - Ionizing (generate charges): dose in Gy = $100$ Rad
  - Non-ionizing (generate defects in solid): fluence in $n_{eq}(1MeV)/cm^2$
- The more inner the detection layer, the harder the radiation (radius$^2$ effect)
- Examples for most inner layers:
  - LHC: $10^{15}$ to $<10^{17}$ $n_{eq}(1MeV)/cm^2$ with 50 to 1 MGy
  - ILC: $<10^{12}$ $n_{eq}(1MeV)/cm^2$ with 5 kGy
1. Motivations & Basic Concepts:

Environmental conditions – 2/2

Timing consideration

- Integration time drives occupancy level (important for finding algorithm)
- Time resolution offers time-stamping of tracks
  - Tracks in one “acquisition event” could be associated to their proper collision event if several have piled-up
- Key question = triggered or not-triggered experiment?

Heat concerns

- Spatial resolution → segmentation → many channels
  - Hot cocktail!
- Readout speed → power dissipation/channel
- Efficient cooling techniques exist BUT
  - add material budget and may not work everywhere (space)

Summary

- Tracker technology driven by environmental conditions: hadron colliders (LHC)
- Tracker technology driven by physics performances: lepton colliders (B factories, ILC), heavy-ion colliders (RHIC, LHC)
- Of course, some intermediate cases: superB factories, CLIC
1. Motivations & Basic Concepts:

For detection layer

- **Detection efficiency**
  - Mostly driven by Signal/Noise
  - **Note:** Noise = signal fluctuation ⊕ readout (electronic) noise
- **Intrinsic spatial resolution**
  - Driven by segmentation (not only)
  - Useful tracking domain $\sigma < 1\text{mm}$
- Linearity and resolution on dE/dx
- **Material budget**
- “Speed” (integration time, time resolution, ...)

For detection systems (multi-layers)

- **Track finding & purity**
- **Two-track resolution**
  - Ability to distinguish two nearby trajectories
  - Mostly governed by signal spread / segments
- **Momentum resolution** $\frac{\Delta p}{p}$
- **Impact parameter resolution**
  - Sometimes called “distance of closest approach” to a vertex
2. Detection technologies

- Intrinsic resolution

- Single layer systems
  - Silicon, gas sensors, scintillator

- Multi-layer systems
  - Drift chamber and TPC

- Tentative simplistic comparison

- Magnets

- Practical considerations

- Leftovers
1. Motivations & Basic Concepts:

- Position measurement comes from segmentation
  - Pitch

- Digital resolution
  \[ \text{Digital resolution} = \frac{\text{pitch}}{\sqrt{12}} \]

- Improvement from signal sharing
  - Position = charge center of gravity
  - Effects generated by
    - Secondary charges spread inside volume
    - Inclined tracks (however, resol. limited at large angles)
  - Potential optimization of segmentation / sharing
    - Work like signal sampling theory (Fourier transform)

- Warnings:
  - Lorentz force from B mimic the effect
  - counterproductive / 2-track resolution

Warnings: 
- Lorentz force from B mimic the effect
- counterproductive / 2-track resolution

ESIPAP - Tracking - J.Baudot
2. Detector Technologies:

- **Basic sensitive element**
  - E-h pairs are generated by ionization in silicon
    - 3.6 eV needed
    - 300 µm thick Si generates ~22000 charges for MIP
      BUT beware of Landau fluctuation
  - Collection: P-N junction = diode
    - Full depletion (10 to 0.5 kV)
      generates a drift field (10^4 V/cm)
    - Collect time ~ 15 ps/µm

- **Silicon strip detectors**
  - sensor “easily” manufactured with pitch down to ~25 µm
  - 1D if single sided
  - Pseudo-2D if double-sided
    - Stereo-angle useful against ambiguities
  - Difficult to go below 100 µm thickness
  - Speed and radiation hardness: LHC-grade
2. Detector Technologies:

**Concept**
- Strips $\rightarrow$ pixels on sensor
- One to one connection from electronic channels to pixels

**Performances**
- Real 2D detector & keep performances of strips
  - Can cope with LHC rate (speed & radiation)
- Pitch size limited by physical connection and #transistors for treatment
  - minimal (today): 50x50 µm$^2$
  - typical: 100x150/400 µm$^2$
  - spatial resolution about 10 µm
- Material budget
  - Minimal (today): 100(sensor)+100(elec.) µm
- Power budget: 10 µW/pixel
2. Detector Technologies:

Concept
- Use industrial CMOS process
  - Implement an array of sensing diode
  - Amplify the signal with transistors near the diode
- Benefit to
  - granularity: pixel pitch down to ~10 µm
  - material: sensitive layer thickness as low as 10-20 µm
- Known as Monolithic Active Pixel Sensors (MAPS)

Sensitive layer
- If undepleted & thin (10-20 µm)
  - Slow (100 ns) thermal drift of charges
  - non-ionizing rad. tolerance \( \lesssim 10^{13} \frac{n_{eq}(1\text{MeV})}{cm^2} \)
- If fully depleted (from 10 to 100 µm)
  - Fast (few ns) field-driven drift of charges
  - non-ionizing rad. tolerance > \( 10^{15} \frac{n_{eq}(1\text{MeV})}{cm^2} \)
2. Detector Technologies:

Concept

- Use industrial CMOS process
  - Implement an array of sensing diode
  - Amplify the signal with transistors near the diode
- Gain in granularity: pitch down to ~ 10 µm
- Gain in sensitive layer thickness ~ 10-20 µm
- For undepleted thin sensitive layer
  - Slow (100 ns) thermal drift of charges
  - non-ionizing rad. tolerance ≲ $10^{13}$ n$_{eq}$(1MeV)/cm$^2$
- For fully depleted thin to thick sensitive layer
  - Fast (few ns) field-driven drift of charges
  - non-ionizing rad. tolerance > $10^{15}$ n$_{eq}$(1MeV)/cm$^2$

Performances

- Spatial resolution 1-10 µm (in 2 dimensions)
- Material budget: ≲ 50 µm
- Power budget: < µW/pixel
- Integration time ≈ 5-100 µs demonstrated
  - ~ 1 µs in development
- Timestamping @ ns level in development
2. Detector Technologies:

**Other active pixel sensors**

- **DEPFET**
  - Fully depleted sensitive layer
  - Large amplification
  - Still require some read-out circuits
    - Not fully monolithic
    - Possibly limited in read-out speed

- **Silicon On Insulator (SOI)**
  - Fully depleted sensitive layer
  - Fully monolithic
  - Electronics similar to MAPS
2. Detector Technologies:

**Basic sensitive element**
- Metallic wire, $1/r$ effect generated an avalanche
- Signal depends on gain (proportional mode) typically $10^4$
- Signal is fast, a few ns

**Gas proportional counters**
- Multi-Wire Proportional Chamber
  - Array of wires
  - 1 or 2D positioning depending on readout
  - Wire spacing (pitch) limited to 1-2 mm
- Straw or drift tube
  - One wire in One tube
  - Extremely fast (compared to Drift Chamber)
  - Handle high rate
  - Spatial resolution <200 µm
  - Left/right ambiguity
2. Detector Technologies:

Micro-pattern gas multipliers

- MSGC
  - Replace wires with lithography micro-structures
  - Smaller anodes pitch 100-200 µm
  - BUT Ageing difficulties due to high voltage and manufacturing not so easy

- GEM
  - Gain $10^5$
  - Hit rate $10^6$ Hz/cm²
2. Detector Technologies:

Wire chambers “advanced”

Micro-pattern gas multipliers

➛ MSGC
  - Replace wires with lithography micro-structures
  - Smaller anodes pitch 100-200 µm
  - BUT Ageing difficulties due to high voltage and manufacturing not so easy

➛ GEM
  - Gain $10^5$
  - Hit rate $10^6$ Hz/cm²

➛ MICROMEGAS
  - Even smaller distance anode-grid
  - Hit rate $10^9$ Hz/cm²

➛ More development
  - Electron emitting foil working in vacuum!
2. Detector Technologies: Drift chambers

- **Basic principle**
  - Mix field and anode wires
    - Generate a drift
  - Pressurize gas to increase charge velocity (few atm)
  - 3D detector
    - 2D from wire position
    - 1D from charge sharing at both ends

- **Spatial Resolution**
  - Related to drift path
    \[ \mu \sqrt{\text{drift length}} \]
  - Typically 100-200 µm

- **Remarks**
  - Could not go to very small radius

Same principle with straw tubes
2. Detector Technologies:

**Benefits**

- Large volume available
- Multi-task: tracking + Part. Identification

**Basic operation principle**

- Gas ionization → charges
- Electric field → charge drift along straight path
- Information collected
  - 2D position of charges at end-cap
  - 3rd dimension from drift time
  - Energy deposited from #charges
- Different shapes:
  - rectangles (ICARUS)
  - Cylinders (colliders)
  - Volumes can be small or very large
End cap readout

- Gas proportional counters
  - Wires+pads, GEM, Micromegas

Performances

- Two-track resolution ~ 1cm
- Transverse spatial resolution ~ 100 - 200 µm
- Longitudinal spatial resolution ~ 0.2 - 1 mm
- Longitudinal drift velocity: 5 to 7 cm/µs
  - ALICE TPC (5m long): 92 µs drift time

Pros
- Nice continuously spaced points along trajectory
- Minimal multiple scattering (inside the vessel)

Cons
- Limiting usage with respect to collision rate
2. Detector Technologies:

**Conclusion on technologies**

**Tentative Comparison**

- **Silicon**
  - Temporal resolution: 10 ns
  - Spatial resolution: <10 μm
  - Material budget: >0.1%X0 per point

- **Gas with micro-pattern**
  - Temporal resolution: <10 ns
  - Spatial resolution: ~0.1%X0 per point

- **Large Gas volume**
  - Temporal resolution: <10 ns
  - Spatial resolution: >50 μm
  - Material budget: A few 0.1%X0 for many point

**Trend**

- Faster collision rates and higher particle multiplicities favour
  - Fast silicon sensors and micro-pattern gas chambers
  - Pixelisation
  - Still large gas ensemble for
    - BelleII (SuperKEKB) -> CDC and ILD (ILC) -> TPC
    - ESIPAP - Tracking - J.Baudot

37
2. Detector Technologies:

- **Solenoid**
  - Field depends on current I, length L, # turns N
    - on the axis \( B = \frac{\mu_0 NI}{\sqrt{L^2 + 4R^2}} \)
    - Typically: 1 T needs 4 to 8 kA
      - superconducting metal to limit heat
  - Field uniformity needs flux return (iron structure)
    - Mapping is required for fitting (remember \( B(x) \)?)
    - Usually performed with numerical integration
  - Calorimetry outside ➔ limited material ➔ superconducting
  - Fringe field calls for compensation

- **Supercondiction**
  - cryo-operation ➔ quenching possible!
  - Magnetic field induces energy: \( E \mu B^2 R^2 L \)
    - Cold mass necessary to dissipate heat in case of quench

<table>
<thead>
<tr>
<th></th>
<th>Field (T)</th>
<th>Radius (m)</th>
<th>Length (m)</th>
<th>Energy (MJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALICE</td>
<td>0.5</td>
<td>6</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>ATLAS</td>
<td>2</td>
<td>2.5</td>
<td>5.3</td>
<td>700</td>
</tr>
<tr>
<td>CMS</td>
<td>4</td>
<td>5.9</td>
<td>12.5</td>
<td>2700</td>
</tr>
<tr>
<td>ILC</td>
<td>4</td>
<td>3.5</td>
<td>7.5</td>
<td>2000</td>
</tr>
</tbody>
</table>
2. Detector Technologies:

- From a detection principle to a detector
  - Build large size or many elements
    - Manufacture infrastructures
    - Characterization capabilities
    - Production monitoring
    - New monolithic silicon pixel detector tend to replace silicon strip technology
  - Integration in the experiment
    - Mechanical support
    - Electrical services (powering & data transmission)
    - Cooling (signal treatment dissipates power)
  - Specific to trackers
    - Internal parts of multi-detectors experiment → limited space
    - Material budget is ALWAYS a concern
    - trade-offs required

Practical considerations
2. Detector Technologies:

- **Silicon drift detectors**
  - Real 2D detectors made of strips
  - 1D is given by drift time

- **Diamond detectors**
  - Could replace silicon for hybrid pixel detectors
  - Very interesting for radiation tolerance

- **Plasma sensor panels**
  - Derived from flat television screen
  - Still in development

- **Charge Coupled Devices (CCD)**
  - Fragile/ radiation tolerance

- **Signal generation**
  - see Ramo’s theorem

- **Nuclear emulsions**
  - One of the most precise ~ 1µm
  - No timing information → very specific applications

- **Scintillators**
  - Extremely fast (100 ps)
  - Could be arranged like straw tubes
  - But quite thick ($X_0 \sim 2$ cm)
3. Standard algorithms

- Finders
- First evaluation of momentum resolution
- Fitters
- Alignment
Global methods

- Transform the coordinate space into **pattern space**
  - “pattern” = parameters used in track model
- Identify the “best” solutions in the new phase space
- Use all points at a time
  - No history effect
- Well adapted to evenly distributed points with same accuracy

Local methods

- Start with a **track seed** = restricted set of points
  - Could require good accuracy from the beginning
- Then extrapolate to next layer-point
  - And so on... **iterative procedure**
- “Wrong” solutions discarded at each iteration
- Possibly sensitive to “starting point”
- Well adapted to redundant information
3. Standard algorithms:

A simple example
- Straight line in 2D: model is \( x = a * z + b \)
- Track parameters \((a, b)\); N measurements \( x_i \) at \( z_i \) (i=1..N)

A more complex example
- Helix in 3D with magnetic field
- Track parameters \((\gamma_0, z_0, D, \tan \lambda, C=R)\)
- Measurements \((r, \varphi, z)\)

Generalization
- Parameters: P-vector \( p \)
- Measurements: N-vector \( c \)
- Model: function \( f(R^P \rightarrow R^N) \)
  \[
  f(p) = c \quad \leftrightarrow \quad \text{propagation}
  \]

Track model

\[
\varphi(r) = \gamma_0 + asin\frac{C r (1 + CD) D / r}{1 + 2CD}
\]

\[
z(r) = z_0 + \frac{\tan \lambda}{C} asin\left(C \sqrt{\frac{r^2 - D^2}{1 + 2CD}}\right)
\]
3. Standard algorithms:

Another view of the helix

- $s =$ track length
- $h =$ sense of rotation
- $\lambda =$ dip angle
- Pivot point ($s=0$):
  - position $(x_0, y_0, z_0)$
  - orientation $\varphi_0$

\[
x(s) = x_o + R \left[ \cos \left( \Phi_o + \frac{hs \cos \lambda}{R} \right) - \cos \Phi_o \right]
\]

\[
y(s) = y_o + R \left[ \sin \left( \Phi_o + \frac{hs \cos \lambda}{R} \right) - \sin \Phi_o \right]
\]

\[
z(s) = z_o + s \sin \lambda
\]
3. Standard algorithms:

- **Track seed = initial segment**
  - Made of few (2 to 4) points
    - One point could be the expected primary vtx
  - Allows to initialize parameter for track model
  - Choose most precise layers first
    - usually inner layers
  - But if high hit density
    - Start farther from primary interaction
    - @ lowest density
    - Limit mixing points from different tracks

- **Extrapolation step**
  - Out or inward (=toward primary vtx) onto the next layer
  - Not necessarily very precise, especially only local model needed
    - Extrapolation uncertainty \( \preceq \) layer point uncertainty
    - Computation speed important
  - Match (associate) nearest point on the new layer
    - Might skip the layer if point missing
    - Might reject a point: if worst track-fit or if fits better with another track
3. Standard algorithms:

Variant with track segments
- First build “tracklets” on natural segments
  - Sub-detectors, or subparts with same resolution
- Then match segments together
- Typical application:
  - Segments large tracker (TPC) with vertex detector (Si)
    - layers dedicated to matching

Variant with track roads
- Full track model used from start

Variant with Kalman filter
- See later

Figure of merit
- $\sigma_{\text{eff}} = \sigma(\text{sensor}) \oplus \sigma(\text{track extrapolation}) = \text{effective spatial resolution}$
- $\rho = \text{background hit density}$
3. Standard algorithms:

Brute force = combinatorial way
- Consider all possible combination of points to make a track
- Keep only those compatible with model
- Usually too time consuming...

Hough transform
- Example straight track:
  - Coord. space \( y = a \times x + b \) \(\Leftrightarrow\) pattern space \( b = y - x \times a \)
  - Each point \((y,x)\) defines a line in pattern space
  - All lines, from points belonging to same straight-track, cross at same point \((a,b)\)
    - In practice:
      discretize pattern space and search for maximum
  - Applicable to circle finder
    - needs two parameters as well \((r, \varphi)\) of center
      if track is assumed to originate from \((0,0)\)
  - More difficult for more than 2 parameters...
### Conformal mapping

- Helix transverse projection = Circle
  - \((x-a)^2 + (y-b)^2 = r^2\)
  - Transform to \(u = x/(x^2+y^2)\), \(v = y/(x^2+y^2)\)
  - Then: \(v = -(a/b)u + (1/2b)\)

### Figure of merit

\[(sensor) \quad z \quad (sensor) \quad \text{bckgrnd}\]
3. Standard algorithms:

- Why do we need to fit?
  - Measurement error
  - Multiple scattering error

- Global fit
  - Assume knowledge of:
    - all track points
    - full correlation matrix
      - difficult if $\sigma_{\text{mult. scatt.}} \gtrsim \sigma_{\text{meas.}}$
  - Least square method

- Iterative fit
  - Iterative process:
    - points included in the fit one by one
    - could be merged with finder step
  - Kalman filter

FITTING drives track extrapolation & momentum res.
3. Standard algorithms:

**Linear model hypothesis**

- P track parameters $\mathbf{p}$, with N measurements $\mathbf{c}$
  \[
  \tilde{\mathbf{c}} = \mathbf{c}_s + A(\tilde{\mathbf{p}} - \tilde{\mathbf{p}}_s) + \mathbf{\varepsilon}
  \]

- $\mathbf{p}_s$ = known starting point (pivot), $A$ = track model NxP matrix,
  $\mathbf{\varepsilon}$ = error vector corresponding to $V$ = covariance NxN matrix

**Sum of squares:**

\[
S(\tilde{\mathbf{p}}) = (\tilde{\mathbf{c}} - \mathbf{c}_s - A(\tilde{\mathbf{p}} - \tilde{\mathbf{p}}_s))^T V^{-1} (\mathbf{c}_s + A(\tilde{\mathbf{p}} - \tilde{\mathbf{p}}_s) - \tilde{\mathbf{c}})
\]

**Best estimator (minimizing variance)**

\[
\frac{dS}{d\tilde{\mathbf{p}}}(\tilde{\mathbf{p}}) = 0 \Rightarrow \tilde{\mathbf{p}} = \tilde{\mathbf{p}}_s + (A^T V^{-1} A)^{-1} A^T V^{-1} (\tilde{\mathbf{c}} - \mathbf{c}_s)
\]

- Variance (= uncertainty) of the estimator:
  \[
  \overline{V_{\tilde{\mathbf{p}}}} = (A^T V^{-1} A)^{-1}
  \]

- Estimator $\mathbf{p}$ follows a $\chi^2$ law with N-P degrees of freedom

**Problem ⇔ inversion of a PnP matrix ($A^T V^{-1} A$)**

- But real difficulty could be computing $V$ (NxN matrix)
  
  ⇐ layer correlations if multiple scattering non-negligible if $\sigma_{\text{mult. scatt.}} \geq \sigma_{\text{meas}}$
3. Standard algorithms:

- **Straight line model**
  - 2D case → D=2 coordinates (z,x)
  - 2 parameters: \( a = \text{slope}, \quad b = \text{intercept at } z=0 \)

- **General case**
  - \( K+1 \) detection planes (i=0...k)
    - located at \( z_i \)
    - Spatial resolution \( \sigma_i \)
  - Useful definitions
    \[
    S_1 = \sum_{i=0}^{K} \frac{1}{i}, \quad S_z = \sum_{i=0}^{K} \frac{z_i}{i}, \quad S_{xz} = \sum_{i=0}^{K} \frac{x_i z_i}{i}, \quad S_{z^2} = \sum_{i=0}^{K} \frac{z_i^2}{i}
    \]
  - Solutions
    \[
    a = \frac{S_1 S_{xz} - S_{xz}^2}{S_1 S_{z^2} - (S_z)^2}, \quad b = \frac{S_x S_{z^2} - S_{z^2} S_{xz}}{S_1 S_{z^2} - (S_z)^2}
    \]
  - Uncertainties
    \[
    a^2 = \frac{S_1^2}{S_1 S_{z^2} (S_z)^2}, \quad b^2 = \frac{S_{z^2}}{S_1 S_{z^2} (S_z)^2}
    \]
    - correlation
    \[
    \text{cov}_{a,b} = \frac{S_z}{S_1 S_{z^2} (S_z)^2}
    \]

- **Case of uniformly distributed \( (K+1) \) planes**
  - \( z_{i+1} - z_i = L/K \) et \( \sigma_i = \sigma \quad \forall i \)
  - \( S_z = 0 \) → a,b uncorrelated
    \[
    a^2 = \frac{12K}{(K+2)L^2} \left( \frac{K}{K+1} \right)^2, \quad b^2 = \left( 1 + \frac{12K}{K+2} \right)^2 \left( \frac{K}{K+1} \right)^2 \]
  - Uncertainties:
    - \( \sigma_a \) and \( \sigma_b \) improve with \( 1/\sqrt{(K+1)} \)
    - \( \sigma_a \) and \( \sigma_b \) improve with \( 1/L \)
    - \( \sigma_b \) improve with \( z_c \)
3. Standard algorithms:

Hypothesis

- $K$ detectors, each with $\sigma$ single point accuracy
- Uniform field over $L$ from dipole
  - Trajectory: $\Delta \alpha = \frac{0.3qBL}{p}$
  - Bending: $\Delta p = p \Delta \alpha$
- Geometrical arrangement optimized for resolution
  - Angular determination on input and output angle:
    $\frac{\sigma_{\text{angular}}}{\sigma_{\text{total}}} = \frac{16}{KL^2}$

Without multiple scattering

- Uncertainty on momentum
  $$\frac{p}{p} = \frac{8}{0.3qBL} \frac{1}{l\sqrt{K}}$$
- Note proportionality to $p$!

Multiple scattering contribution

- Additional term on $\sigma_\alpha$ almost directly from smult.scatt
  $$\frac{\sigma_{\text{multiple}}}{\sigma_{\text{total}}} = \frac{13.6 \text{ (MeV/c)}}{p}$$
Hypothesis

- K detectors uniformly distributed each with $\sigma$ single point accuracy
- Uniform field over path length L

Without multiple scattering

- Uncertainty on transverse momentum (Glückstern formula)

$$\frac{p_T}{p_T} = \frac{\sqrt{720}}{0.3q} \frac{1}{BL^2} \frac{1}{\sqrt{K + 6}} p_T$$

- Works well with large $K > 20$
3. Standard algorithms:

- **Dimensions**
  - P parameters for track model
  - D “coordinates” measured at each point (usually $D < P$)
  - K measurement points (# total measures: $N = K \times D$)

- **Starting point**
  - Initial set of parameters: first measurements
  - With large uncertainties if unknowns

- **Iterative method**
  - Propagate to next layer = prediction
    - Using the **system equation**
      $$\tilde{p}_k = G \tilde{p}_{k-1} + \omega_k$$
    - $G$ = PxP matrix, $\omega$ = perturbation associated with covariance PxP matrix $V_\omega$
    - Update the covariance matrix with additional uncertainties
      $V_{k|k-1} = V_{k-1} + V_k$
  - Add new point to update parameters and covariance, using the **measure equation**
    $$\tilde{m}_k = H \tilde{p}_k + \varepsilon_k$$
    - $H$ = DxP matrix, $\varepsilon$ = measure error associated with **diagonal** covariance DxD matrix $V_m$
    - Weighted means of prediction and measurement using variance $\Leftrightarrow \chi^2$ fit
  - Iterate...

\[
\begin{align*}
\tilde{p}_k &= \left( V_{k|k-1}^{-1} \tilde{p}_{k|k-1} + H^T V_m^{-1} m_k \right) \cdot \left( V_{k|k-1}^{-1} + H^T V_m^{-1} H \right)^{-1}
\end{align*}
\]
3. Standard algorithms:

○ Forward and backward filters
  - Forward estimate of $p_k$: from 1→k-1 measurements
  - Backward estimate of $p_k$: from k+1→K measurements
  - Independent estimates ➔ combination with weighted mean = smoother step

○ Computation complexity
  - only PxP, DxP or DxD matrices computation (≪NxN)

○ Mixing with finder
  - After propagation step: local finder
  - Some points can be discarded if considered as outliers in the fit (use $\chi^2$ value)

○ Include exogenous measurements
  - Like dE/dx, correlated to momentum
  - Additional measurement equation
    \[
    \tilde{m}'_k = H' \tilde{p}_k + \tilde{e}'_k
    \]
    \[
    \tilde{p}_k = \left( V_{klk-1}^{-1} + H' V_{m_k}^{-1} \tilde{m}_k + H' V_{m'_k}^{-1} \tilde{m}'_k \right) \cdot \left( V_{klk-1}^{-1} + H' V_{m_k}^{-1} H + H' V_{m'_k}^{-1} H' \right)^{-1}
    \]
Let’s come back to one initial & implicit hypothesis

- “We know where the point are located.”
- True to the extent we know where the detector is!
- BUT, mechanical instability (magnetic field, temperature, air flow...) and also drift speed variation (temperature, pressure, field inhomogeneity...) limit our knowledge
- Periodic determination of positions and deformations needed = alignment

Initially assumption for detector positions & tracks built from these assumptions

Note hit position relative to detector are the same tracks reconstructed are not even close to reality...
3. Standard algorithms:

Alignment parameters
- Track model depends on additional “free” parameters, i.e. the sensor positions

Methods
- Global alignment:
  - Fit the new params. to minimize the overall $\chi^2$ of a set of tracks (Millepede algo.)
  - Beware: many parameters could be involved (few $10^3$ can easily be reached)
- Local alignment:
  - Use tracks reconstructed with reference detectors
  - Align other detectors by minimizing the “residual” (track-hit distance) width

For both cases
- Use a set of well know tracks and tracking-”friendly” environment to avoid bias
  - Muons (very traversing) and no magnetic field
  - Low multiplicity events
4. Advanced methods (brief illustrations)

- Why?
- Neural network
- Cellular automaton
4. Advanced methods

Shall we do better?
- Higher track/vertex density, less efficient the classical method
- Allows for many options and best choice

Adaptive features
- **Dynamic change** of track parameters during finding/fitting
- Measurements are weighted according to their uncertainty
  - Allows to take into account several “normally excluded” info
- Many hypothesis are handled simultaneously
  - But their number decrease with iterations (annealing like behavior)
- Non-linearity
- Often CPU-time costly (is that still a problem?)

Examples
- Neural network, Elastic nets, Gaussian-sum filters, Deterministic annealing, Cellular automaton

Adaptive methods
4. Advanced methods

Cellular automaton

- **Initialization**
  - built any cell (= segment of 2 points)

- **Iterative step**
  - associate neighbour cells (more inner)
  - Raise “state” with associated cells
  - Kill lowest state cells

J. Lettenbichler et al., 2013

0 (black), 1 (red), 2 (orange), 3 (green), 4 (cyan)
5. Deconstructing some tracking systems

- CMS (colliders)
- AMS, ANTARES (telescopes)
5. Some tracking systems:

- **Superconducting Coil**, 4 Tesla
- **CALORIMETERS**
  - ECAL 76k scintillating PbWO4 crystals
  - HCAL Plastic scintillator/brass sandwich
- **IRON YOKE**
- **TRACKER** Pixels
  - Silicon Microstrips
  - 210 m² of silicon sensors
  - 9.6 M channels
- **MUON BARREL**
  - Drift Tube Chambers (DT)
  - Resistive Plate Chambers (RPC)
- **MUON ENDCAPS**
  - Cathode Strip Chambers (CSC)
  - Resistive Plate Chambers (RPC)

**Specifications**
- Total weight: 12500 t
- Overall diameter: 15 m
- Overall length: 21.6 m
- 2900 scientists from 182 institutes from 38 countries
5. Some tracking systems:

The trackerS

- Double Sided
- Single Sided

\( \eta = 0.9 \)

\( \eta = 2.5 \)
5. Some tracking systems:

- Alignment residual width
5. Some tracking systems:

- Taking a picture of the material budget
  - Using secondary vertices from $\gamma \rightarrow e^+e^-$

- Measuring it by data/simulation comparison
5. Some tracking systems:

- Tracking algorithm = multi-iteration process

![CMS Simulation Graph](image)
5. Some tracking systems:

- Tracking efficiency
5. Some tracking systems:

- Tracking efficiency
  - Single, isolated muons

![Graph showing efficiency vs. #eta and p_T for different muon p_T values and regions.](image)
5. Some tracking systems:

- Tracking efficiency

  - All pions
5. Some tracking systems:

- Tracking purity
  - All pions
5. Some tracking systems:

- Tracking resolution

\[ d_0 = \text{transverse impact parameter} \]
5. Some tracking systems:

- Tracking resolution

ALICE figure
5. Some tracking systems:

Impact parameter resolution

\[ \sigma_{ip} \propto \sqrt{R^2_{\text{ext}} \sigma^2_{\text{int}} + R^2_{\text{int}} \sigma^2_{\text{ext}}} \oplus \frac{R_{\text{int}} \sigma_{\theta(mn)}}{p \sin^{3/2}(\theta)} \]
5. Some tracking systems:

AMS: A TeV precision, multipurpose particle physics spectrometer in space.

Particles and nuclei are defined by their charge ($Z$) and energy ($E \sim P$).

- **TRD**
  - Identify $e^+, e^-$
  - Silicon Tracker
    - $Z, P$
  - ECAL
    - $E$ of $e^+, e^-, \gamma$

- **TOF**
  - $Z, E$

- **Magnet**
  - $\pm Z$

- **RICH**
  - $Z, E$

$Z, P$ are measured independently by the Tracker, RICH, TOF and ECAL.

G. Ambrosi, June 20th 2014
5. Some tracking systems:

Silicon strip sketch

Fig. 5. The effective position resolution (weighted average of two Gaussian widths) in the y-coordinate for different inclination angles (top), the Maximum Detectable Rigidity (MDR, 100% rigidity measurement error) as a function of the inclination angle estimated for 1TV proton incidence with the simulation (middle), and the inclination angle distribution in the geometric acceptance of the tracker (bottom).
5. Some tracking systems:
Summary

Fundamental characteristics of any tracking & vertexing device:
- (efficiency), granularity, material budget, power dissipation, “timing”, radiation tolerance
- All those figures are intricated: each technology has its own limits

Many technologies available
- None is adapted to all projects (physics + environment choose, in principle)
- Developments are ongoing for upgrades & future experiments
  - Goal is to extent limits of each techno. → convergence to a single one?

Reconstruction algorithms
- Enormous boost (variety and performances) in the last 10 years
- Each tracking system has its optimal algorithm

Development trend
- Always higher hit rates call for more data reduction
- Tracking info in trigger → high quality online tracking/vertexing

Link with:
- PID: obvious with TPC, TRD, topological reco.
- Calorimetry: Particle flow algorithm, granular calo. using position sensors
References

- R. Frühwirth, M. Regler, R. K. Bock, H. Grote, D. Notz 
  Data Analysis Techniques for High-Energy Physics 

- P. Billoir 
  Statistics for trajectometry, 
  proceedings of SOS 2012, doi:10.1051/epjconf/20135503001

- ...and of course the Particle Data Group review 
  http://pdg.web.cern.ch, “Reviews, Tables, Plots” section

- D. Green 
  The Physics of Particle Detectors 
  ed. Cambridge University Press 2005 
  (some sections describing tracking)
Detector technologies


- Fabio Sauli
  *Gaseous Radiation Detectors: Fundamentals and Applications*
  ed. Cambridge University Press 2014

- Helmut Spieler,
  *Semiconductor Detector Systems*,
  ed. Oxford Univ. Press 2005

- Leonardo Rossi, Peter Fischer, Tilman Rohe and Norbert Wermes
  *Pixel Detectors: From Fundamentals to Applications*,
  ed. Springer 2006
Reconstruction algorithm & fit


Contributions from experiments


- G.Piacquadio, ATLAS Alignment, Tracking and Physics Performance Results, proceedings of VERTEX 2010, PoS(VERTEX 2010)015


- S.Amerio, Online Track Reconstruction at Hadron Collider, Proceedings of ICHEP 2010, PoS(ICHEP 2010)481

Was not discussed

- Particle interaction with matter
- The readout electronics
- Cooling systems
- The magnets to produce the mandatory magnetic field for momentum measurement
- Vertexing
Backups
Backups:

OPAL drift chamber
Backups:

ALICE - TPC
(ALICE) TPC $dE/dx$
Backups:

ICARUS - TPC
Backups: NA-50 fixed target
Backups:

ATLAS tracking setup
Backups:

ATLAS tracking setup
Backups:

ALICE setup

1. L3 Magnet
2. HMPID
3. TOF
4. Dipole Magnet
5. Muon Filter
6. Tracking Chambers
7. Trigger Chambers
8. Absorber
9. TPC
10. PHOS
11. ITS
More position sensitive detectors

Backups:

DEPFET

Silicon drift

CCD

MICROMEGAS
Was not discussed

- Particle interaction with matter
- The readout electronics
- Cooling systems
- The magnets to produce the mandatory magnetic field for momentum measurement
- Vertexing