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Tracking
Hypothesis:

- Two sensors
  - perfect positions
  - Infinitely thin
- 1 straight tracks
  - 2 parameters \((a, b)\)

Estimation of track parameters

- Assuming track model is straight

- No uncertainty!

\[
a = \frac{x_1}{z_1} \frac{x_0}{z_0}, \quad b = \frac{x_0z_1}{z_1} \frac{x_1z_0}{z_0}
\]
What are we talking about?

Hypothesis:

- Two sensors
  - Positions with UNCERTAINTY $\sigma_{\text{det}}$
  - Infinitely thin
- 1 straight tracks
  - 2 parameters ($a, b$)

Estimation of track parameters

- Assuming track model is straight
- Uncertainties from error propagation

$$a = \frac{z_1}{z_1} \times_0 \times_1, \quad b = \frac{z_0 z_1}{z_0} \times_0 \times_1$$

$$a = \frac{\sqrt{2}}{z_1} \times_0 \times_1 \quad \text{det}, \quad b = \frac{\sqrt{z_1^2 + z_0^2}}{z_1} \times_0 \times_1 \quad \text{det}$$

$$\text{cov}_{a, b} = \frac{\sqrt{z_1 + z_0}}{z_1} \times_0 \times_1 \quad \text{det}$$
Hypothesis:

- More than two sensors
  - Positions with uncertainty $\sigma_{\text{det}}$
  - Infinitely thin
- 1 straight tracks
  - 2 parameters ($a, b$)

Estimation of track parameters

- Assuming track model is straight
  - Need FITTING PROCEDURE least square
  - Need covariance matrix of measurements (here diagonal)
- Uncertainties from error propagation
  - Detail depends on geometry
  - Both estimation & uncertainties improve

$$a = \frac{S_1 S_{xz}}{S_1 S_{z^2}} \frac{S_x S_z}{(S_z)^2}, \quad b = \frac{S_x S_{z^2}}{S_1 S_{z^2}} \frac{S_z S_{xz}}{(S_z)^2}$$

$$\sigma_a^2 = \frac{S_1}{S_1 S_{z^2}} \frac{S_z}{(S_z)^2}, \quad \sigma_b^2 = \frac{S_z^2}{S_1 S_{z^2}} \frac{S_z S_{xz}}{(S_z)^2}$$

$$\text{COV}_{a,b} = \frac{S_z}{S_1 S_{z^2}} \frac{S_z S_{xz}}{(S_z)^2}$$

See LSM on straight tracks later
What are we talking about?

**Hypothesis:**

- More than two sensors
  - Positions with uncertainty $\sigma_{det}$
  - With some THICKNESS $\rightarrow$ physics effect
- 1 straight tracks
  - 2 parameters $(a, b)$

**Estimation of track parameters**

- Assuming track model is straight
  - Need fitting procedure least square
  - Need covariance matrix of measurements $\rightarrow$ NON DIAGONAL terms
- Uncertainties from error propagation
  - same estimators but increased uncertainties

What are we talking about?

\[
a = \frac{S_1 S_{xz}}{S_1 S_{z^2}} \frac{S_x S_z}{(S_z)^2}, \quad b = \frac{S_x S_{z^2}}{S_1 S_{z^2}} \frac{S_z S_{xz}}{(S_z)^2}
\]

**Complex covariant matrix expression**

- correlation between sensors
- Various implementations possible
What are we talking about?

**Hypothesis:**
- More than two sensors
  - Positions with uncertainty $\sigma_{\text{det}}$
  - With some thickness
- MANY straight tracks
  - Still 2 parameters (a,b)...per track!
  - But may change along track path

**New step = FINDING**
- Which hits to which tracks ?
  - Strongly depends on geometry

**Estimation of track parameters**
- Happens after finder
- Uncertainties involve correlation
Lecture outline

1. Basic concepts
2. Position sensitive detectors
3. Standard algorithms
4. Advanced algorithms
5. Optimizing a tracking system
6. References
1. Motivations & basic concepts

- Motivations
- Types of measurements
- The 2 main tasks
- Environmental considerations
- Figures of merit
1. Motivations & Basic Concepts

- Understanding an event
  - Individualize tracks \( \approx \) particles
  - Measure their properties
  - LHC: \( \sim 1000 \) particles per 25 ns “event”

- Track properties
  - **Momentum** \( \Leftrightarrow \) curvature in B field
    - Reconstruct invariant masses
    - Contribute to jet energy estimation
  - **Energy** \( \Leftrightarrow \) range measurement
    - Limited to low penetrating particle
  - **Mass** \( \Leftrightarrow \) dE/dx measurement
  - **Origin** \( \Leftrightarrow \) vertexing (connecting track)
    - Identify decays
    - Measure flight distance
  - **Extension** \( \Leftrightarrow \) particle flow algorithm (pfa)
    - Association with calorimetric shower
1. Motivations & Basic Concepts

Magnetic field curves trajectories
\[ \frac{d\vec{p}}{dt} = q \vec{v} \cdot \vec{B} \]
- Rewritten with position \((x)\) and path length \((l)\) → basic equation:
- In \(B=4T\) a 10 GeV/c particle will get a sagitta of 1.5 cm @ 1m

Fixed-target experiments
- Dipole magnet on a restricted path segment
- Measurement of deflection (angle variation)

Collider experiment
- Barrel-type with axial \(B\) over the whole path
- Measurement of curvature (sagitta)
\[ \frac{p_T}{q} = \frac{0.3 \cdot B(T)L}{\Delta \alpha} \]

Other arrangements
- Toroidal \(B\)... not covered

Two consequences
- Position sensitive detectors needed
- Perturbation effects on trajectories limit precision on track parameters
1. Motivations & Basic Concepts

Identifying through topology

- Short-lived weakly decaying particles
  - Charm $c\tau \sim 120$ $\mu$m
  - Beauty $c\tau \sim 470$ $\mu$m
  - tau, strange/charmed/beauty particle

Exclusive reconstruction

- Decay topology with secondary vertex
- Exclusive = all particles associated

Inclusive “kink” reconstruction

- Some particles are invisible ($\nu$)
1. Motivations & Basic Concepts

- **Inclusive reconstruction**
  - Selecting parts of the daughter particles = flavor tagging
  - based on impact parameter (IP)
  - $\sigma_{IP} \sim 20\text{-}100 \, \mu m$ requested

- **Definition of impact parameter (IP)**
  - Also DCA = distance of closest approach from the trajectory to the primary vertex
  - Full 3D or 2D (transverse plane $d_{\rho}$) +1D (beam axis)
  - Sign extremely useful for flavor-tagging

Sign defined by charge + traj. Position /VP

Sign defined by angle dca / jet momentum
Finding the event origin

- Where did the collision did occur?
  - Primary vertex
- (life)Time dependent measurements
  - CP-asymmetries @ B factories ($\Delta z \approx 60-120 \mu m$)
- Case of multiple collisions / event
  - $\gg 10$ vertex @ LHC

Remarks for collider

- Usually no measurement below 1-2 cm / primary vertex
  - Due to beam-pipe maintaining vacuum
- Requires extrapolation $\Rightarrow$ expect “unreducible” uncertainties
1. Motivations & Basic Concepts

- **Usually not a tracker task**
  - CALORIMETERs (see lecture by Isabelle)
  - Indeed calorimeters gather material to stop particles while trackers try to avoid material (multiple scattering)
  - however...calorimetry tries to improve granularity

- **Particle flow algorithm**
  - Colliders (pp and ee)

- **Energy evaluation by counting particles**
  - Clearly heretic for calorimetry experts
  - Requires to separate $E_{\text{deposit}}$ in dense environment

- **Range measurement for low energy particles**
  - Stack of tracking layers
  - Modern version of nuclear emulsion

NOT COVERED
1. Motivations & Basic Concepts

- Reminder on the physics (see other courses)
  - Coulomb scattering mostly on nuclei
  - Molière theory description as a centered gaussian process
    - the thinner the material, the less true → large tails

- In-plane description (defined by vectors \( p_{\text{in}}, p_{\text{out}} \))
  - Corresponds to \((\phi, \theta)\) with \( p_{\text{in}} = p_z \) and
    \[
    p_{\text{out}}^2 = p_{\text{out},z}^2 + p_{\text{out},T}^2 \Rightarrow \begin{cases}
    p_{\text{out}} \cos \theta = p_{\text{out},z} \\
    p_{\text{out},T} = p_{\text{out}} \sin \theta = p_{\text{out}} \theta
    \end{cases}
    \]
    (note: \([0,2]\) uniform)

\[
q = 13.6 \text{ (MeV/c)} \times \frac{1}{p} \times z \times \sqrt{\frac{\text{thickness}}{X_0}} \left[ 1 + 0.038 \ln \left( \frac{\text{thickness}}{X_0} \right) \right]
\]

\( X_0 = \text{radiation length} \)

Same definition as in calorimetry
... though this is accidental
1. Motivations & Basic Concepts

**In-space description (defined by fixed x/y axes)**

→ Corresponds to \((\theta_x, \theta_y)\) with

\[
\begin{aligned}
p_{\text{out, } T}^2 &= p_{\text{out, } x}^2 + p_{\text{out, } y}^2 \\
p_{\text{out}} \sin \theta_x &\approx p_{\text{out}} \theta_x \\
p_{\text{out}} \sin \theta_y &\approx p_{\text{out}} \theta_y
\end{aligned}
\]

\[\theta_x \text{ and } \theta_y \text{ are independent gaussian processes}\]

\[x = y = \frac{\sqrt{2}}{2}\]

\[
\begin{aligned}
q^2 &= q_x^2 + q_y^2 \\
\sqrt{q^2} &= \sqrt{q_x^2} + \sqrt{q_y^2} \text{ and } \sqrt{q_x} = \sqrt{q_y} = \sqrt{q}
\end{aligned}
\]

\[\theta \in [p_{\text{out, } T}, p_{\text{out}}] \text{ plane}
\]

\[\phi \in [p_{\text{out, } x}, p_{\text{out, } T}] \text{ plane}
\]

\[\theta_x \in [p_{\text{out}}, p_{\text{out, } x}] \text{ plane}
\]

\[\theta_y \in [p_{\text{out}}, p_{\text{out, } y}] \text{ plane}
\]
Important remark when combining materials

- Total thickness $T = \sum T_i$, each material $(i)$ with $X_0(i)$

- Definition of effective radiation length $\rightarrow$

- Consider single gaussian process

and never do variance addition (which minimize deviation)
1. Motivations & Basic Concepts

Impact on tracking algorithm
- The track parameters evolves along the track!
- May drive choice of reconstruction method

Photon conversion
- Alternative definition of radiation length
  probability for a high-energy photon to generate a pair over a path \( dx \):
  \[
  \gamma \rightarrow e^+e^- = \text{conversion vertex}
  \]
- Generate troubles:
  - Additional unwanted tracks
  - Decrease statistics for electromagnetic calorimeter

Remember this simple case

\[
\text{Prob} = \frac{dx}{9} \frac{7}{X_0}
\]

CMS “picture” of material budget through photon conversion vertices (silicon tracker only)
1. Motivations & Basic Concepts

The collider paradigm

- Basic inputs from detectors
  - Succession of 2D or 3D points (or track segments)
    ➔ Who’s who?

- 2 steps process
  - Step 1: track identification = finding = pattern recognition
    • Associating a set of points to a track
  - Step 2: track fitting
    • Estimating trajectory parameters ➔ momentum

- Both steps require
  - Track model (signal, background)
  - Knowledge of measurement uncertainties
  - Knowledge of materials traversed (Eloss, mult. scattering)

- Vertexing needs same 2 steps
  - Identifying tracks belonging to same vertex
  - Estimating vertex properties (position + 4-vector)
Telescope mode
- Single particle at a time
  - Sole nuisance = noise
- Trigger from beam
  - Often synchronous
- Goal = get the incoming direction

The astroparticle way
- Similar to telescope mode
- No synchronous timing
- Ex: deep-water $\nu$ telescopes

=> For 2 last cases: mostly a fitting problem
- Usually with straight track model
Life in a real experiment is tough (for detectors of course)

- Chasing small cross-sections ➔ large luminosity and/or energy
- Short interval between beam crossing
  - LHC: 25 ns (and >10 collisions / crossing)
  - CLIC: 5 ns (but not continuous)
- Large amount of particles (could be > $10^7$ part/cm$^2$/s) ➔ background, radiation
  - makes the finding more complicated
- Vacuum could be required (space, very low momentum particles (CBM, LHCb))

Radiation tolerance

- Two types of energy loss
  - Ionizing (generate charges): dose in Gy = 100 Rad
  - Non-ionizing (generate defects in solid): fluence in $n_{eq}(1MeV)/cm^2$
- The more inner the detection layer, the harder the radiation (radius$^2$ effect)
- Examples for most inner layers:
  - LHC: $10^{15}$ to $<10^{17}$ $n_{eq}(1MeV)/cm^2$ with 50 to 1 MGy
  - ILC: $<10^{12}$ $n_{eq}(1MeV)/cm^2$ with 5 kGy
1. Motivations & Basic Concepts:

Environmental conditions – 2/2

○ Timing consideration
  - Integration time drives occupancy level (important for finding algorithm)
  - Time resolution offers time-stamping of tracks
    - Tracks in one “acquisition event” could be associated to their proper collision event if several have piled-up
  - Key question = triggered or not-triggered experiment?

○ Heat concerns
  - Spatial resolution → segmentation → many channels
  - Readout speed → power dissipation/channel
  - Efficient cooling techniques exist BUT add material budget and may not work everywhere (space)

○ Summary
  - Tracker technology driven by environmental conditions: hadron colliders (LHC)
  - Tracker technology driven by physics performances: lepton colliders (B factories, ILC), heavy-ion colliders (RHIC, LHC)
  - Of course, some intermediate cases: superB factories, CLIC
1. Motivations & Basic Concepts:

- For detection layer
  - Detection efficiency
    - Mostly driven by Signal/Noise
    - **Note:** Noise = signal fluctuation $\oplus$ readout (electronic) noise
  - Intrinsic spatial resolution
    - Driven by segmentation (not only)
    - Useful tracking domain $\sigma < 1\text{mm}$
  - Linearity and resolution on dE/dx
  - Material budget
  - “Speed” (integration time, time resolution, ...)

- For detection systems (multi-layers)
  - Track finding & purity
  - Two-track resolution
    - Ability to distinguish two nearby trajectories
    - Mostly governed by signal spread / segments
  - Momentum resolution $\frac{\langle p \rangle}{p}$
  - Impact parameter resolution
    - Sometimes called “distance of closest approach” to a vertex

Figures of Merit
2. Detection technologies

- Intrinsic resolution
- Single layer systems
  - Silicon, gas sensors, scintillator
- Multi-layer systems
  - Drift chamber and TPC
- Tentative simplistic comparison
- Magnets
- Practical considerations
- Leftovers
1. Motivations & Basic Concepts:

- Position measurement comes from segmentation
  - Pitch

- Digital resolution
  \[ \text{pitch} = \frac{\text{pitch}}{\sqrt{12}} \]

- Improvement from signal sharing
  - Position = charge center of gravity
  - Effects generated by
    - Secondary charges spread inside volume
    - Inclined tracks (however, resol. limited at large angles)
  - Potential optimization of segmentation / sharing
    - Work like signal sampling theory (Fourier transform)

- Warnings:
  - Lorentz force from B mimic the effect
  - counterproductive / 2-track resolution
2. Detector Technologies:

- **Basic sensitive element**
  - E-h pairs are generated by ionization in silicon
    - 3.6 eV needed
    - 300 µm thick Si generates ~22000 charges for MIP
      BUT beware of Landau fluctuation
  - Collection: P-N junction = diode
    - Full depletion (10 to 0.5 kV)
      generates a drift field (10⁴ V/cm)
    - Collect time ~ 15 ps/µm

- **Silicon strip detectors**
  - sensor “easily” manufactured with pitch down to ~25 µm
  - 1D if single sided
  - Pseudo-2D if double-sided
    - Stereo-angle useful against ambiguities
  - Difficult to go below 100 µm thickness
  - Speed and radiation hardness: LHC-grade
2. Detector Technologies:

**Concept**
- Strips $\rightarrow$ pixels on sensor
- One to one connection from electronic channels to pixels

**Performances**
- Real 2D detector & keep performances of strips
  - Can cope with LHC rate (speed & radiation)
- Pitch size limited by physical connection and #transistors for treatment
  - minimal (today): 50x50 $\mu$m$^2$
  - typical: 100x150/400 $\mu$m$^2$
  - spatial resolution about 10 $\mu$m
- Material budget
  - Minimal (today): 100(sensor)+100(elec.) $\mu$m
- Power budget: 10 $\mu$W/pixel
2. Detector Technologies:

CMOS Pixel Sensor (CPS)

Concept
- Use industrial CMOS process
  - Implement an array of sensing diode
  - Amplify the signal with transistors near the diode
- Benefit to
  - granularity: pixel pitch down to \( \sim 10 \, \mu m \)
  - material: sensitive layer thickness as low as 10-20 \( \mu m \)
- Known as Monolithic Active Pixel Sensors (MAPS)

Sensitive layer
- If undepleted & thin (10-20 \( \mu m \))
  - Slow (100 ns) thermal drift of charges
  - non-ionizing rad. tolerance \( \lesssim 10^{13} \, n_{eq(1MeV)}/cm^2 \)
- If fully depleted (from 10 to 100 \( \mu m \))
  - Fast (few ns) field-driven drift of charges
  - non-ionizing rad. tolerance \( > 10^{15} \, n_{eq(1MeV)}/cm^2 \)
2. Detector Technologies:

Concept
- Use industrial CMOS process
  - Implement an array of sensing diode
  - Amplify the signal with transistors near the diode
- Gain in granularity: pitch down to $\sim 10 \, \mu m$
- Gain in sensitive layer thickness $\sim 10-20 \, \mu m$
- For undepleted thin sensitive layer
  - Slow (100 ns) thermal drift of charges
  - non-ionizing rad. tolerance $\lesssim 10^{13} \, n_{eq}(1 \, \text{MeV})/\text{cm}^2$
- For fully depleted thin to thick sensitive layer
  - Fast (few ns) field-driven drift of charges
  - non-ionizing rad. tolerance $> 10^{15} \, n_{eq}(1 \, \text{MeV})/\text{cm}^2$

Performances
- Spatial resolution 1-10 $\mu m$ (in 2 dimensions)
- Material budget: $\lesssim 50 \, \mu m$
- Power budget: $< \mu W$/pixel
- Integration time $\approx 5-100 \, \mu s$ demonstrated
  - $\sim 1 \, \mu s$ in development
- Timestamping @ ns level in development
2. Detector Technologies:

- **DEPFET**
  - Depleted p-channel FET
  - Fully depleted sensitive layer
  - Large amplification
  - Still require some read-out circuits
    - Not fully monolithic
    - Possibly limited in read-out speed

- **Silicon On Insulator (SOI)**
  - Fully depleted sensitive layer
  - Fully monolithic
  - Electronics similar to MAPS
2. Detector Technologies:

**Basic sensitive element**
- Metallic wire, $1/r$ effect generated an avalanche
- Signal depends on gain (proportional mode)
  typically $10^4$
- Signal is fast, a few ns

**Gas proportional counters**
- Multi-Wire Proportional Chamber
  - Array of wires
  - 1 or 2D positioning depending on readout
  - Wire spacing (pitch) limited to 1-2 mm
- Straw or drift tube
  - One wire in One tube
  - Extremely fast (compared to Drift Chamber)
  - Handle high rate
  - Spatial resolution <200 µm
  - Left/right ambiguity

Electric fields line around anode wires
2. Detector Technologies:

Micro-pattern gas multipliers

- MSGC
  - Replace wires with lithography micro-structures
  - Smaller anodes pitch 100-200 µm
  - BUT Ageing difficulties due to high voltage and manufacturing not so easy

- GEM
  - Gain $10^5$
  - Hit rate $10^6$ Hz/cm²
2. Detector Technologies:

Micro-pattern gas multipliers

- MSGC
  - Replace wires with lithography micro-structures
  - Smaller anodes pitch 100-200 µm
  - BUT Ageing difficulties due to high voltage and manufacturing not so easy

- GEM
  - Gain $10^5$
  - Hit rate $10^6$ Hz/cm²

- MICROMEGAS
  - Even smaller distance anode-grid
  - Hit rate $10^9$ Hz/cm²

- More development
  - Electron emitting foil working in vacuum!
2. Detector Technologies:

- **Basic principle**
  - Mix field and anode wires
    - Generate a drift
  - Pressurize gas to increase charge velocity (few atm)
  - 3D detector
    - 2D from wire position
    - 1D from charge sharing at both ends

- **Spatial Resolution**
  - Related to drift path
    \[ \mu \sqrt{\text{drift length}} \]
  - Typically 100-200 µm

- **Remarks**
  - Could not go to very small radius
2. Detector Technologies:

**Benefits**
- Large volume available
- Multi-task: tracking + Part. Identification

**Basic operation principle**
- Gas ionization $\rightarrow$ charges
- Electric field $\rightarrow$ charge drift along straight path
- Information collected
  - 2D position of charges at end-cap
  - 3rd dimension from drift time
  - Energy deposited from #charges
- Different shapes:
  - rectangles (ICARUS)
  - Cylinders (colliders)
  - Volumes can be small or very large
2. Detector Technologies:

End cap readout
- Gas proportional counters
  - Wires+pads, GEM, Micromegas

Performances
- Two-track resolution $\sim 1$cm
- Transverse spatial resolution $\sim 100 - 200 \mu$m
- Longitudinal spatial resolution $\sim 0.2 - 1$ mm
- Longitudinal drift velocity: 5 to 7 cm/µs
  - ALICE TPC (5m long): 92 µs drift time

Pro
- Nice continuously spaced points along trajectory
- Minimal multiple scattering (inside the vessel)

Cons
- Limiting usage with respect to collision rate
Conclusion on technologies

Tentative Comparison

- Faster collision rates and higher particle multiplicities favour:
  - Fast silicon sensors and micro-pattern gas chambers
  - Pixelisation
  - Still large gas ensemble for
    - BelleII (SuperKEKB) -> CDC and ILD (ILC) -> TPC

Trend
2. Detector Technologies:

**Solenoid**
- Field depends on current I, length L, # turns N
  - on the axis \[ B = \frac{\mu_0 NI}{\sqrt{L^2 + 4R^2}} \]
  - Typically: 1 T needs 4 to 8 kA
    ➔ superconducting metal to limit heat
- Field uniformity needs flux return (iron structure)
  - Mapping is required for fitting (remember \( B(x) \)?)
  - Usually performed with numerical integration
- Calorimetry outside ➔ limited material ➔ superconducting
- Fringe field calls for compensation

**Supercondiction**
- cryo-operation ➔ quenching possible!
- Magnetic field induces energy: \( E \mu B^2 R^2 L \)
  - Cold mass necessary to dissipate heat in case of quench

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From a detection principle to a detector

- Build large size or many elements
  - Manufacture infrastructures
  - Characterization capabilities
  - Production monitoring
  - New monolithic silicon pixel detector tend to replace silicon strip technology

- Integration in the experiment
  - Mechanical support
  - Electrical services (powering & data transmission)
  - Cooling (signal treatment dissipates power)

- Specific to trackers
  - Internal parts of multi-detectors experiment → limited space
  - Material budget is ALWAYS a concern
  - ⇒ trade-offs required
2. Detector Technologies:

- **Silicon drift detectors**
  - Real 2D detectors made of strips
  - 1D is given by drift time

- **Diamond detectors**
  - Could replace silicon for hybrid pixel detectors
  - Very interesting for radiation tolerance

- **Plasma sensor panels**
  - Derived from flat television screen
  - Still in development

- **Charge Coupled Devices (CCD)**
  - Fragile/ radiation tolerance

- **Signal generation**
  - see Ramo’s theorem

- **Nuclear emulsions**
  - One of the most precise ~ 1µm
  - No timing information → very specific applications

- **Scintillators**
  - Extremely fast (100 ps)
  - Could be arranged like straw tubes
  - But quite thick ($X_0 \sim 2$ cm)
3. Standard algorithms

- Finders
- First evaluation of momentum resolution
- Fitters
- Alignment
3. Standard algorithms:

**Global methods**
- Transform the coordinate space into *pattern space*
  - “pattern” = parameters used in track model
- Identify the “best” solutions in the new phase space
- Use all points at a time
  - No history effect
- Well adapted to evenly distributed points with same accuracy

**Local methods**
- Start with a track seed = restricted set of points
  - Could require good accuracy from the beginning
- Then extrapolate to next layer-point
  - And so on... *iterative procedure*
- “Wrong” solutions discarded at each iteration
- Possibly sensitive to “starting point”
- Well adapted to redundant information
### A simple example
- Straight line in 2D: model is \( x = a*z + b \)
- Track parameters \((a, b)\); N measurements \(x_i\) at \(z_i\) \((i=1..N)\)

### A more complex example
- Helix in 3D with magnetic field
- Track parameters \((\gamma_0, z_0, D, \tan\lambda, \text{C}=\text{R})\)
- Measurements \((r, \varphi, z)\)

### Generalization
- Parameters: \(P\)-vector \(p\)
- Measurements: \(N\)-vector \(c\)
- Model: function \(f(\mathcal{R}^P \rightarrow \mathcal{R}^N)\)

\[
\begin{align*}
\varphi(r) &= \gamma_0 + \sin \left( \frac{Cr (1 + CD)D/r}{1 + 2CD} \right) \\
z(r) &= z_0 + \frac{\tan\lambda}{C} \sin \left( C \sqrt{\frac{r^2 - D^2}{1 + 2CD}} \right)
\end{align*}
\]
Another view of the helix

- $s =$ track length
- $h =$ sense of rotation
- $\lambda =$ dip angle
- Pivot point ($s=0$):
  - position $(x_0, y_0, z_0)$
  - orientation $\phi_0$

\[
x(s) = x_0 + R \left[ \cos \left( \Phi_0 + \frac{hs \cos \lambda}{R} \right) - \cos \Phi_0 \right]
\]
\[
y(s) = y_0 + R \left[ \sin \left( \Phi_0 + \frac{hs \cos \lambda}{R} \right) - \sin \Phi_0 \right]
\]
\[
z(s) = z_0 + s \sin \lambda
\]
3. Standard algorithms:

- **Track seed = initial segment**
  - Made of few (2 to 4) points
    - One point could be the expected primary vtx
  - Allows to initialize parameter for track model
  - Choose *most precise* layers first
    - usually inner layers
  - But if high hit density
    - Start farther from primary interaction @ *lowest density*
    - Limit mixing points from different tracks

- **Extrapolation step**
  - Out or inward (=toward primary vtx) onto the next layer
  - Not necessarily very precise, especially *only local model* needed
    - Extrapolation uncertainty \( \leq \) layer point uncertainty
    - Computation speed important
  - Match (associate) nearest point on the new layer
    - Might skip the layer if point missing
    - Might reject a point: if worst track-fit or if fits better with another track
3. Standard algorithms:

- **Variant with track segments**
  - First build “tracklets” on natural segments
    - Sub-d Detectors, or subparts with same resolution
  - Then match segments together
  - Typical application:
    - Segments large tracker (TPC) with vertex detector (Si)
      - layers dedicated to matching

- **Variant with track roads**
  - Full track model used from start

- **Variant with Kalman filter**
  - See later

- **Figure of merit**
  - $\sigma_{\text{eff}} = \sigma(\text{sensor}) \oplus \sigma(\text{track extrapolation}) = \text{effective spatial resolution}$
  - $\rho = \text{background hit density}$
3. Standard algorithms:

- **Brute force = combinatorial way**
  - Consider all possible combination of points to make a track
  - Keep only those compatible with model
  - Usually too time consuming...

- **Hough transform**
  - Example straight track:
    - Coord. space $y = a^*x + b \iff$ pattern space $b = y - x^*a$
    - Each point $(y,x)$ defines a line in pattern space
    - All lines, from points belonging to same straight-track, cross at same point $(a,b)$
    - In practice:
      discretize pattern space and search for maximum
  - Applicable to circle finder
    - needs two parameters as well ($r, \varphi$ of center)
      if track is assumed to originate from $(0,0)$
  - More difficult for more than 2 parameters...
3. Standard algorithms:

Conformal mapping

- Helix transverse projection = Circle
  - \((x-a)^2 + (y-b)^2 = r^2\)
  - Transform to \(u = x/(x^2+y^2), \ v = y/(x^2+y^2)\)
  - Then: \(v = -(a/b) \ u + (1/2b)\)

Figure of merit

\[(sensor)_z (sensor)_{bckgrnd}\]
3. Standard algorithms:

Why do we need to fit?

- Measurement error
- Multiple scattering error

Global fit

- Assume knowledge of:
  - all track points
  - full correlation matrix
    - difficult if $\sigma_{\text{mult. scatt.}} \gtrapprox \sigma_{\text{meas.}}$
- Least square method

Iterative fit

- Iterative process:
  - points included in the fit one by one
  - could be merged with finder step
- Kalman filter

FITTING drives track extrapolation & momentum res.
Linear model hypothesis
- \( p \) track parameters, with \( N \) measurements \( c \)

\[
\tilde{c} = \tilde{c}_s + A(\tilde{p} - \tilde{p}_s) + \tilde{e}
\]

- \( p_s \) = known starting point (pivot), \( A \) = track model \( N \times P \) matrix, \( \varepsilon \) = error vector corresponding to \( V \) = covariance \( N \times N \) matrix

Sum of squares:
\[
\frac{\text{(model - measure)}^2}{\text{uncertainty}^2} = S(\tilde{p}) = (\tilde{c}_s + A(\tilde{p} - \tilde{p}_s) - \tilde{c})^T V^{-1} (\tilde{c}_s + A(\tilde{p} - \tilde{p}_s) - \tilde{c})
\]

Best estimator (minimizing variance)
\[
\frac{dS}{dp}(\tilde{p}) = 0 \quad \Rightarrow \quad \tilde{p} = \tilde{p}_s + (A^T V^{-1} A)^{-1} A^T V^{-1} (\tilde{c} - \tilde{c}_s)
\]

- Variance (= uncertainty) of the estimator:

\[
V_{\tilde{p}} = (A^T V^{-1} A)^{-1}
\]

- Estimator \( \tilde{p} \) follows a \( \chi^2 \) law with \( N - P \) degrees of freedom

Problem \( \Leftrightarrow \) inversion of a \( P \times P \) matrix \( (A^T V^{-1} A) \)
- But real difficulty could be computing \( V \) (\( N \times N \) matrix)

\( \Leftrightarrow \) layer correlations if multiple scattering non-negligible if \( \sigma_{\text{mult. scatt.}} \geq \sigma_{\text{meas}} \)
3. Standard algorithms:

- **Straight line model**
  - 2D case \( \rightarrow \) D=2 coordinates \((z,x)\)
  - 2 parameters: \( a = \text{slope}, \ b = \text{intercept at } z=0 \)

- **General case**
  - \( K+1 \) detection planes \((i=0\ldots k)\)
    - located at \( z_i \)
    - Spatial resolution \( \sigma_i \)
  - Useful definitions
    - \( S_1 = \sum_{i=0}^{K} \frac{1}{2}, \ S_z = \sum_{i=0}^{K} \frac{z_i}{2}, \ S_{xz} = \sum_{i=0}^{K} \frac{x_i z_i}{2}, \ S_z^2 = \sum_{i=0}^{K} \frac{z_i^2}{2} \)
  - Solutions
    - \( a = \frac{S_1 S_{xz}}{S_1 S_z^2 (S_z^2)^2}, \ b = \frac{S_z S_{xz}}{S_1 S_z^2 (S_z^2)^2} \)
  - Uncertainties
    - \( a^2 = \frac{S_1^2}{S_1 S_z^2 (S_z^2)^2}, \ b^2 = \frac{S_z^2}{S_1 S_z^2 (S_z^2)^2} \)
    - \( \text{correlation } \text{cov}_{a,b} = \frac{S_z}{S_1 S_z^2 (S_z^2)^2} \)

- **Case of uniformly distributed \((K+1)\) planes**
  - \( z_{i+1} - z_i = L/K \) et \( \sigma_i = \sigma \) \( \forall i \)
  - \( S_z = 0 \) \( \rightarrow \) \( a, b \) uncorrelated
    - \( a^2 = \frac{12K}{(K+2)L^2} K^2, \ b^2 = \left(1+12 \frac{K}{K+2} \frac{z_c^2}{L^2}\right)^2 \frac{1}{K+1} \)
  - Uncertainties:
    - \( \sigma_a \) and \( \sigma_b \) improve with \( 1/\sqrt{K+1} \)
    - \( \sigma_a \) and \( \sigma_b \) improve with \( 1/L \)
    - \( \sigma_b \) improve with \( z_c \)
3. Standard algorithms:

**Hypothesis**
- K detectors, each with $\sigma$ single point accuracy
- Uniform field over $L$ from dipole
  - Trajectory: $\Delta \alpha = \frac{0.3qBL}{p}$
  - Bending: $\Delta p = p \Delta \alpha$
- Geometrical arrangement optimized for resolution
  - Angular determination on input and output angle: $\left(\frac{2}{K} \frac{l^2}{16}\right)^2$

**Without multiple scattering**
- Uncertainty on momentum
  $$\frac{p}{p} = \frac{8}{1} \frac{1}{0.3q BL l \sqrt{K}} p$$
- Note proportionality to $p$!

**Multiple scattering contribution**
- Additional term on $\sigma_\alpha$ almost directly from smult.scatt
  $$\frac{13.6 \text{ (MeV/c)}}{p} z$$
3. Standard algorithms:

- Hypothesis
  - K detectors uniformly distributed each with $\sigma$ single point accuracy
  - Uniform field over path length $L$

- Without multiple scattering
  - Uncertainty on transverse momentum (Glückstern formula)

$$\frac{p_T}{p_T} = \frac{\sqrt{720}}{0.3q} \frac{1}{BL^2} \frac{1}{\sqrt{K + 6}} p_T$$

- Works well with large $K > 20$
3. Standard algorithms:

Dimensions
- P parameters for track model
- D “coordinates” measured at each point (usually D<P)
- K measurement points (# total measures: N = KxD)

Starting point
- Initial set of parameters: first measurements
- With large uncertainties if unknowns

Iterative method
- Propagate to next layer = prediction
  - Using the system equation \( \vec{p}_k = G \vec{p}_{k-1} + \tilde{\omega}_k \)
  - \( G = P \times P \) matrix, \( \tilde{\omega} = \) perturbation associated with covariance \( P \times P \) matrix \( V_\omega \)
  - Update the covariance matrix with additional uncertainties (ex: material budget between layers)
    \( V_{k|k-1} = V_{k-1} + V_k \)
- Add new point to update parameters and covariance, using the measure equation \( \vec{m}_k = H \vec{p}_k + \tilde{\epsilon}_k \)
  - \( H = D \times P \) matrix, \( \tilde{\epsilon} = \) measure error associated with diagonal covariance \( D \times D \) matrix \( V_m \)
  - Weighted means of prediction and measurement using variance \( \Leftrightarrow \chi^2 \) fit
- Iterate...

\[
\vec{p}_k = \left( V_{k|k-1}^{-1} \vec{p}_{k|k-1} + H^T V_m^{-1} \vec{m}_k \right) \cdot \left( V_{k|k-1}^{-1} + H^T V_m^{-1} H \right)^{-1}
\]
3. Standard algorithms:

- Forward and backward filters
  - Forward estimate of $p_k$: from $1 \rightarrow k-1$ measurements
  - Backward estimate of $p_k$: from $k+1 \rightarrow K$ measurements
  - Independent estimates $\rightarrow$ combination with weighted mean = smoother step

- Computation complexity
  - only $P \times P$, $D \times P$ or $D \times D$ matrices computation ($\ll N \times N$)

- Mixing with finder
  - After propagation step: local finder
  - Some points can be discarded if considered as outliers in the fit (use $\chi^2$ value)

- Include exogenous measurements
  - Like $dE/dx$, correlated to momentum
  - Additional measurement equation $\bar{m'}_k = H' \bar{p}_k + \bar{e'}_k$

$$\bar{p}_k = \left(V^{-1}_{klk-1} \bar{p}_{klk-1} + H^T V^{-1}_{m_k} \bar{m}_k + H'^T V^{-1}_{m'_k} \bar{m'}_k \right) \cdot \left(V^{-1}_{klk-1} + H^T V^{-1}_{m_k} H + H'^T V^{-1}_{m'_k} H'\right)^{-1}$$
Let’s come back to one initial & implicit hypothesis

- “We know were the point are located.”
- True to the extent we know were the detector is!
- BUT, mechanical instability (magnetic field, temperature, air flow...) and also drift speed variation (temperature, pressure, field inhomogeneity...) limit our knowledge
- Periodic determination of positions and deformations needed = alignment

Note hit position relative to detector are the same tracks reconstructed are not even close to reality...
3. Standard algorithms:

Alignment parameters
- Track model depends on additional “free” parameters, i.e. the sensor positions

Methods
- Global alignment:
  - Fit the new params. to minimize the overall $\chi^2$ of a set of tracks (Millepede algo.)
  - Beware: many parameters could be involved (few $10^3$ can easily be reached)

- Local alignment:
  - Use tracks reconstructed with reference detectors
  - Align other detectors by minimizing the “residual” (track-hit distance) width

For both cases
- Use a set of well know tracks and tracking-”friendly” environment to avoid bias
  - Muons (very traversing) and no magnetic field
  - Low multiplicity events
4. Advanced methods
(brief illustrations)

- Why?
- Neural network
- Cellular automaton
4. Advanced methods

Shall we do better?
- Higher track/vertex density, less efficient the classical method
- Allows for many options and best choice

Adaptive features
- **Dynamic change** of track parameters during finding/fitting
- Measurements are weighted according to their uncertainty
  - Allows to take into account several “normally excluded” info
- Many hypothesis are handled simultaneously
  - But their number decrease with iterations (annealing like behavior)
- Non-linearity
- Often CPU-time costly (is that still a problem?)

Examples
- Neural network, Elastic nets, Gaussian-sum filters, Deterministic annealing, Cellular automaton
Cellular automaton

- Initialization
  - built any cell (= segment of 2 points)
- Iterative step
  - associate neighbour cells (more inner)
  - Raise “state” with associated cells
  - Kill lowest state cells

J. Lettenbichler et al., 2013

0 (black), 1 (red), 2 (orange), 3 (green), 4 (cyan)
5. Deconstructing some tracking systems

- CMS (colliders)
- AMS, ANTARES (telescopes)
5. Some tracking systems:

- **Superconducting Coil**, 4 Tesla
- **Calorimeters**
  - ECAL 76k scintillating PbWO4 crystals
  - HCAL Plastic scintillator/brass sandwich
- **Tracker**
  - Pixels
  - Silicon Microstrips
  - 210 m² of silicon sensors
  - 9.6 M channels
- **Muon Barrel**
  - Drift Tube Chambers (DT)
  - Resistive Plate Chambers (RPC)
- **Muon Endcaps**
  - Cathode Strip Chambers (CSC)
  - Resistive Plate Chambers (RPC)

**Dimensions**
- Total weight: 12500 t
- Overall diameter: 15 m
- Overall length: 21.6 m

2900 scientists from 182 institutes from 38 countries
5. Some tracking systems:

- The trackers
5. Some tracking systems:

- Alignment residual width
5. Some tracking systems:

- Taking a picture of the material budget
  - Using secondary vertices from $\gamma \rightarrow e^+e^-$

- Measuring it by data/simulation comparison
5. Some tracking systems:

- Tracking algorithm = multi-iteration process
5. Some tracking systems:

- Tracking efficiency

![Graph showing tracking efficiency]
5. Some tracking systems:

- Tracking efficiency
  - Single, isolated muons
5. Some tracking systems:

- Tracking efficiency
  - All pions

- Graph showing CMS simulation efficiency vs. $\eta$ (left) and $p_T$ (right) for different pion momenta and regions.
5. Some tracking systems:

- Tracking purity
  - All pions

![Graph showing tracking purity for different pion energies and regions.](image)
5. Some tracking systems:

- Tracking resolution

$d_0 = \text{transverse impact parameter}$
5. Some tracking systems:

- Tracking resolution

ALICE figure
5. Some tracking systems:

Impact parameter resolution

\[ \sigma_{ip} \propto \sqrt{\frac{R_{ext}^2 \sigma_{int}^2 + R_{int}^2 \sigma_{ext}^2}{R_{ext} - R_{int}}} + \frac{R_{int} \sigma_{\theta(int)}}{p \sin^{3/2}(\theta)} \]
5. Some tracking systems:

AMS: A TeV precision, multipurpose particle physics spectrometer in space.

- **TRD**
  - Identify $e^+$, $e^-$

- **Silicon Tracker**
  - $Z$, $P$

- **ECAL**
  - $E$ of $e^+$, $e^-$, $\gamma$

- **TOF**
  - $Z$, $E$

- **Magnet**
  - $\pm Z$

- **RICH**
  - $Z$, $E$

Particles and nuclei are defined by their charge ($Z$) and energy ($E \sim P$).

$Z$, $P$ are measured independently by the Tracker, RICH, TOF and ECAL.
5. Some tracking systems:

Fig. 5. The effective position resolution (weighted average of two Gaussian widths) in the y-coordinate for different inclination angles (top), the Maximum Detectable Rigidity (MDR, 100% rigidity measurement error) as a function of the inclination angle estimated for 1 TV proton incidence with the simulation (middle), and the inclination angle distribution in the geometric acceptance of the tracker (bottom).
5. Some tracking systems:
Summary

Fundamental characteristics of any tracking & vertexing device:
- (efficiency), granularity, material budget, power dissipation, “timing”, radiation tolerance
- All those figures are intricated: each technology has its own limits

Many technologies available
- None is adapted to all projects (physics + environment choose, in principle)
- Developments are ongoing for upgrades & future experiments
  - Goal is to extent limits of each techno. → convergence to a single one?

Reconstruction algorithms
- Enormous boost (variety and performances) in the last 10 years
- Each tracking system has its optimal algorithm

Development trend
- Always higher hit rates call for more data reduction
- Tracking info in trigger → high quality online tracking/vertexing

Link with:
- PID: obvious with TPC, TRD, topological reco.
- Calorimetry: Particle flow algorithm, granular calo. using position sensors
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Reconstruction algorithm & fit


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Was not discussed

- Particle interaction with matter
- The readout electronics
- Cooling systems
- The magnets to produce the mandatory magnetic field for momentum measurement
- Vertexing
Backups:

OPAL drift chamber
Backups:

ALICE - TPC

ALICE
(ALICE) TPC $dE/dx$
ICARUS - TPC

Backups:
NA-50 fixed target

Backups:
Backups:

ATLAS tracking setup

- Barrel semiconductor tracker
- Pixel detectors
- Barrel transition radiation tracker
- End-cap transition radiation tracker
- End-cap semiconductor tracker
Backups:

ATLAS tracking setup

[Diagram showing ATLAS tracking setup with labels for Solenoid coil, TRT(barrel), SCT(barrel), TRT(end-cap), SCT(end-cap), Pixel, ID end-plate, Cryostat, Pixel support tube, Pixel PP1, Beam-pipe, and R parameters.]
Backups:

ALICE setup

1. L3 MAGNET
2. HMPID
3. TOF
4. DIPOL MAGNET
5. MUON FILTER
6. TRACKING CHAMBERS
7. TRIGGER CHAMBERS
8. ABSORBER
9. TPC
10. PHOS
11. ITS
Backups:

CMS

Key:
- Muon
- Electron
- Charged Hadron (e.g. Pion)
- Neutral Hadron (e.g. Neutron)
- Photon

Transverse slice through CMS

Silicon Tracker

Electromagnetic Calorimeter

Hadron Calorimeter

Superconducting Solenoid

Iron return yoke interspersed with Muon chambers
More position sensitive detectors

Backups:

**DEPFET**

**Silicon drift**

**CCD**

**MICROMEGAS**
Was not discussed

- Particle interaction with matter
- The readout electronics
- Cooling systems
- The magnets to produce the mandatory magnetic field for momentum measurement
- Vertexing