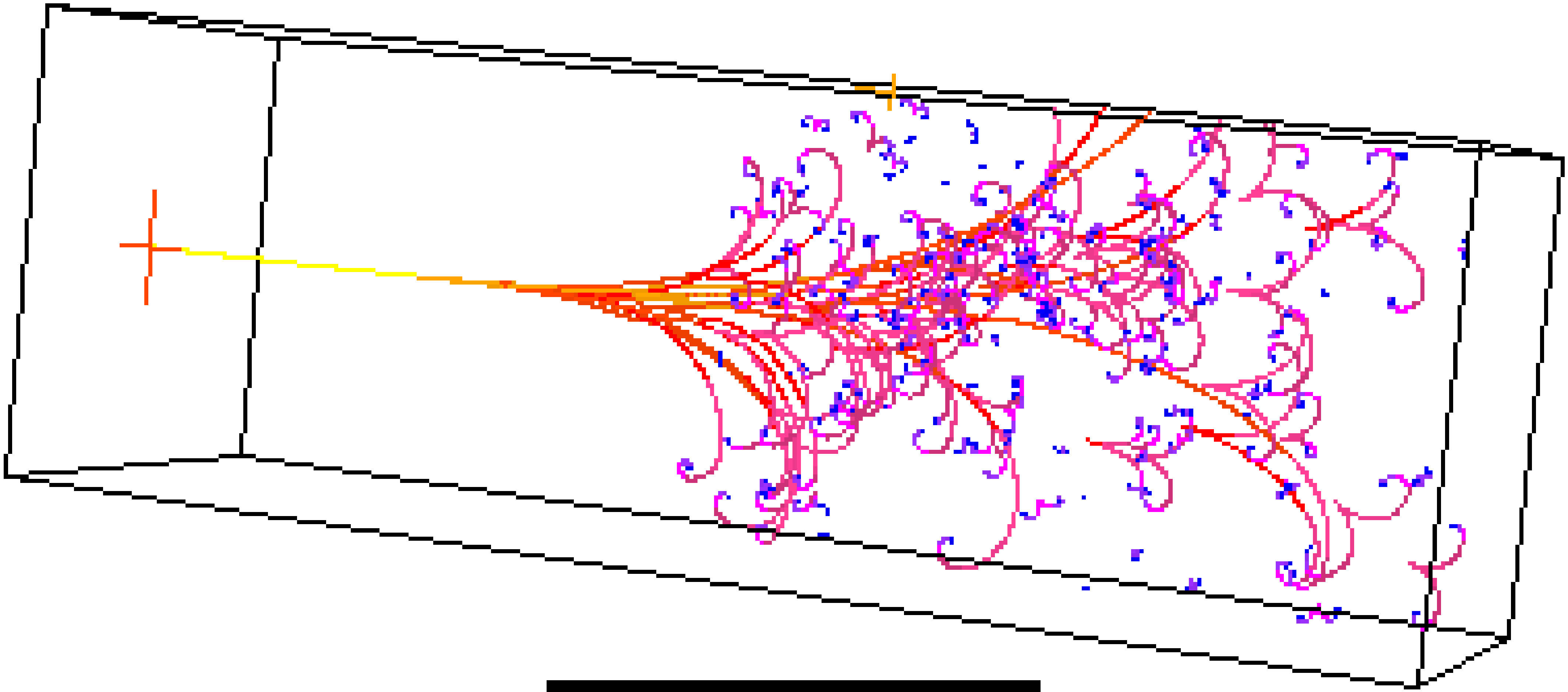
Physics of Electromagnetic Showers



1. Particles interact with matter
depends on particle and material

Glossary

Table 27.1: Summary of variables used in this section. The kinematic variables β and γ have their usual meanings.

Symbol	Definition	Units or Value
α	Fine structure constant ($e^2/4\pi\epsilon_0\hbar c$)	1/137.035 999 11(46)
M	Incident particle mass	MeV/c^2
E	Incident part. energy γMc^2	MeV
T	Kinetic energy	MeV
m_ec^2	Electron mass $\times$ c^2	0.510 998 918(44) MeV
r_e	Classical electron radius $e^2/4\pi\epsilon_0 m_ec^2$	2.817 940 325(28) fm
N_A	Avogadro's number	6.022 1415(10) $\times$ 10^{23} mol^{-1}
ze	Charge of incident particle	
Z	Atomic number of absorber	
A	Atomic mass of absorber	g mol^{-1}
K/A	$4\pi N_A r_e^2 m_e c^2/A$	0.307 075 MeV g^{-1} cm^2 for A = 1 g mol^{-1}
I	Mean excitation energy	eV (*Nota bene!*)
$\delta(\beta\gamma)$	Density effect correction to ionization energy loss	
$\hbar\omega_p$	Plasma energy ($\sqrt{4\pi N_e r_e^3}\ m_ec^2/\alpha$)	$\sqrt{\rho\,\langle Z/A\rangle}$ $\times$ 28.816 eV (ρ in g cm^{-3})
N_e	Electron density	(units of r_e)$^{-3}$
w_j	Weight fraction of the jth element in a compound or mixture	
n_j	$\propto$ number of jth kind of atoms in a compound or mixture	
—	$4\alpha r_e^2 N_A/A$	(716.408 g cm^{-2})$^{-1}$ for $A = 1$ g mol^{-1}
X_0	Radiation length	g cm^{-2}
E_c	Critical energy for electrons	MeV
$E_{\mu c}$	Critical energy for muons	GeV
E_s	Scale energy $\sqrt{4\pi/\alpha}\ m_ec^2$	21.2052 MeV
R_M	Molière radius	g cm^{-2}

Electromagnetic Showers

An ElectroMagnetic (EM) shower is a cascade of secondary electrons/positrons and photons initiated by the interaction with matter (ie, energy loss) of an incoming of electron/positron or photon.

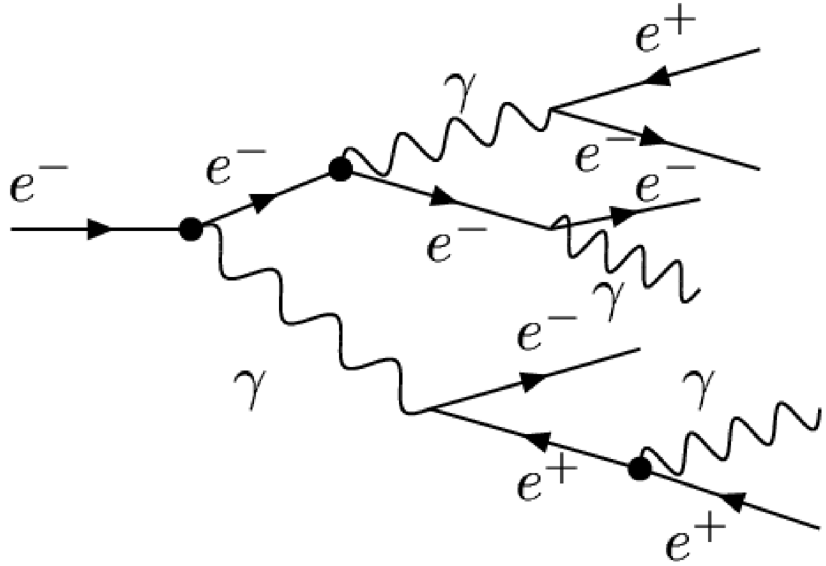
➢ The main energy loss mechanism are:

- Ionization
- Bremsstrahlung

} for e+/e-

- Compton scattering
- Pair creation
- Photo-electric effect

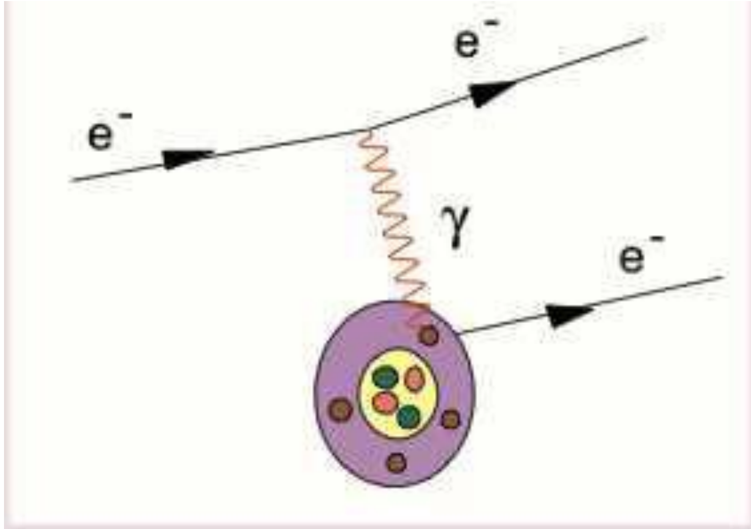
} for γ



Ionization

- Interaction of charged particles with electron cloud of atoms (loss of electrons, atoms -> ions)

- **Dominant process at low energy**



- **Bethe-Bloch formula** (general)

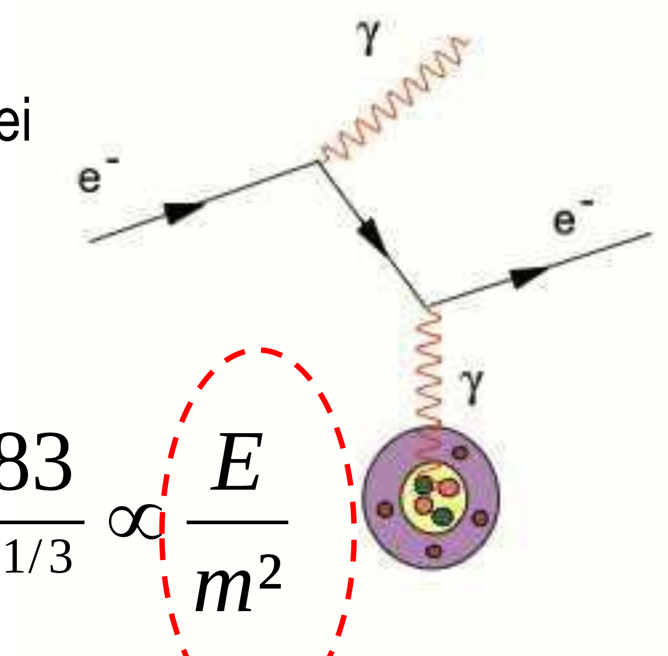
$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] \quad (\mathrm{MeV.g^{-1}.cm^2})$$

Energy loss depends:

- quadratic ally on the charge and velocity of the incident particle (but not on its mass)
- Linearly on the material (through electron density)
- Logarithmically on the material (through mean ionization I)

Bremsstrahlung

- Radiation of real photons in the Coulomb field of the atomic nuclei
- **Dominant process at high energy**



$$\left(-\frac{dE}{dx}\right)_{rad} = 4\alpha N_A \frac{Z^2}{A} z^2 \left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mc^2}\right)^2 E \ln\frac{183}{Z^{1/3}} \propto \frac{E}{m^2}$$

- Important for electrons, much less for muons (apart from ultra-relativistic)

$$\left(-\frac{dE}{dx}\right)_{rad} = 4\alpha N_A \frac{Z^2}{A} r_e^{\,2} E \ln\frac{183}{Z^{1/3}}$$ (for electrons)

Radiation length

- Conveniently re-written as: $\left(\frac{dE}{dx}\right)_{rad} = \frac{E}{X_0}$

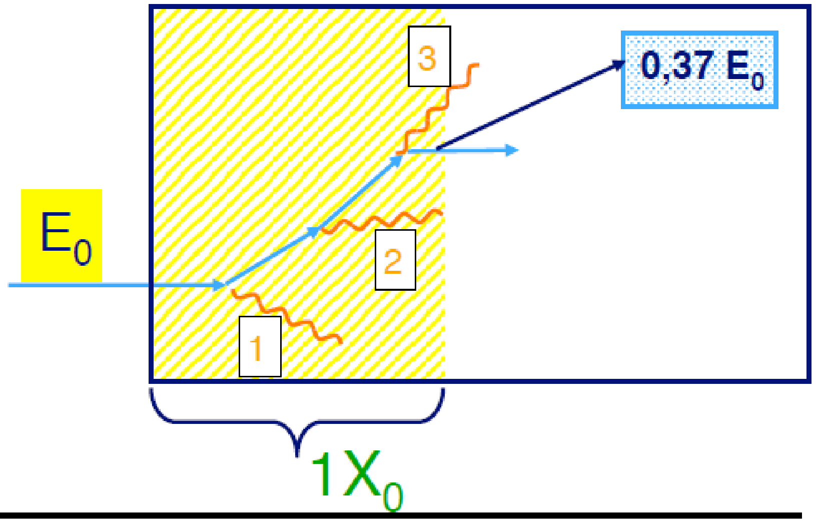
$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^{\,2} \ln\frac{183}{Z^{1/3}}}$$

Radiation Length

- **Definition:** mean distance over which the incident electron loses all BUT 1/e ≈ 37% of its incident energy via radiation (ie, it radiated ≈63% of its incident energy)

$$\left(\frac{dE}{dx}\right)_{rad} = \frac{E}{X_0} \Rightarrow \frac{dE}{E} = \frac{dx}{X_0}$$

$$\Rightarrow \boxed{E = E_0 e^{-x/X_0}}$$



- Useful approximation: $X_0 \approx \frac{180A}{Z^2}$ (g.cm^{-2}) Also in cm (taking into account density)

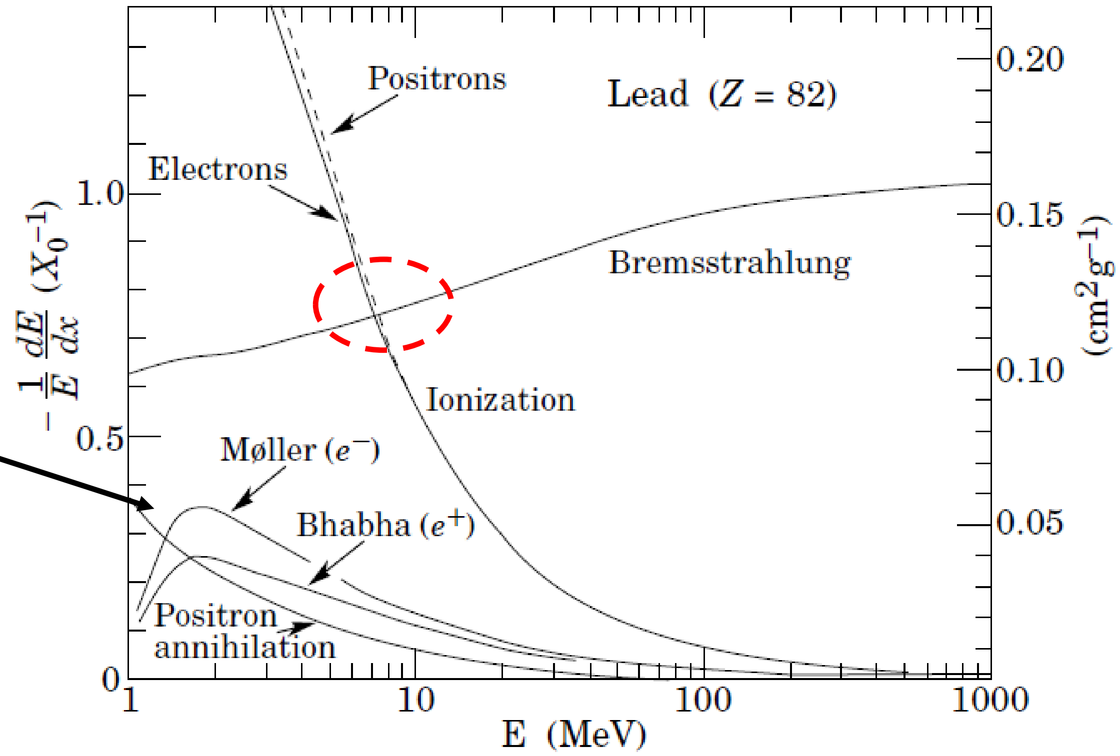
- Examples:

Material	W	Pb	Cu	Al	Stainless Steel	PbWO4	(dry) Air	(liquid) Water
Z	74	82	29	13	-	-	-	-
X_0 (cm)	0,35	0,56	1,4	8,9	1,76	0,89	30390	36,08

Critical Energy

Fractional energy loss for electrons/positrons in Lead

Other processes (Bhabha, meller, ...) neglected in HEP (most of the time)



- Radiation (ionization) dominant at high (low) energies
- Crossing point: $\left(\frac{dE}{dx}\right)_{rad}(E_C) = \left(\frac{dE}{dx}\right)_{ioniz}(E_C)$ E_C: **critical energy**

Strongly material dependent (scales as 1/Z)

- **Examples:**

Material	W	Pb	(liquid) Ar	Cu
Z	74	82	29	13
E_C(MeV)	8,4	7,1	37	20,2

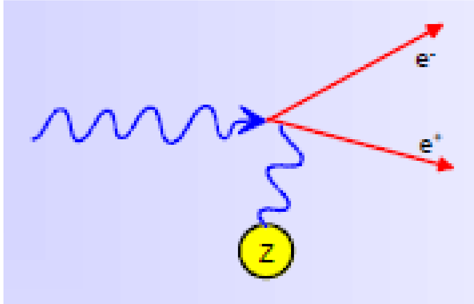
$$E_C(solid) = \frac{610\ \mathrm{MeV}}{Z + 1.24}$$

$$E_C(liquid) = \frac{710\ \mathrm{MeV}}{Z + 0.92}$$

Photons: Pair production

- Can only occurs in the Coulomb field of a nucleus (or an electron) if $E_\gamma > 2m_e c^2$

$$\gamma + nucleus \rightarrow e^+ e^- + nucleus$$



$$\sigma_{pair} \approx 4\alpha r_e^{\,2} Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} \right) \approx \frac{7}{9} \frac{A}{N_A} \frac{1}{X_0}$$

- Mean free path of photon before it creates a pair

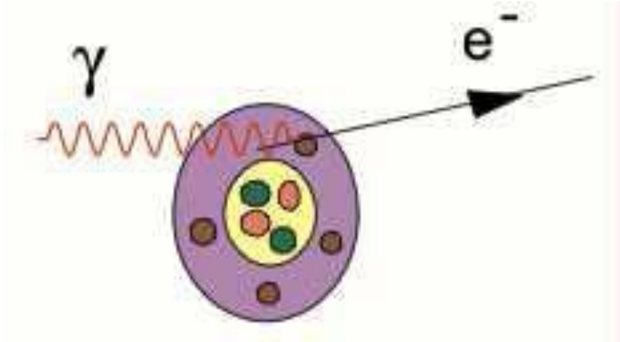
$$\lambda_{pair} \approx \frac{9}{7} X_0$$

- **Remarks:**
  - $\sigma_{pair} \propto Z(Z+1)$
  - Photons have a high penetrating power than electrons
  - Pair creation is independent of incident energy (for $E_\gamma > 1$ GeV)
  - e+e- is **emitted in photon direction**

Photons: Photo-Electric effect

- Photon extract an electron from the atom

$$\gamma + atom \rightarrow atom^* + e^-$$



$$\sigma_{pe} \approx Z^5 \alpha^4 \left(\frac{m_e c^2}{E_\gamma} \right)^{7/2}$$

- **Remarks:**
  - $\sigma_{pe} \propto Z^5, E^{-3.5}$
  - Electrons are emitted (more or less) **isotropically**

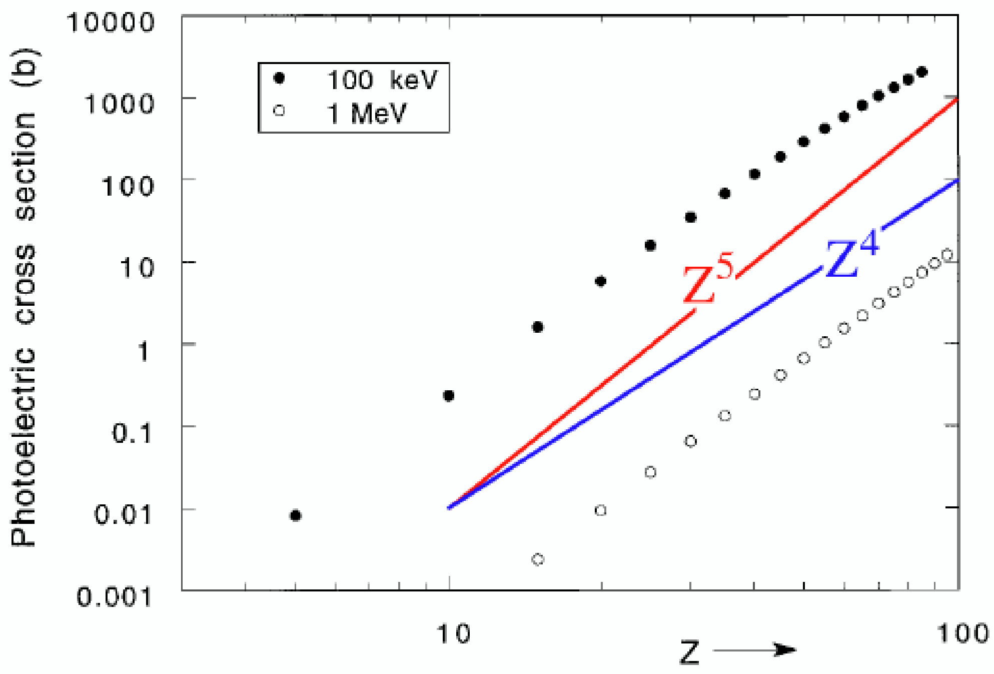
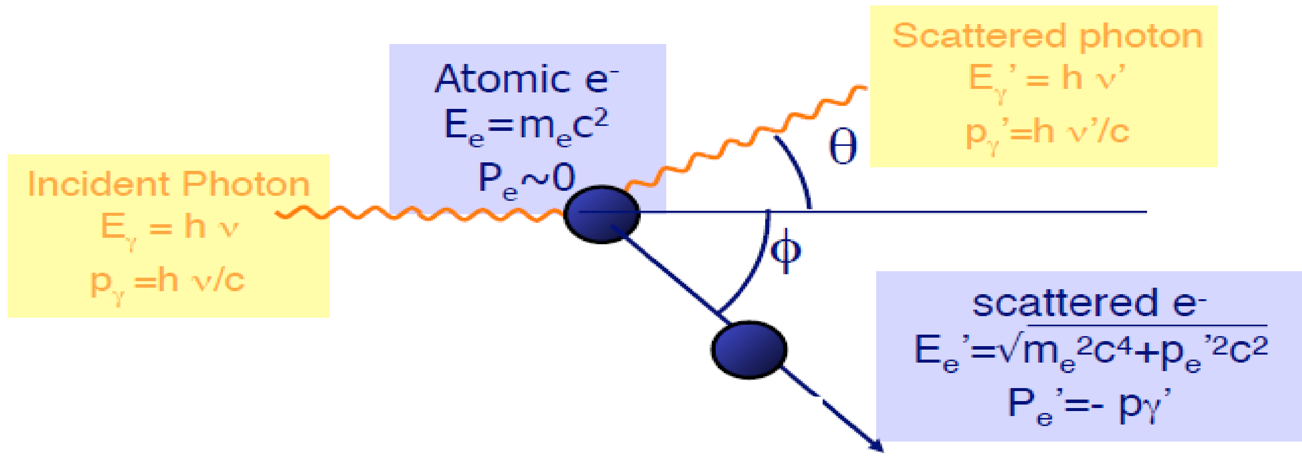


FIG. 2.3. Cross section for the photoelectric effect as a function of the Z value of the absorber. Data for 100 keV and 1 MeV γs.

Photons: Compton scattering

$$\gamma + e \rightarrow \gamma' + e'$$

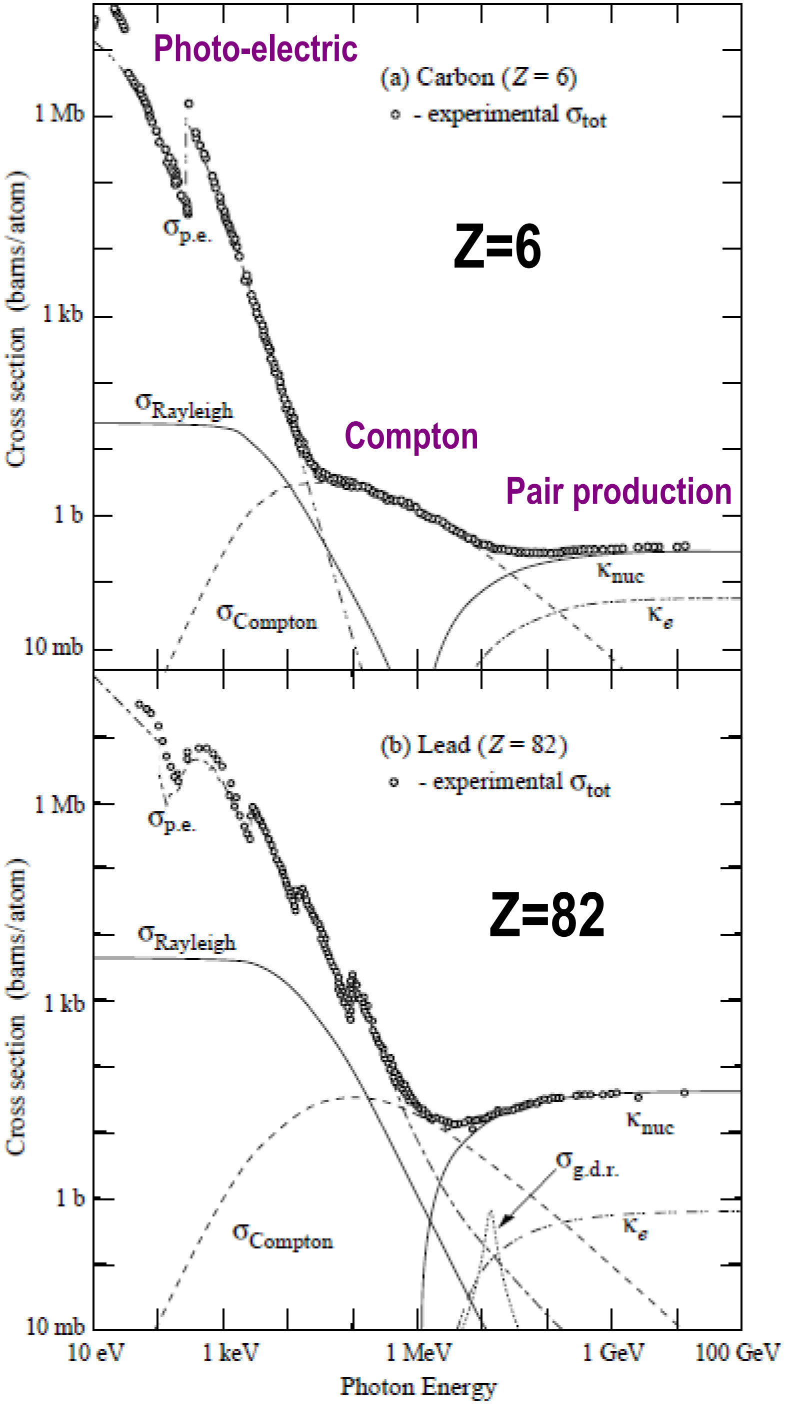


$$\sigma_{Compton} \approx Z \frac{\ln E_\gamma}{E_\gamma}$$

- **Remarks:**
  - $\sigma_{Compton} \propto Z, E^{-1}$
  - Electrons are emitted (more or less) **isotropically**

Photons: importance of the processes

γ Total cross-section vs E_γ



- Photo-electric: dominant at very low energy
- Compton: dominant for Eγ~100 KeV – 5 GeV
- Pair Production: dominant at higher energies

Photons: Angular Distributions

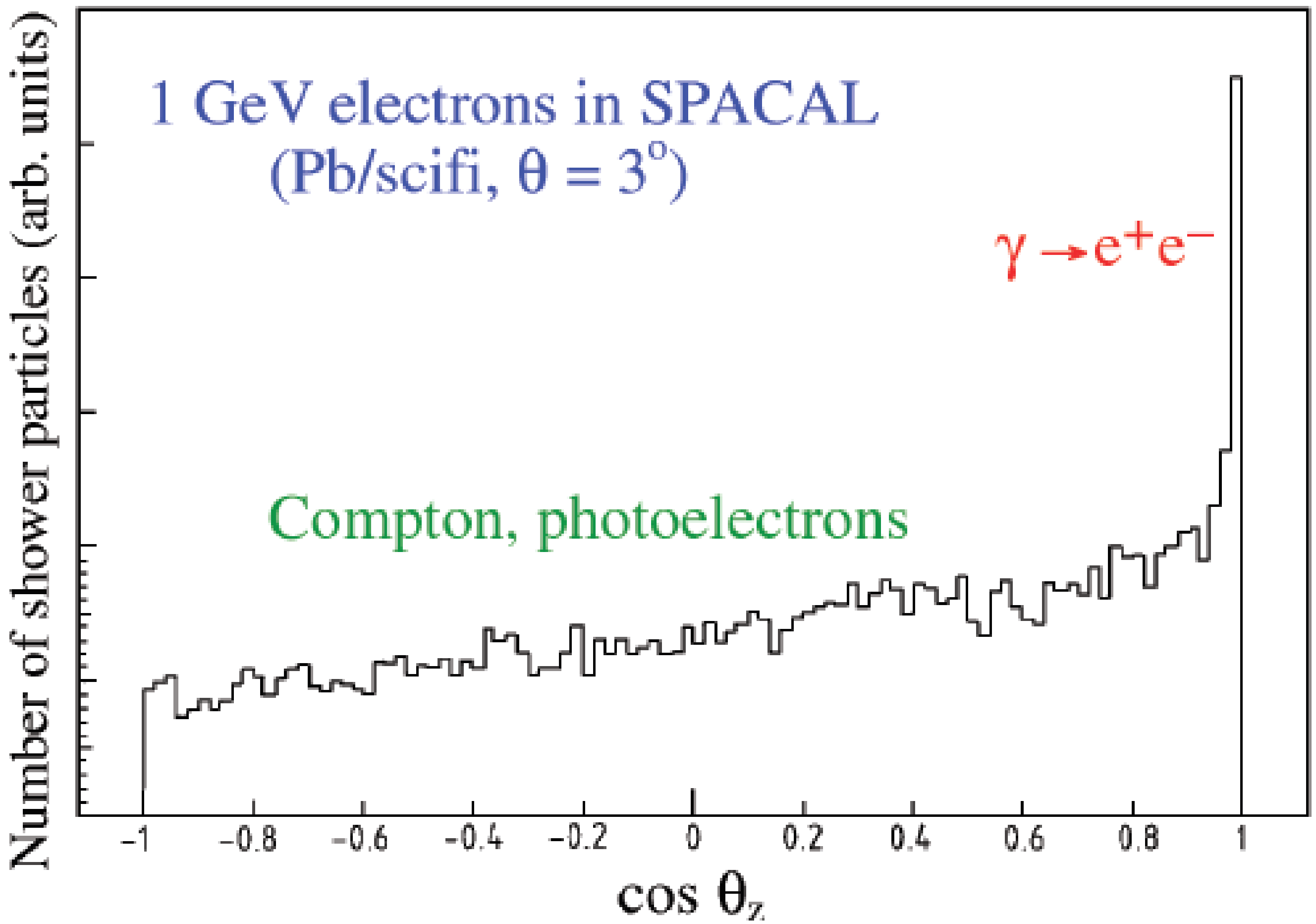


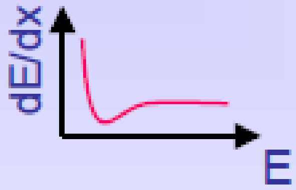
Fig. 11: Angular distribution of the shower particles (e^+, e^-) through which the energy of a 1 GeV electron is absorbed in a lead-based calorimeter [7].

Summary for Electrons & Photons

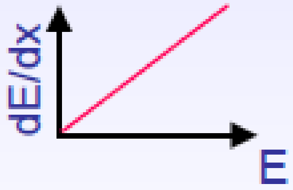
Reminder: basic electromagnetic interactions

e^+ / e^-

- Ionisation

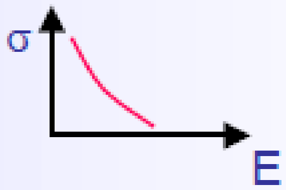


- Bremsstrahlung

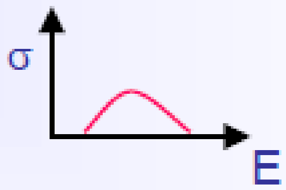


γ

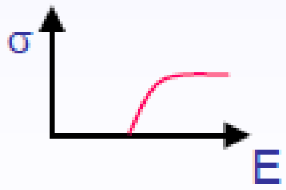
- Photoelectric effect



- Compton effect



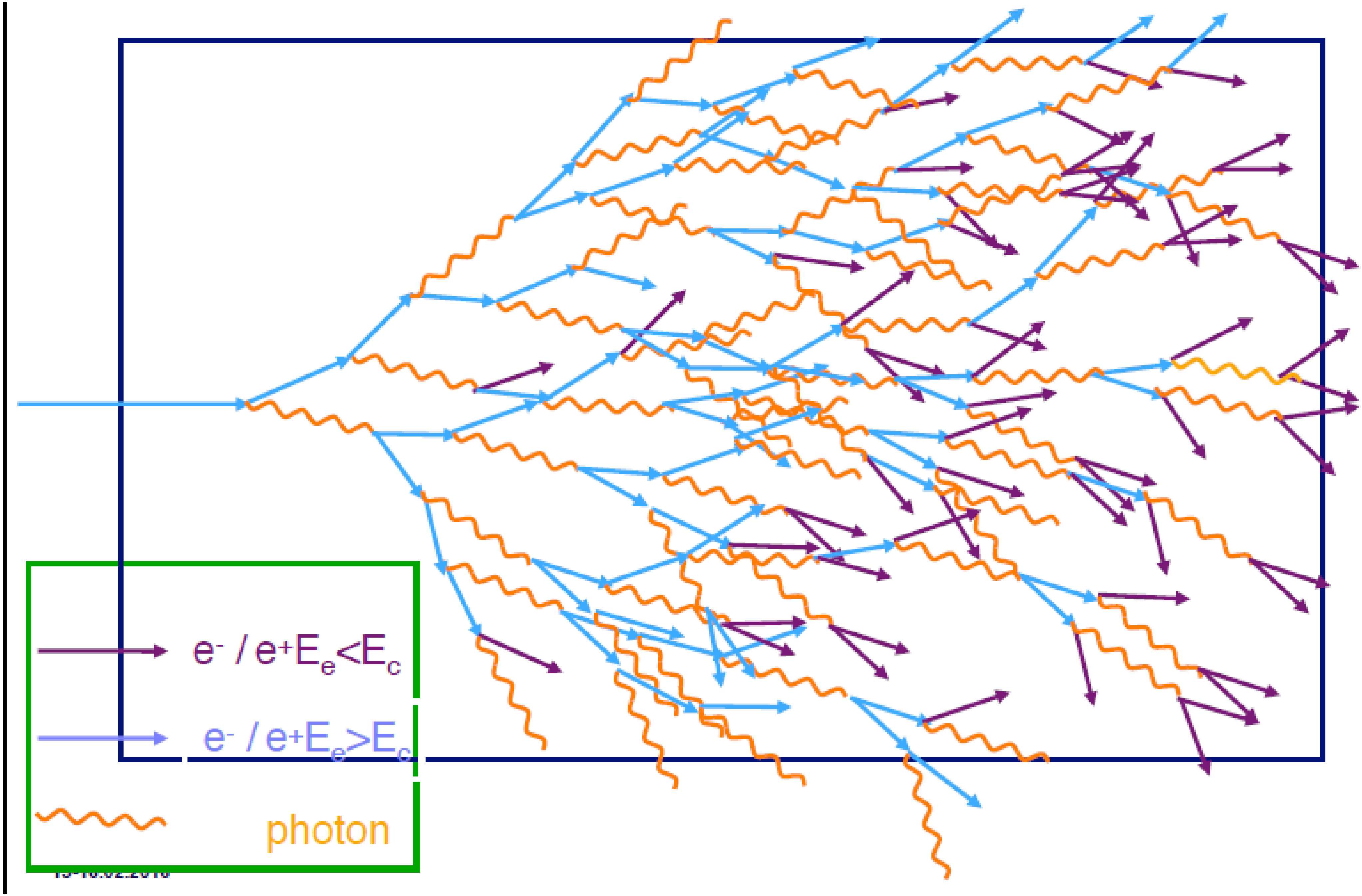
- Pair production



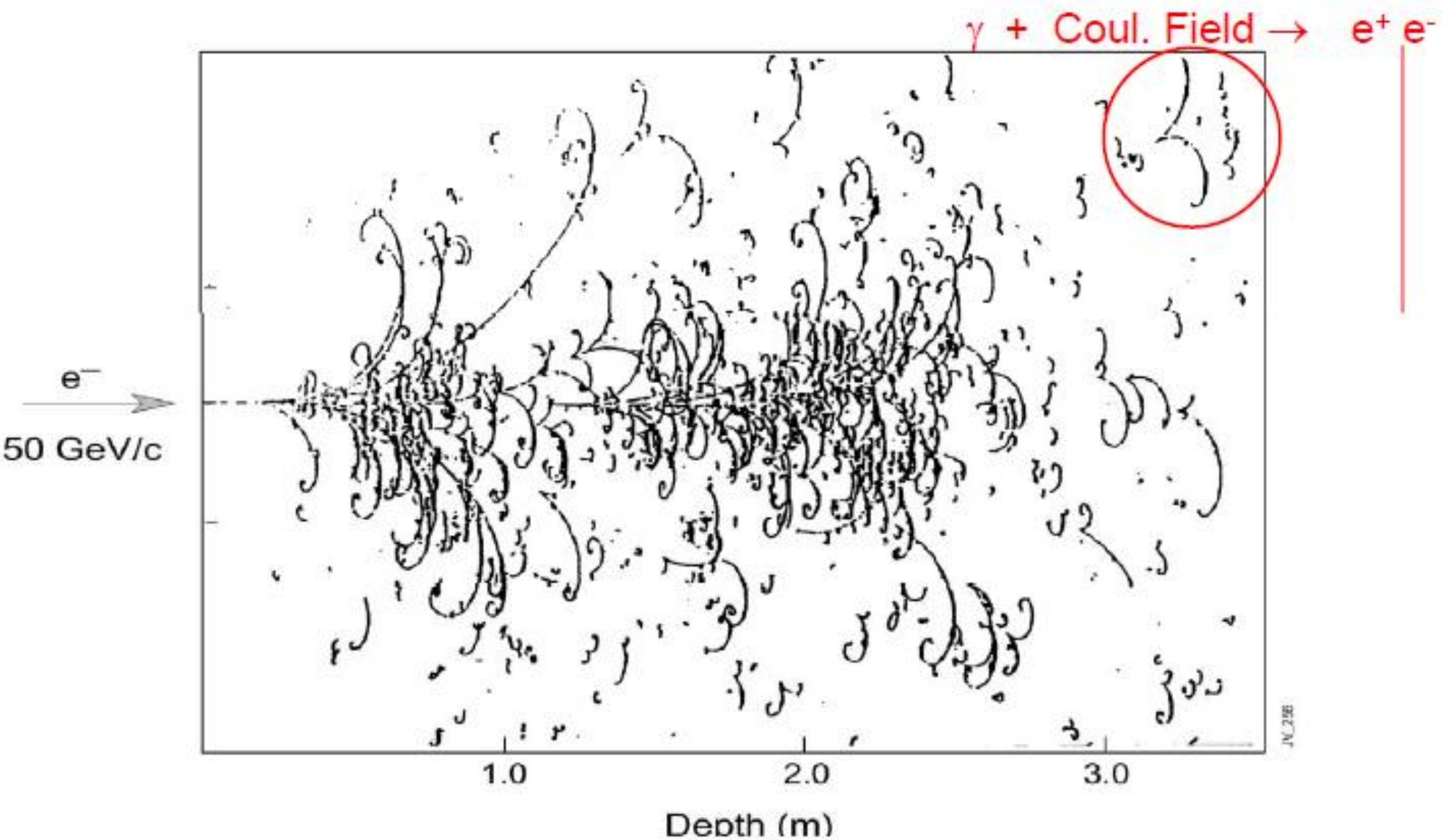
Electromagnetic shower: summary

- High-energy electrons or photons interact with dense material from calorimeter:
  cascade of secondary particles
- The number of cascade particles is **proportional to the energy** deposited by the incident particle
- The role of the calorimeter is to **count** these cascade particles
- The relative occurrence of the various processes creating the cascade particles **depends on Z.**
  - Above 1 GeV, bremsstrahlung radiation and pair production dominates
  - The shower develops like this until secondary particles reaches E_C where loss by ionization dominated
  - Below E_C, the number of secondary particles slowly decreases as electrons (photons) are stopped (absorbed)
- **The shower development is governed by the "radiation length" X_0**

Electromagnetic shower: “powerpoint” example



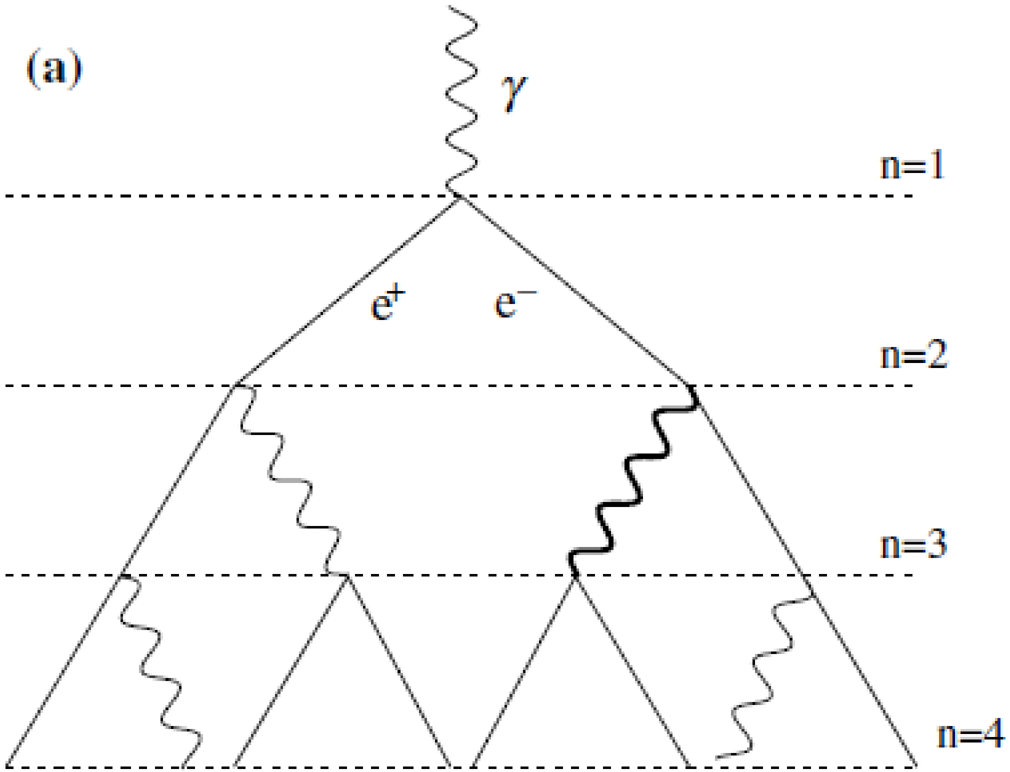
Electromagnetic Shower: real example



Big European Bubble Chamber filled with Ne:H_2 = 70%:30%, 3T Field, L=3.5 m, X_0≈34 cm, 50 GeV incident electron

EM shower: a simple model

- "Simple" approach from **Heitler**
- **Assumptions:**
  - Only 2 dominant processes (brem, pair production) for $E>E_C$ (energy loss via ionization/excitation below)
  - Assume X_0 as a generation length
  - Energy equally shared between the production of each interaction



1 incident photon with E_0

After 1 X_0: 2 electrons with $E=E_0/2$

After 2 X_0, $e\rightarrow\gamma e'$ with $E'=E_0/4$

…

After tX0, number of particles $N(t) = 2^t$ with $E(t)=E_0/2^t$

Maximum number of particles reached at $E=E_C$:

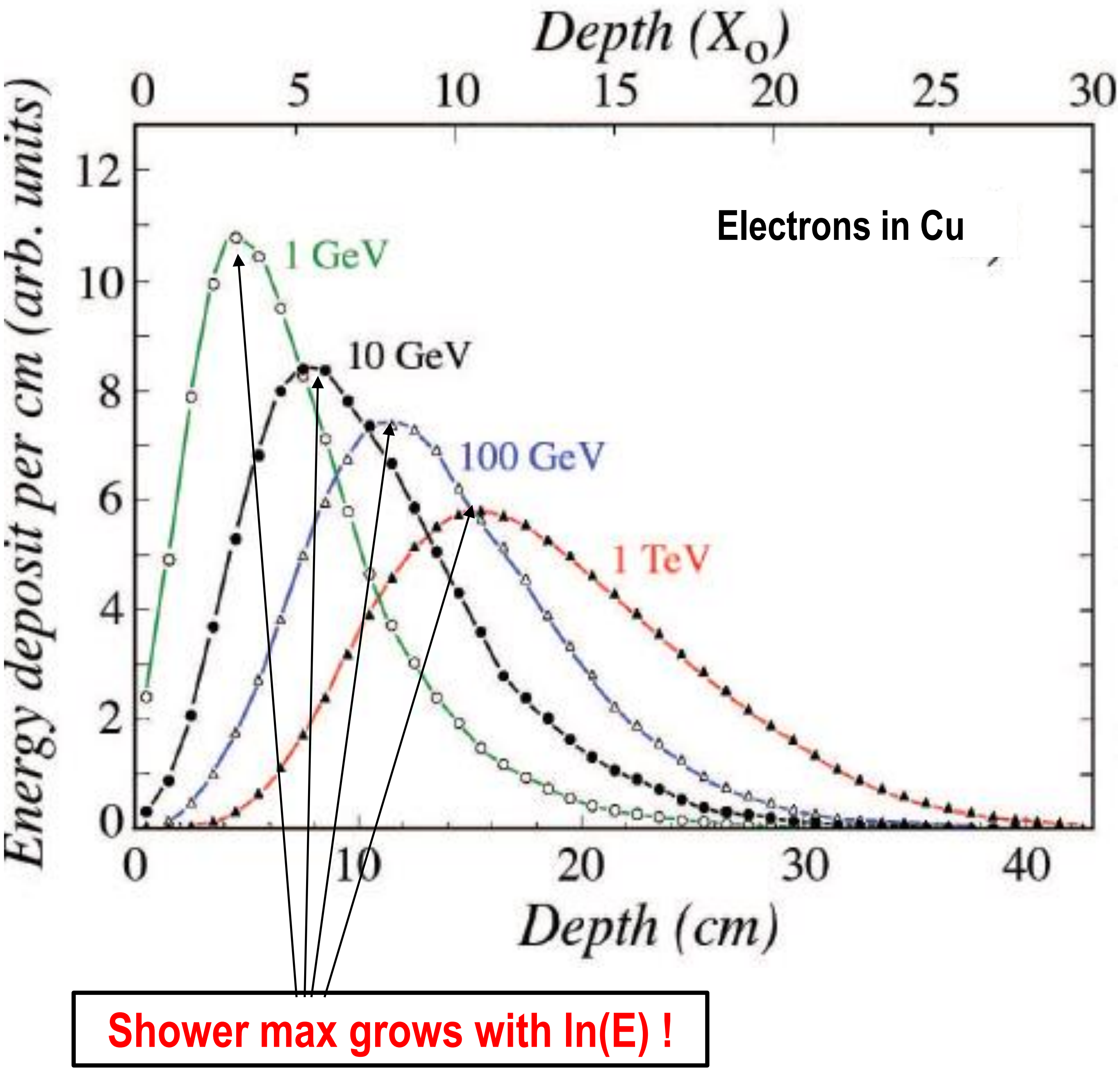
$E(t_{max})=E_C$ $E_0/2t_{max}=E_C$

Shower maximum

$$t_{max} = \frac{\ln E_0 / E_C}{\ln 2}$$

$$N(t_{max}) \approx \frac{E_0}{E_C}$$

EM shower: Longitudinal profile



$$\frac{dE}{dt} \propto E_0 b \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)}$$

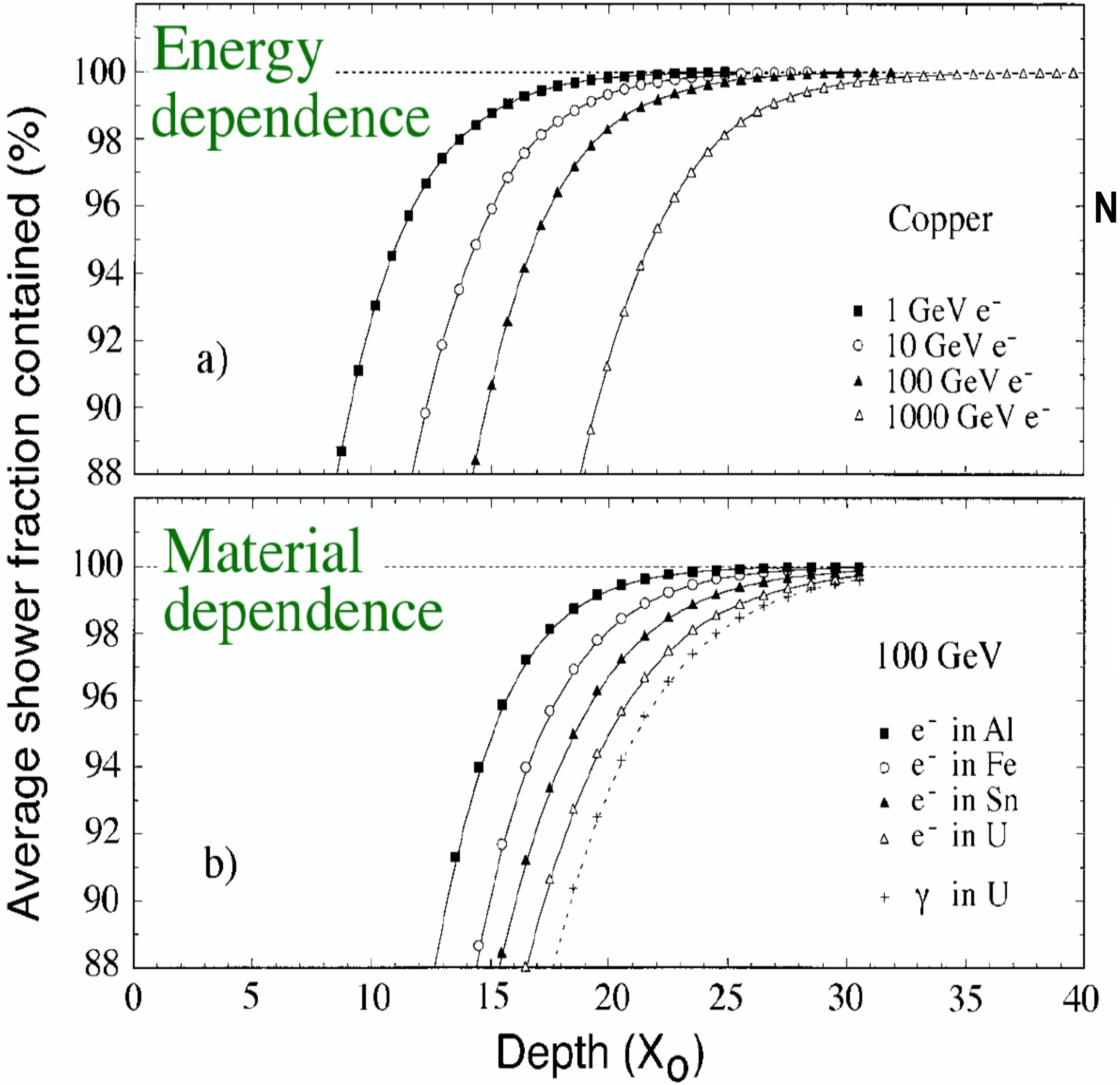
Shower energy development parametrisation

b: material

E.Longo & I.Sestili

(NIM128 (1975))

EM shower: longitudinal containment



Need about 25-30 X_0 to contain shower
(depending on the energy of interest, material)

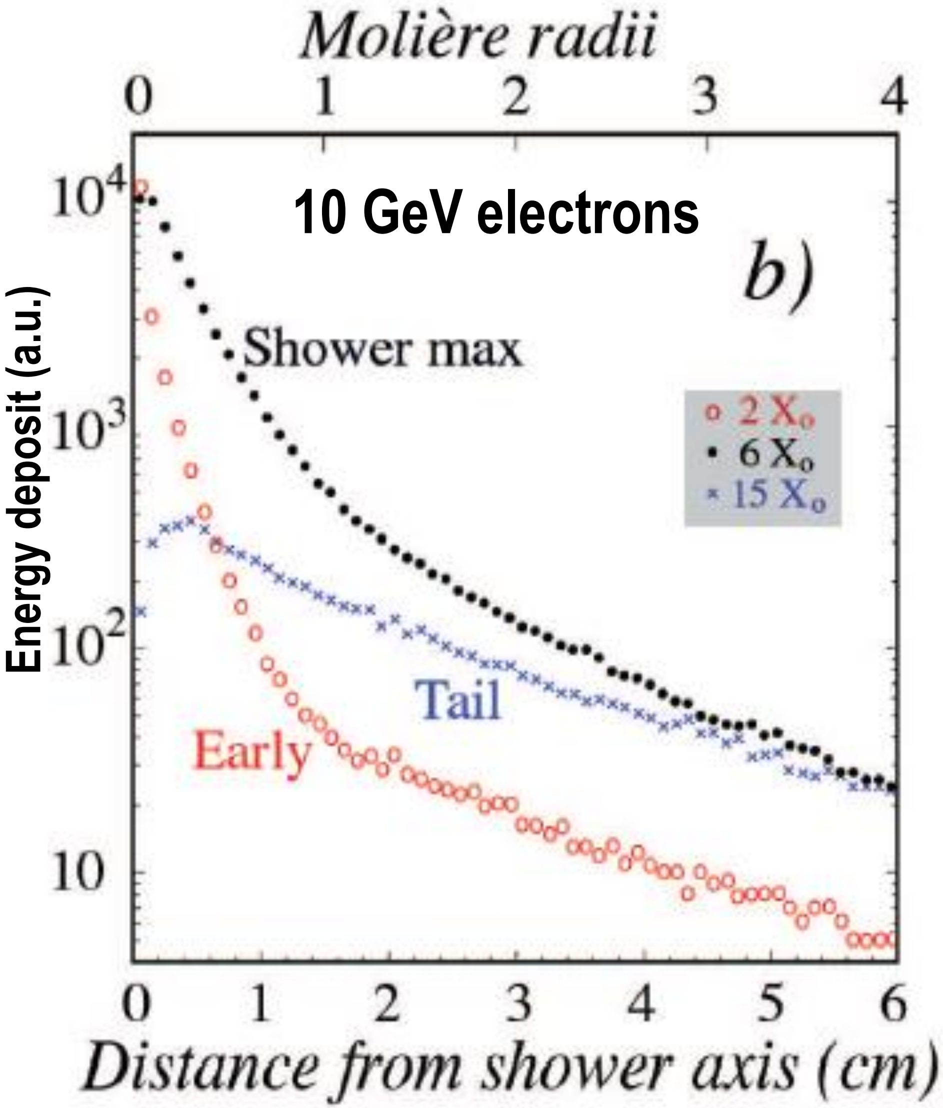
Longitudinal containment:

$$t_{95\%} = t_{max} + 0.08Z + 9.6$$

Calorimeter can be compact !

EM shower: lateral profile

- Lateral shower width determined by:
  - Multiple scattering of e+/e- (early, up to shower max) => "core"
  - Compton γ away from axis (beyond shower max) => "halo"



Radial distributions of EM showers in Cu at various depth

The EM shower gets **wider with increasing depth…**

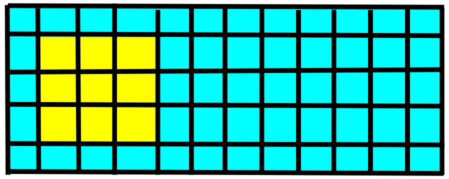
Lateral profile **independent of energy.**

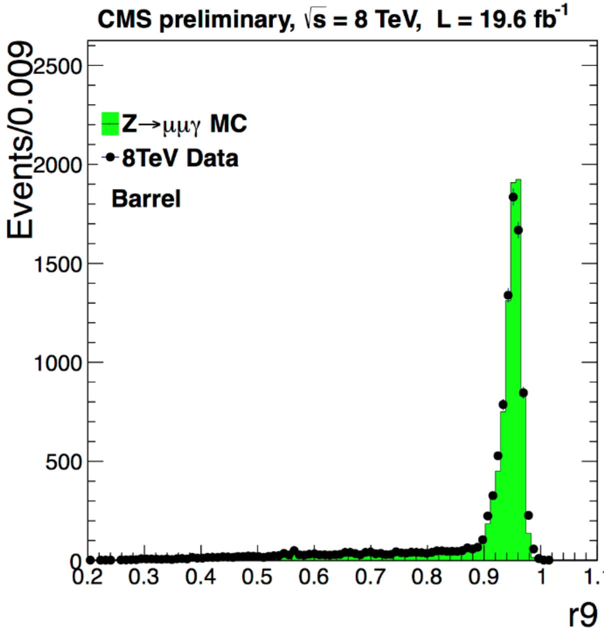
EM Shower Simulations

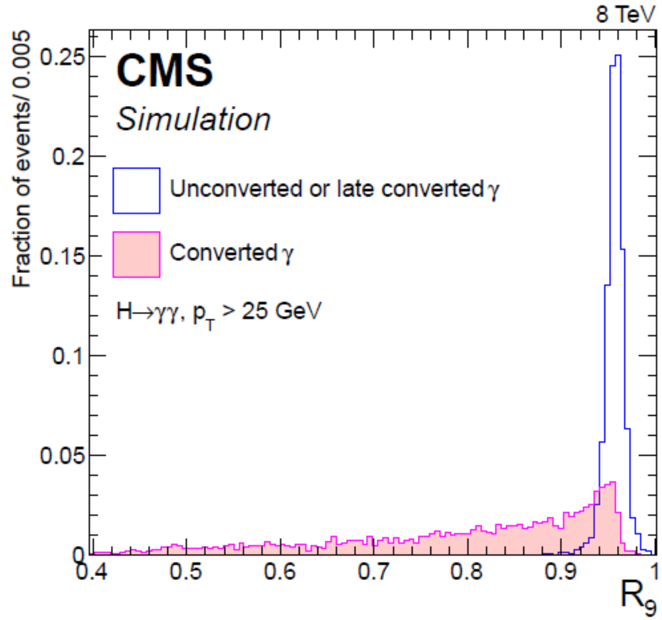
- Electromagnetic processes are well understood and can be very well reproduced by MC simulation:
  - A key element in understanding detector performance and particle ID

- **CMS in situ measurement**

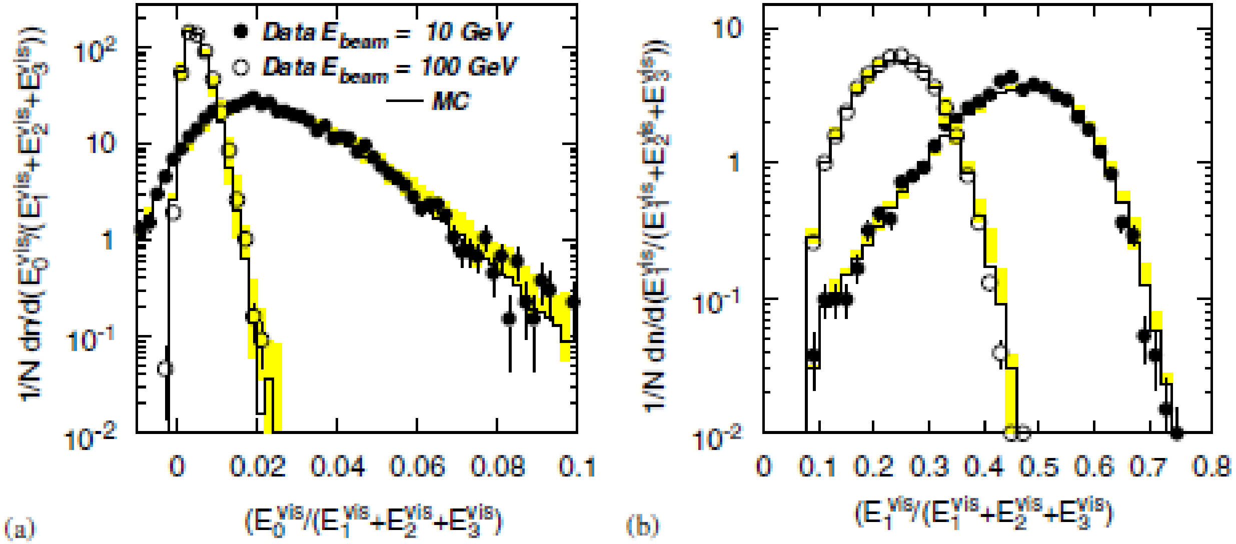
$$R_9 = E_{3\times3}/E_{SC}$$

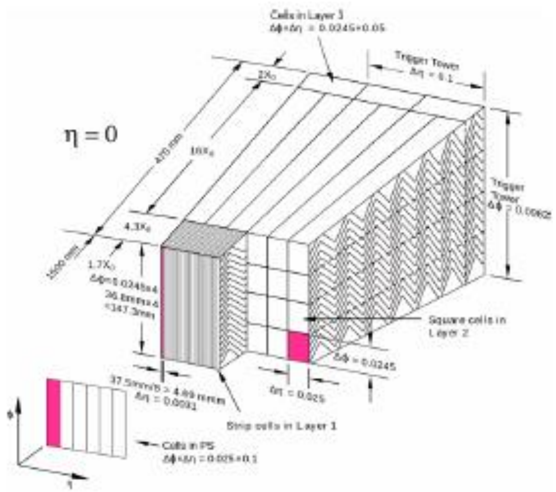






- **ATLAS test beam**





EM shower: Moliere Radius

- **Moliere radius:** characteristic of a material giving the scale of the transverse dimension of an EM shower

$$R_M = \frac{21MeV}{E_C} X_0$$ (g.cm^{-2})

Scales as A/Z, while X0 scales as A/Z². much less dependent on material than X_0 !

- 90% of shower energy contained in a cylinder of $1R_m$
- 95% of shower energy contained in a cylinder of $2R_m$
- 99% of shower energy contained in a cylinder of $3.5R_m$

Calorimeter properties of some material

Material	Z	Density [g cm^{-3}]	E_c [MeV]	X_0 [mm]	R_M [mm]	λ_{int} [mm]	$(dE/dx)_{mip}$ [MeV cm^{-1}]
C	6	2.27	83	188	48	381	3.95
Al	13	2.70	43	89	44	390	4.36
Fe	26	7.87	22	17.6	16.9	168	11.4
Cu	29	8.96	20	14.3	15.2	151	12.6
Sn	50	7.31	12	12.1	21.6	223	9.24
W	74	19.3	8.0	3.5	9.3	96	22.1
Pb	82	11.3	7.4	5.6	16.0	170	12.7
^{238}U	92	18.95	6.8	3.2	10.0	105	20.5
Concrete	-	2.5	55	107	41	400	4.28
Glass	-	2.23	51	127	53	438	3.78
Marble	-	2.93	56	96	36	362	4.77
Si	14	2.33	41	93.6	48	455	3.88
Ge	32	5.32	17	23	29	264	7.29
Ar (liquid)	18	1.40	37	140	80	837	2.13
Kr (liquid)	36	2.41	18	47	55	607	3.23
Polystyrene	-	1.032	94	424	96	795	2.00
Plexiglas	-	1.18	86	344	85	708	2.28
Quartz	-	2.32	51	117	49	428	3.94
Lead-glass	-	4.06	15	25.1	35	330	5.45
Air 20°, 1 atm	-	0.0012	87	304 m	74 m	747 m	0.0022
Water	-	1.00	83	361	92	849	1.99

EM shower: Energy Resolution

Calorimeter's resolution is determined by fluctuations.

- Ideally, if all N secondary particles are detected: $E \propto N \Rightarrow \sigma_E/E \propto \sigma(N)/N$

Fluctuation in N follow Poissonian distribution

$\Rightarrow \sigma(N)/N \propto \sqrt{N} / N \propto 1/\sqrt{N}$

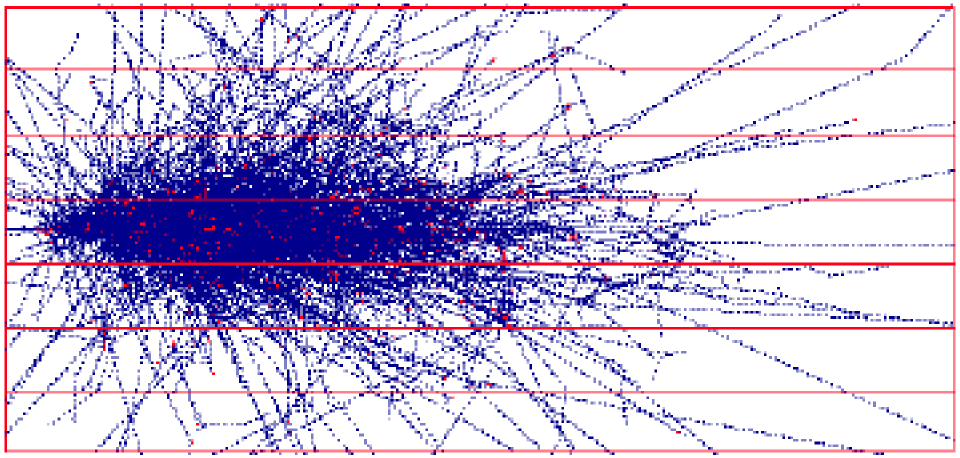
- **Intrinsic limit / ultimate resolution: determined by fluctuations of number of shower particles.**

- In reality, only a fraction f_S of secondary particles can be detected (via ionization, Cherenkov, scintillation …)
- $N_{max} = N_{tot} / E_{th}$,
  where E_{th} is the threshold energy of the detector, ie, the minimal energy to produce a detectable signal (100 eV for plastic scintillators, ~3 eV for semi-conductors…)

$$\frac{\sigma(E)}{E} \propto \frac{1}{\sqrt{E}} \frac{1}{\sqrt{f_S}}$$

- Other type of fluctuations may impact resolution, eg:
  - Signal **quantum** fluctuations (photoelectron statistics,….)
  - Shower **leakage**,
  - **Instrumental** effects (electronic noise, light attenuation, structural non-uniformity)
  - **Sampling** fluctuations (in sampling calorimeters)

Homogenous Calorimeter



All the energy is deposited in the active medium

- Excellent energy resolution
- No longitudinal segmentation

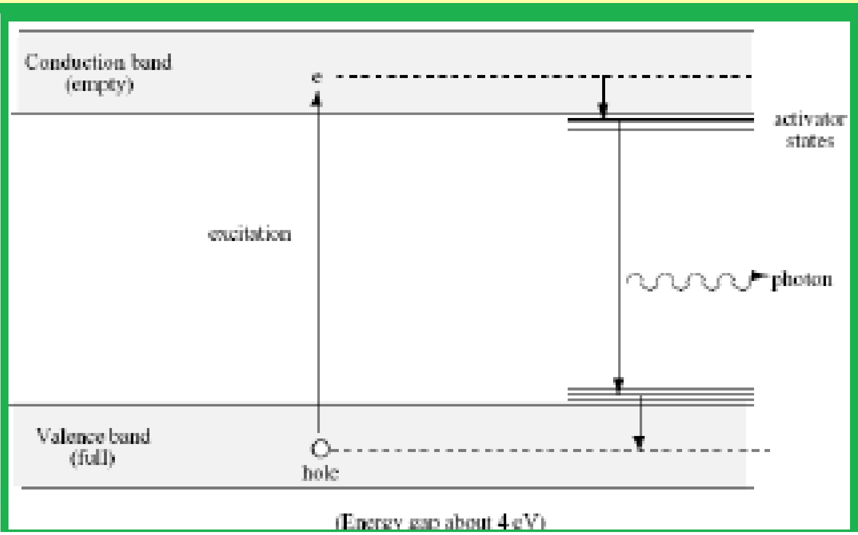
All $e^{\pm}$ with $E_{kin} > E_{th}$ produce a signal

Scintillating crystals

$E_{th} \approx \beta . E_{gap} \sim \text{eV}$

$\rightarrow 10^2 \div 10^4\ \gamma/\text{MeV}$

$\sigma/E \sim (1 \div 3)\%/\sqrt{E}\ (\text{GeV})$

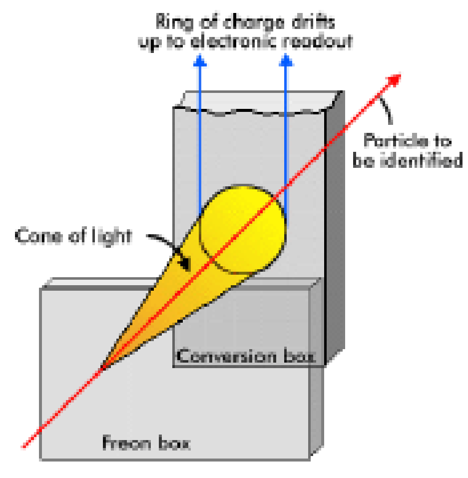


Cerenkov radiators

$\beta > 1/n \rightarrow E_{th} \approx 0.7\ \text{MeV}$

$\rightarrow 10 \div 30\ \gamma/\text{MeV}$

$\sigma/E \sim (5 \div 10)\%/\sqrt{E}\ (\text{GeV})$



Example

Take a Lead Glass crystal

E_c = 15 MeV

produces Cerenkov light

Cerenkov radiation is produced par $e^{\pm}$ with $\beta > 1/n$, i.e $E > 0.7$MeV

Take a 1 GeV electron

At maximum 1000 MeV/0.7 MeV $e^{\pm}$ will produce light

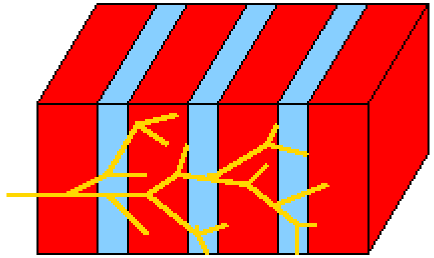
Fluctuation $1/\sqrt{1400} = 3\%$

In addition, one has to take into account the photon detection efficiency which is typically 1000 photo-electrons/GeV: $1/\sqrt{1000} \sim 3\%$

Final resolution $\sigma/E \sim 5\%/\sqrt{E}$

Sampling Calorimeters

- **Sampling Calorimeters:**
  - Sandwich of **high-Z absorber** (Pb, W, Ur,…) and **low-Z active media** (liquid, gaz, …)
    - Ex: ATLAS (Pb/LAr), DØ (Ur/LAr), …



- **Longitudinal segmentation**
- Energy resolution limited by fluctuations in energy deposited in the active layers (ie, the number n_{ch} of charged particles crossing the active layers)

- n_{ch} increases linearly with incident energy and fineness of the sampling:
  $n_{ch} \propto E / t$, where t=thickness of each absorber layer

For independent sampling:

$$\frac{\sigma(E)}{E} \propto \frac{1}{\sqrt{n_{ch}}} \propto \sqrt{\frac{t}{E}}$$ (stochastic contribution only)

For fixed active layers thickness, the resolution should improves as absorber thickness decreases.

Resolution of sampling calorimeters

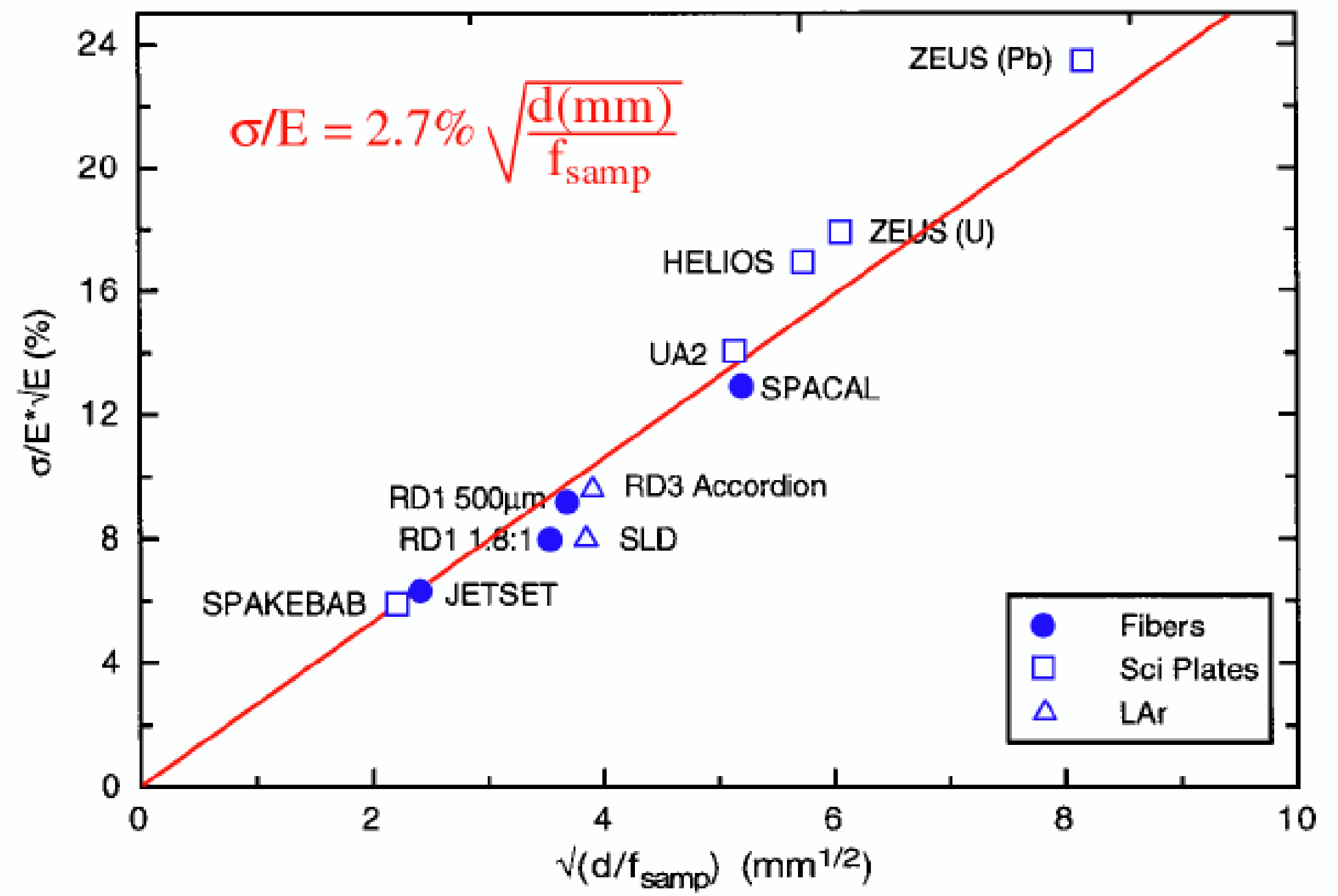


FIG. 4.8. The em energy resolution of sampling calorimeters as a function of the parameter $(d/f_{\rm samp})^{1/2}$, in which d is the thickness of an active sampling layer (*e.g.* the diameter of a fiber or the thickness of a scintillator plate or a liquid-argon gap), and $f_{\rm samp}$ is the sampling fraction for mips [Liv 95].

Sampling fluctuations in EM calorimeters determined by sampling **fraction** (f_{samp}) and sampling **frequency**

f_{samp}: energy deposited in active layers over total energy
d: thickness of active layer

Calorimeter: Energy Resolution

- Calorimeter resolution can be parameterized by the following formula:

$$\frac{\sigma}{E} = \frac{S}{\sqrt{E}} \oplus \frac{N}{E} \oplus C$$

$\oplus$: quadratic sum

Stochastic term (S):

- Accounts for any kind of Poisson-like fluctuations (number of secondary particles generated by processes, quantum, sampling, etc…)

Noise term (N): relevant at **low energy**

- Electronics noise from readout system
- At Hadron colliders: contributions from pile-up (from low energy particles generated by additional interactions): fluctuations of energy entering the measurement area from other source than primary particle.

Constant term (C): dominant at **high energy**

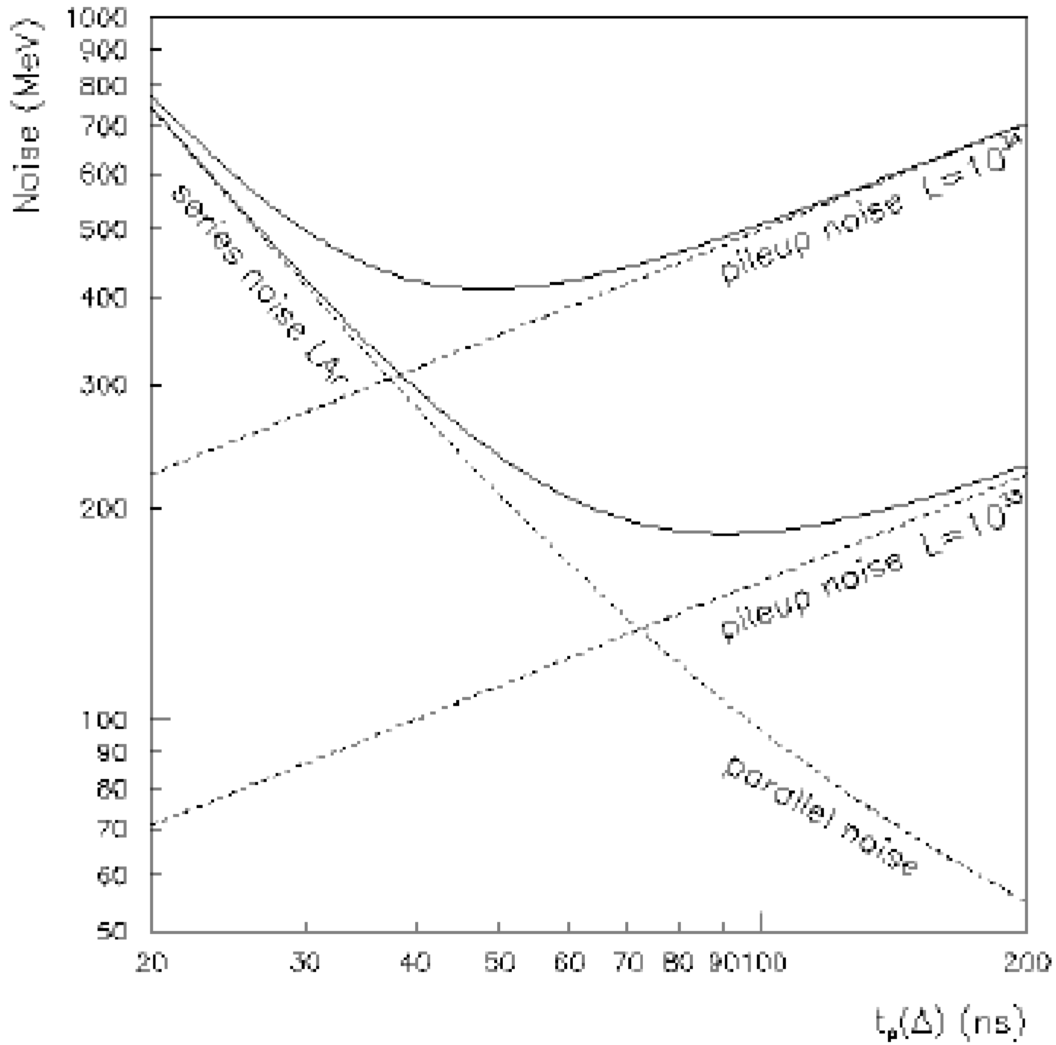
- Imperfections in construction, non-uniformity of signal collection, fluctuations in longitudinal energy containment, loss of energy in dead material, etc…

Noise Term

Electronics noise vs pile-up noise
(example from LAr ATLAS calorimeter)

Electronics integration time was optimized, taking into account both contributions for LHC nominal luminosity ($L=10^{34}$ $cm^{-2}s^{-1}$)

At this luminosity, contribution from noise to an electron is typically ~300-400 MeV



Constant Term

➢ The constant term describes the level of uniformity of the calorimeter response vs position, time, temperature (and not corrected for)

c = (leakage)⊕(intercalibration)⊕(system instability)⊕(nonuniformity)

To have c ~ 0.5 % all contributions must stay below 0.3 %

➢ **Leakage:**
- **Non-Poissonian fluctuations**
- For a given average containment, **longitudinal fluctuations larger than lateral ones.**
- Front face: Negligible
- Rear face:
  - Dangerous
  - Increase as ln(E)
  - Can be removed/attenuated if sufficient X0

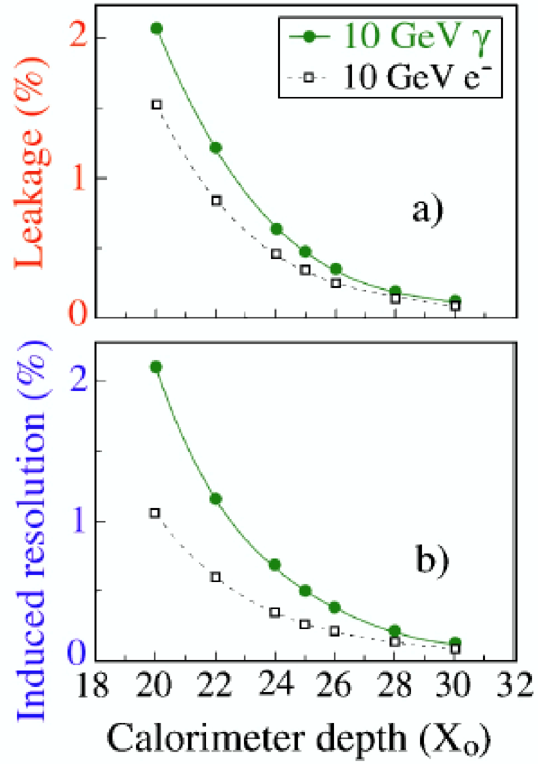
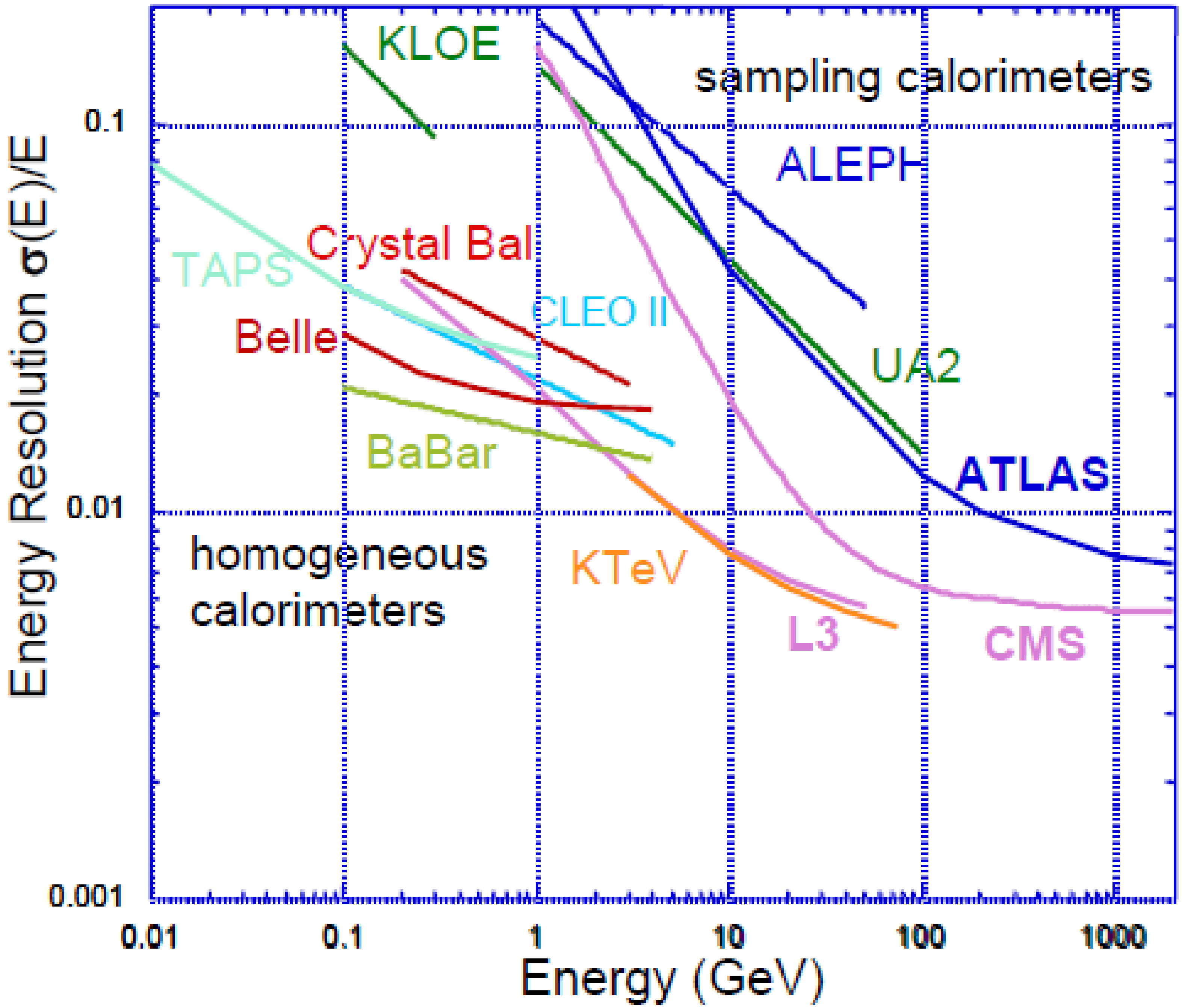


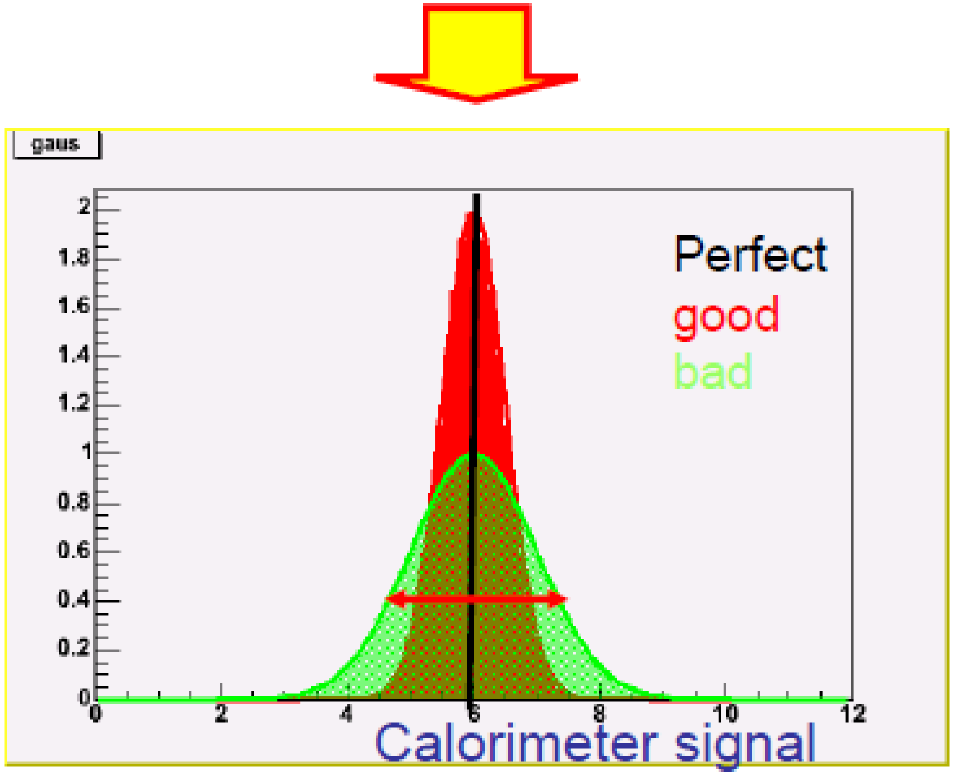
Figure 5: The average fraction of the shower energy carried by particles escaping the calorimeter through the back plane (a) and the relative increase in the energy resolution caused by this effect (b), for showers induced by 10 GeV electrons and 10 GeV γs developing in blocks of tin with different thicknesses, ranging from $20X_0$ to $30X_0$. Results from EGS4 Monte Carlo calculations.

Calorimeters: a comparison

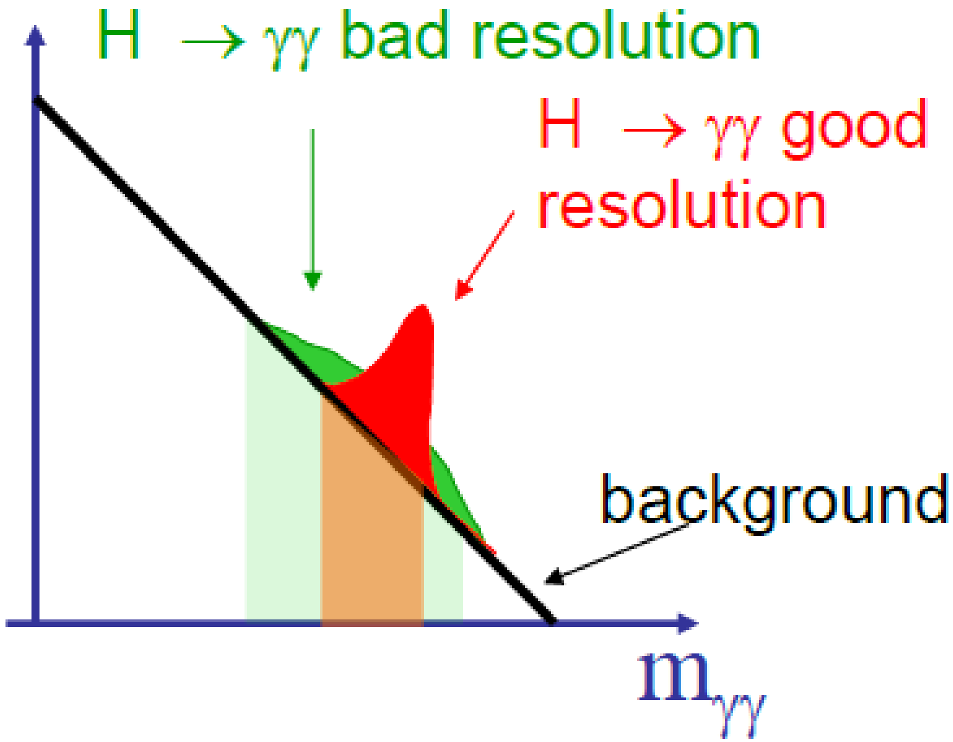


Why precision matter so much?

Response to monochromatic source of energy E



σ(calo) defines the energy resolution for energy E.



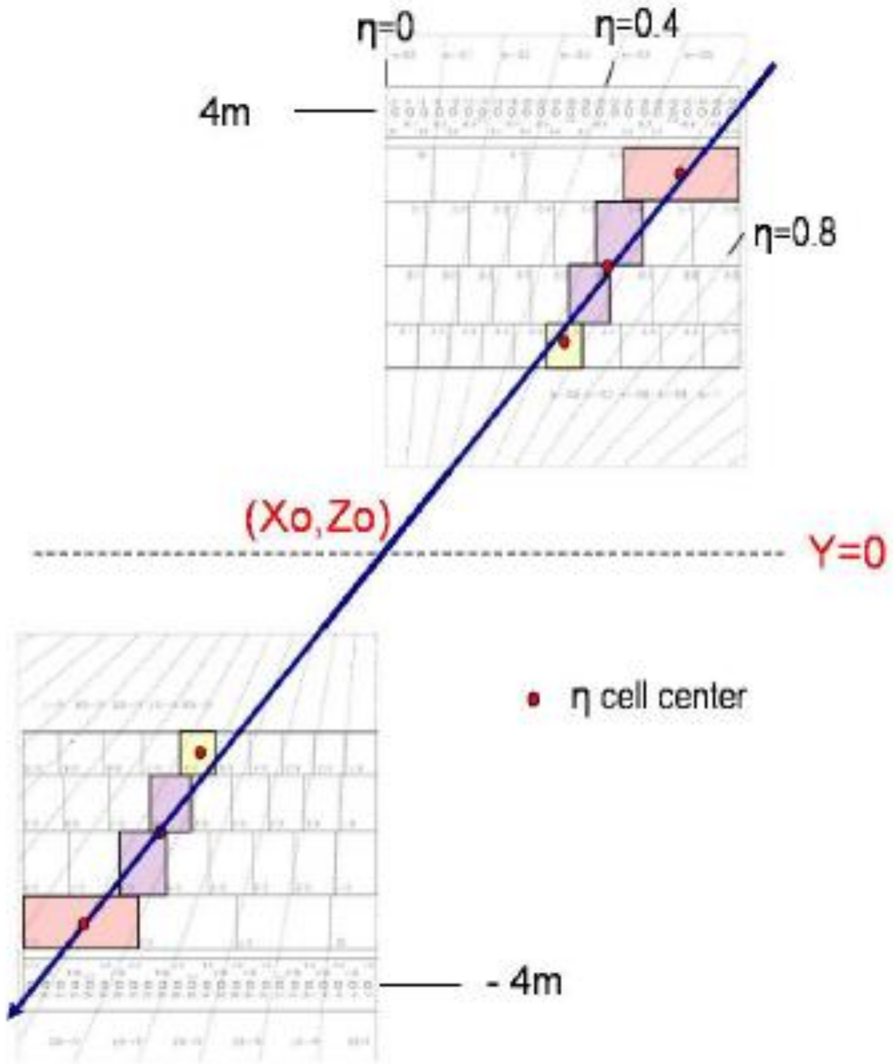
Signal = constant

integrated B $\propto \sigma_{\gamma\gamma} \rightarrow$

$S/\sqrt{B} \propto 1/\sqrt{\sigma\gamma\gamma}$

… but $\sigma_{\gamma\gamma} = f(\sigma calo)$

What about muons ?



Muons vs electrons

Muons are charged leptons, like electrons… but much heavier !

$m_{e \sim 0.511}$ MeV/c²

$m_\mu \sim 105{,}66$ MeV/c²

$m_e/m_\mu \sim 200$

$(m_e/m_\mu)^2 \sim 4000$

- Loss of energy via brem ?
  Remember:

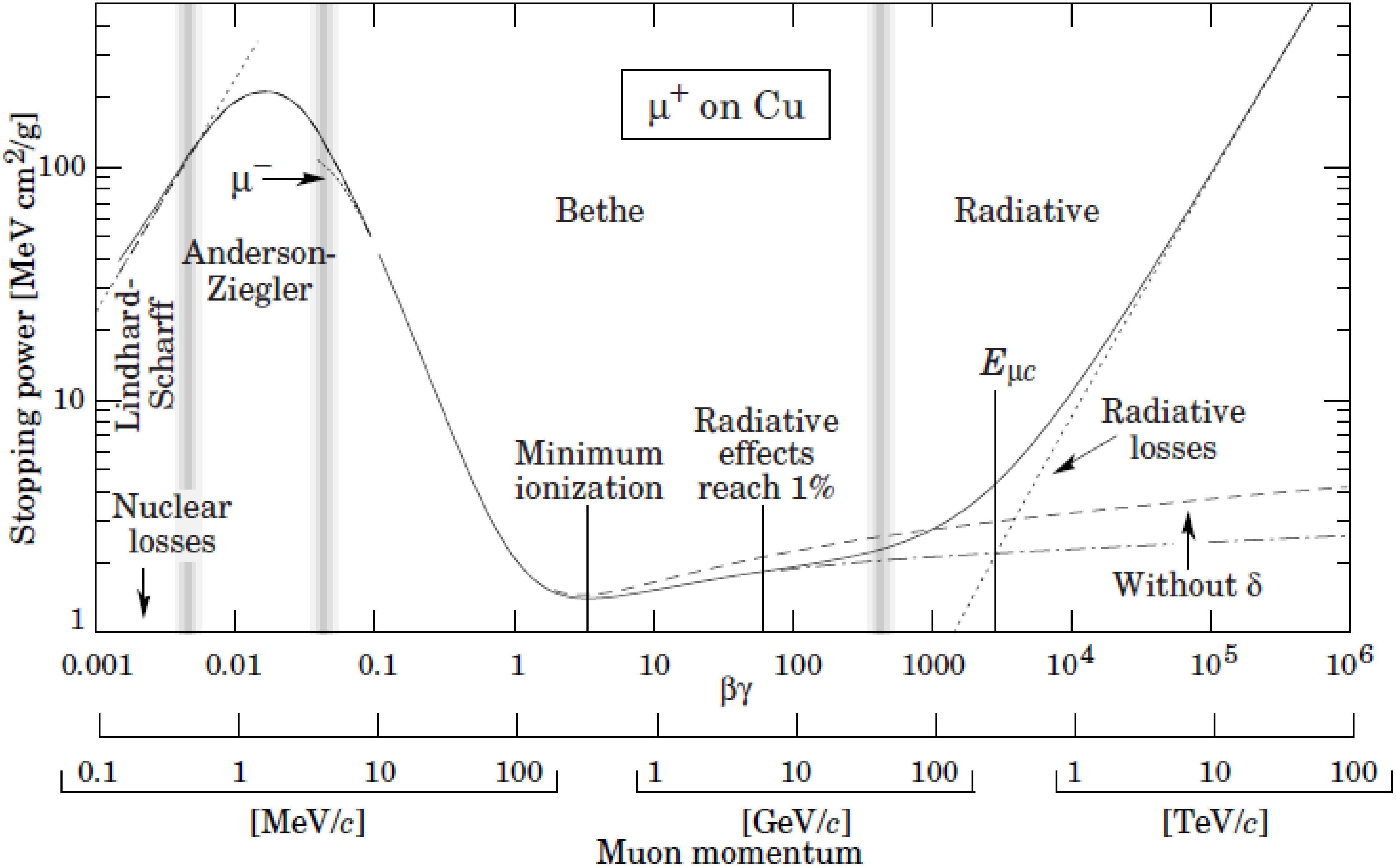
$$\left(-\frac{dE}{dx}\right)_{rad} \propto \frac{E}{m^2}$$

Much less important than for electrons…

Main mechanism for muons is ionization => no "shower" !

E_C (e-) in Cu: 20 MeV
E_C (μ) in Cu: 1 TeV…

Muon energy loss in Cu



Muons in calorimeter

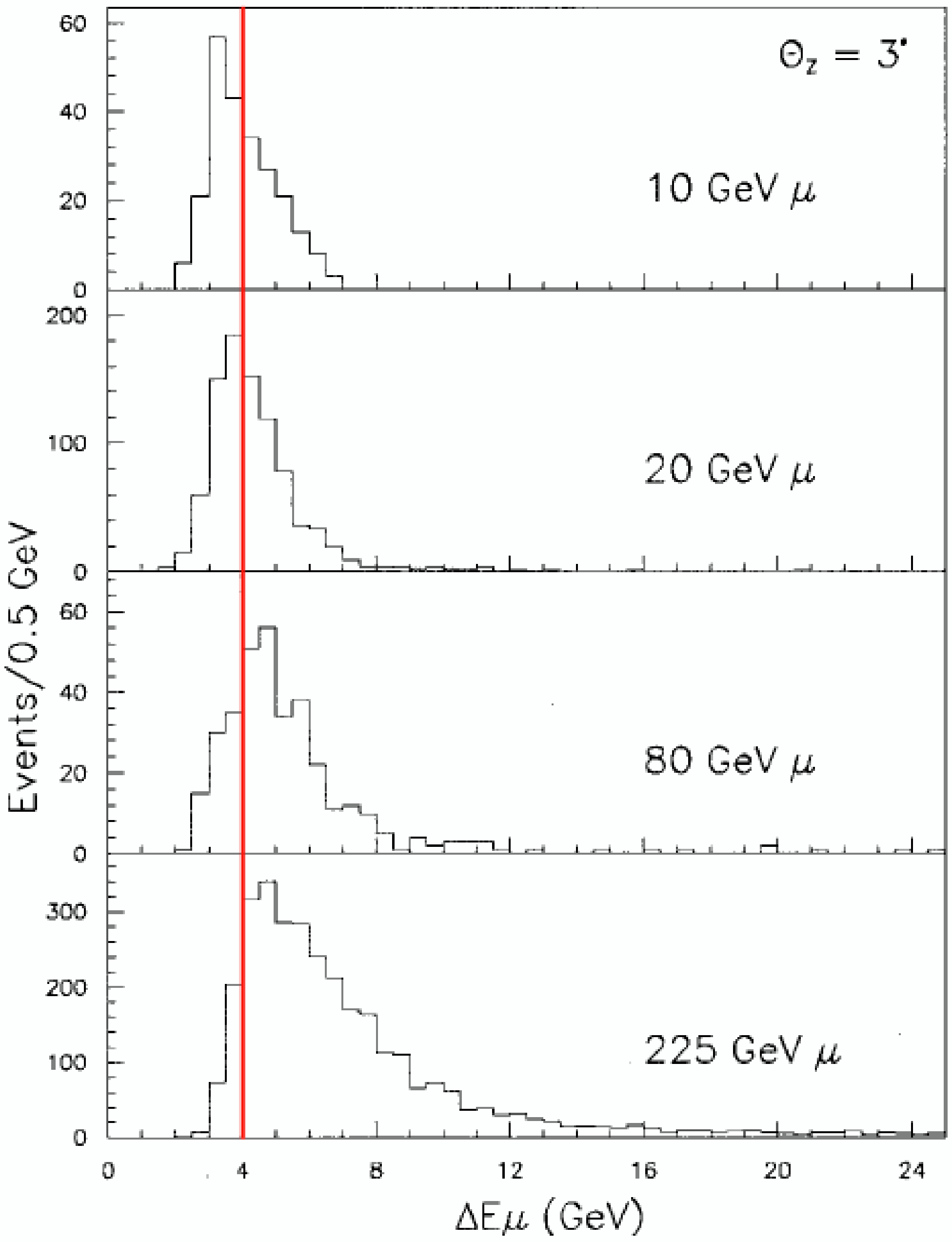


FIG. 2.19. Signal distributions for muons of 10, 20, 80 and 225 GeV traversing the $9.5\lambda_{int}$ deep SPACAL detector at $\theta_z = 3^\circ$. From [Aco 92c].

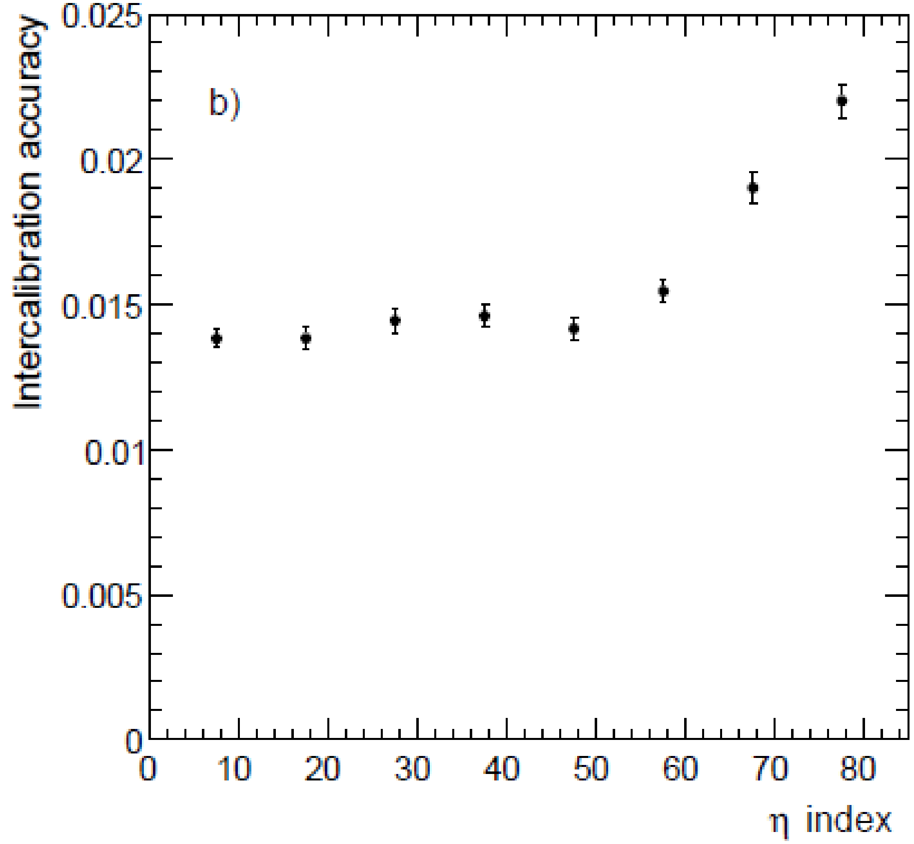
- Muons are NOT "mip" (Minimum Ionizing Particles) !
- Effect of radiation can be seen, especially at high energy and in high-Z material.
  - In Pb (Z=82), E_C (μ) =250 GeV (vs 6 MeV for e-)
- Muon energy deposit in matter NOT proportional to their energy

Muons for calorimeter

- Energy deposits from muons in calorimeter:
  - Very little (except for catastrophic loss from radiation)
  - Well known
  - Local

⇒ Muons heavily used to assess:
  - Calorimeter response uniformity (low energy), dead cells,…
  - Analyze the calorimeter geometry,

- **Cosmic muons are essential part of commissioning of calorimeters !**



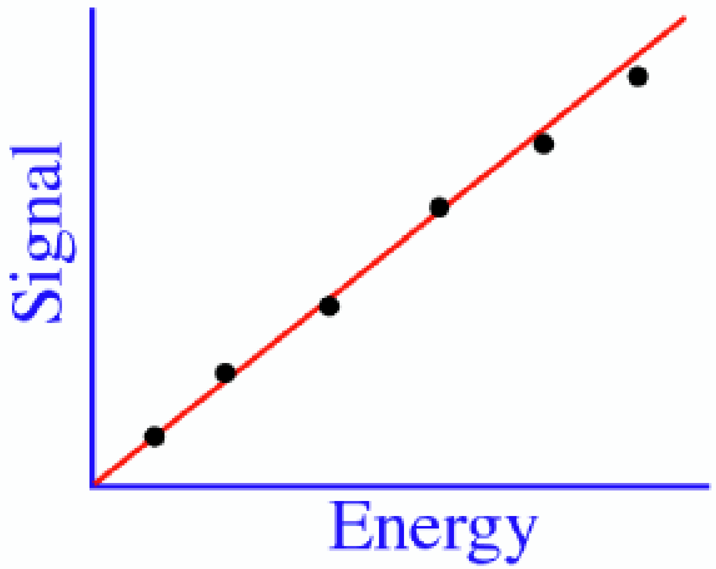
<u>Ex:</u> CMS ECAL
The intercalibration precision ranges from 1.4% in the central region to 2.2% at the high η end of the ECAL barrel **BEFORE real collisions !**

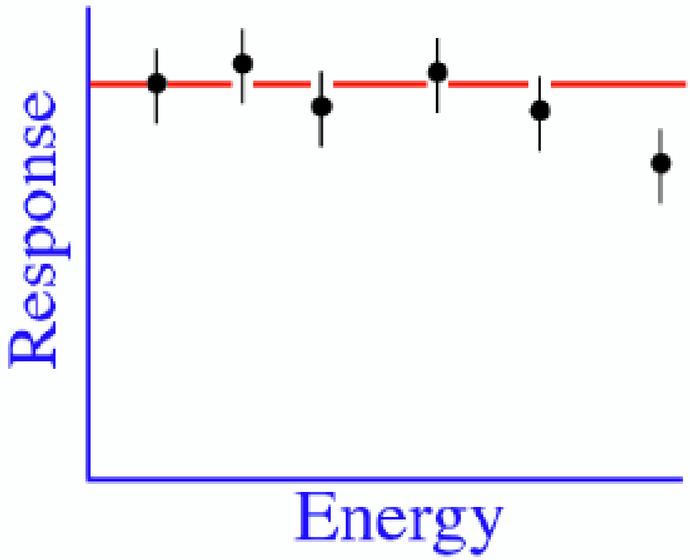
BACK UP SLIDES

LINEARITY

Response: mean signal per unit of deposited energy
e.g. # of photons electrons/GeV, pC/MeV, μA/GeV

➔A linear calorimeter has a constant response





Electromagnetic calorimeters are in general linear.
All energies are deposited via ionisation/excitation of the absorber.

RADIATION LENGTH

Approximation

$$X_0 \approx \frac{180A}{Z^2} \quad g.cm^{-2}$$

Energy loss by radiation

$$< E(x) > = E_0 \, e^{-\frac{x}{X_0}}$$

γ Absorption ($e^+ e^-$ pair creation)

$$< I(x) > = I_0 \, e^{-\frac{7}{9}\frac{x}{X_0}}$$

For compound material

$$1/X_0 = \Sigma \, w_j / X_j$$