Quantifying the evidence for the current speed-up of the Universe with low and intermediate-redshift data. A more model-independent approach

arXiv:1810.02278, JCAP 1905 (2019) 026

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Outline

I. Motivation and goal of this work

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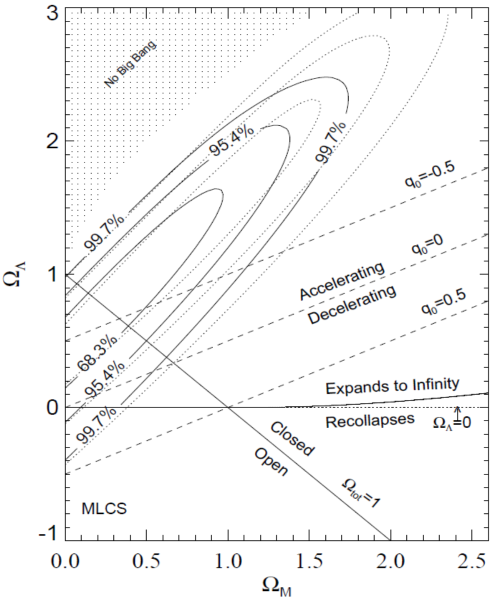
As we will see, it is not easy to fulfill the last requirement. In fact, it is impossible to obtain this information in a fully model or parametrization-independent way.

1998-1999: Discovery of the speed-up of the Universe

High-Z Supernova Search Team and the Supernova Cosmology Project collaborations:

- SnIa samples contained individuals up to $z = 0.97$ and 0.83, respectively, and also included low-redshift SnIa of $z < 0.15$ from the Calán/Tololo Supernova Survey and, in the first case, from the CfA sample too.
- $P(\Lambda > 0) = 99\%$, and restricting the analysis to the purely flat-ΛCDM, they found $\sim 3 - 4\sigma$ evidence in favor of the current positive acceleration of the Universe.

See arXiv:astro-ph/9805201, arXiv:astro-ph/9812133



Perlmutter et al. (1999)

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$$q(z) = q_0 + q_1 z$$

They find:

- Evidence for positive acceleration: $P(q_0 < 0) = 99.2\%$
- Evidence for the existence of a deceleration-acceleration transition point: $P(q_1 > 0) = 99.8\%$
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Second aim of the paper: try to quantify q_0 in a fairer way.

Fitting results for some simple nested models, using the SnIa from the Pantheon+MCT compilation and CCH

Model	$\Omega_m^{(0)}$	H_0 [km s^{-1}Mpc^{-1}]	w_0	w_1	q_0	$\chi^2_{\rm min}$
ΛCDM	0.295 ± 0.021	70.45 ± 2.36	-1	0	-0.554 ± 0.032	15.74
XCDM	0.306 ± 0.051	70.31 ± 2.42	-1.03 ± 0.15	0	-0.578 ± 0.099	15.74
CPL	0.301 ± 0.104	70.34 ± 2.47	-1.04 ± 0.16	0.10 ± 1.78	-0.494 ± 0.195	15.74

- CPL:

$$w(z) = w_0 + \frac{w_1 z}{1+z}$$

- XCDM:

$$w(z) = w_0$$

- ΛCDM:

$$w(z) = -1$$

Short introduction to Cosmography

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Taking into account that:

$$q(z) = -1 + \frac{1+z}{E(z)}\frac{dE}{dz}\,,$$

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$$E(z) = 1 + (1+q_0)z + \frac{1}{2}(j_0 - q_0^2)z^2 + \frac{1}{6}(3q_0^3 + 3q_0^2 - 3j_0 - 4q_0j_0 - s_0)z^3 + \ldots$$

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- In order to find the DE density I can make use of the covariant energy conservation equation

$$-(1+z)\sum_i \frac{d\rho_i}{dz} + 3\sum_i [\rho_i(z) + p_i(z)] = 0\,,$$

but of course in order to integrate the equation we have to specify the source function that sets the interaction between DE and the matter sector.

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  but of course in order to integrate the equation we have to specify the source function that sets the interaction between DE and the matter sector.
- We can relate, therefore, a particular form of $q(z)$ with a whole family of DE models.

$q(z)$-parametrization	H_0 [km s^{-1}Mpc^{-1}]	q_0	q_1	q_2	$\chi^2_{\rm min}$
q_0	72.29 ± 2.37	-0.288 ± 0.036	–	–	32.64
$q_0 + q_1 z$	70.35 ± 2.47	-0.503 ± 0.063	0.66 ± 0.16	–	16.26
$q_0 + q_1 z/(1+z)$	70.55 ± 2.46	-0.611 ± 0.084	1.50 ± 0.36	–	15.74
$q_0 + q_1 z/(1+z) + q_2 z^2/(1+z)^2$	70.49 ± 2.51	-0.59 ± 0.20	1.33 ± 1.89	0.31 ± 3.24	15.74

- Similar problems to the case of using concrete cosmological models are encountered.
- Notice that the error bars found for q_0 in the two-parameter expansions (the preferred cases) are quite different to the ones derived in the ΛCDM, for instance.
- In this work we want to mitigate these problems, by removing part of the subjectivity on the choice of models/parametrizations.

Some examples papers based on:

- Concrete parametrizations of the deceletarion or jerk functions: astro-ph/0106051, astro-ph/0402512, astro-ph/0603053, astro-ph/0512586, astro-ph/0605683, astro-ph/0612196, 0706.0546, 0805.1261, 0904.3550, 0811.0981, 0905.4552, 0811.2379, 1105.1871, 1005.2986, 1203.3213, 1109.4574, 1303.1620, 1305.5190, 1601.05172, 1505.03814, 1610.07337, 1712.01075, 1805.02854, 1811.05400.
- Individual truncated cosmographical series: gr-qc/0703122, 0710.1887, 0905.4552, 0911.1249, 1009.0963, 1610.08972, 1712.01075, 1808.06623, 1809.04043, 1903.11433.
- Alternative expansions of the luminosity distance: 1505.04043.
- Concrete cosmological models, including parametrizations of the DE density or the DE EoS parameter: astro-ph/9805201, astro-ph/9812133, astro-ph/0104455, astro-ph/0106051, astro-ph/0309368, astro-ph/0402512, astro-ph/0605683, 1506.01354, 1610.08972, 1611.00999, arXiv:1702.08244.

II. Strategy. Data. WFR method

IIa. Data

Cosmic Chronometers

The Hubble function can be written in terms of the redshift as:

$$H(z) = \frac{-1}{1+z}\frac{dz}{dt}$$

If we have a pair of passively evolving-galaxies and we have access to their spectra, then it is principle possible to obtain their associated redshifts and also dt by making use of the so-called stellar population synthesis (SPS) techniques.

z_i	$H^{\rm ori}(z_i)$ [km s^{-1}Mpc^{-1}]	$H^{\rm pro}(z_i)$ [km s^{-1}Mpc^{-1}]	
0.07	69.0 ± 19.6	69.0 ± 19.7	[73]
0.09	69.0 ± 12.0	69.0 ± 12.1	[74]
0.12	68.6 ± 26.2	68.6 ± 26.3	[73]
0.17	83.0 ± 8.0	83.0 ± 8.3	[75]
0.1791	75.0 ± 4.0	77.8 ± 8.1	[76]
	81.0 ± 5.0		
0.1993	75.0 ± 5.0	77.7 ± 8.7	[76]
	81.0 ± 6.0		
0.2	72.9 ± 29.6	72.9 ± 29.7	[73]
0.27	77.0 ± 14.0	77.0 ± 14.1	[75]
0.28	88.8 ± 36.6	88.8 ± 36.7	[73]
0.3519	83.0 ± 14.0	85.2 ± 16.9	[76]
	88.0 ± 16.0		
0.3802	83.0 ± 13.5	86.0 ± 15.6	[77]
	89.3 ± 14.1		
0.4	95.0 ± 17.0	95.0 ± 17.2	[75]
0.4004	77.0 ± 10.2	79.8 ± 12.3	[77]
	82.8 ± 10.6		
0.4247	87.1 ± 11.2	90.3 ± 13.6	[77]
	93.7 ± 11.7		
0.4497	92.8 ± 12.9	96.1 ± 15.3	[77]
	99.7 ± 13.4		
0.47	89.0 ± 49.6	89.0 ± 49.6	[78]
0.4783	80.9 ± 9.0	83.8 ± 10.8	[77]
	86.6 ± 8.7		
0.48	97.0 ± 62.0	97.0 ± 62.0	[79]
0.5929	104.0 ± 13.0	106.7 ± 16.4	[76]
	110.0 ± 15.0		
0.6797	92.0 ± 8.0	94.6 ± 11.9	[76]
	98.0 ± 10.0		
0.7812	105.0 ± 12.0	96.3 ± 21.0	[76]
	88.0 ± 11.0		
0.8754	125.0 ± 17.0	124.5 ± 17.3	[76]
	124.0 ± 17.0		
0.88	90.0 ± 40.0	90.0 ± 40.1	[79]
0.9	117.0 ± 23.0	117.0 ± 23.2	[75]
1.037	154.0 ± 20.0	132.5 ± 45.8	[76]
	113.0 ± 15.0		
1.3	168.0 ± 17.0	168.0 ± 17.5	[75]
1.363	160.0 ± 33.6	160.0 ± 33.8	[80]
1.43	177.0 ± 18.0	177.0 ± 18.5	[75]
1.53	140.0 ± 14.0	140.0 ± 14.4	[75]
1.75	202.0 ± 40.0	202.0 ± 40.3	[75]
1.965	186.5 ± 50.4	186.5 ± 50.6	[80]

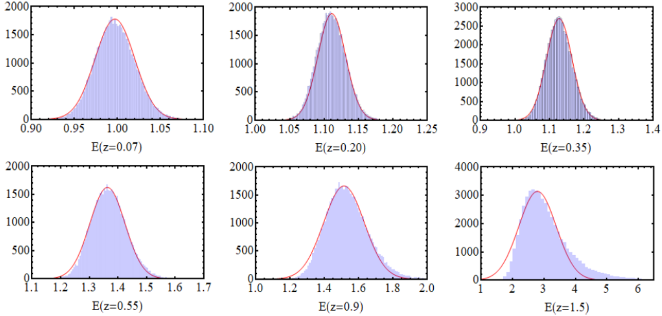
$$H^{\rm pro}(z_i) = \frac{\displaystyle\sum_{j=1}^{2} \frac{H_j^{\rm ori}(z_i)}{\sigma_j^2(z_i)}}{\displaystyle\sum_{j=1}^{2} \sigma_j^{-2}(z_i)}$$

$$\sigma_j(z_i) = \sqrt{\tilde{\sigma}_j^2(z_i) + [H_1^{\rm ori}(z_i) - H_2^{\rm ori}(z_i)]^2 + [0.025 H_j^{\rm ori}(z_i)]^2}$$

Pantheon+MCT sample of SnIa

Scolnic et al. arXiv:1710.00845, Riess et al. arXiv:1710.00844

z_i	$E(z_i)$	Correlation matrix					
0.07	0.997 ± 0.023	1.00					
0.20	1.111 ± 0.020	0.39	1.00				
0.35	1.128 ± 0.037	0.53	−0.14	1.00			
0.55	1.364 ± 0.063	0.37	0.37	−0.16	1.00		
0.90	1.52 ± 0.12	0.01	−0.08	0.17	−0.39	1.00	
1.50	2.78 ± 0.59	−0.03	−0.07	−0.07	0.13	−0.16	1.00



BAO data

I have considered the radial component of the anisotropic BAOs obtained from the measurement of:

- The power spectrum and bispectrum from the Baryon Oscillation Spectroscopic Survey (BOSS) data release 12 galaxies [Gil-Marín et al. (2017)], $H(z=0.32)r_s(z_d) = (11.55 \pm 0.38) \cdot 10^3\,\mathrm{km\,s^{-1}}$ and $H(z=0.57)r_s(z_d) = (14.02 \pm 0.22) \cdot 10^3\,\mathrm{km\,s^{-1}}$.
- The complete Sloan Digital Sky Survey (SDSS) III Lyα-quasar auto and cross-correlation functions [Bourboux et al. 2017], $c/[H(z=2.40)r_s(z_d)] = 8.94 \pm 0.22$.
- The SDSS-IV extended BOSS data release 14 quasar sample [Gil-Marín et al. (2018)], $H(z=1.52)r_s(z_d) = (24.0 \pm 1.8) \cdot 10^3\,\mathrm{km\,s^{-1}}$.

The theoretical expression of the sound horizon at the redshift of the radiation drag z_d reads,

$$r_s(z_d) = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)}\,dz\,.$$

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$$H_0 = (70 \pm 5)\,\mathrm{km/s/Mpc} \qquad ; \qquad \rho = -0.56$$

IIb. Weighted Function Regression (WFR) method

We construct a family of fitting functions from the original Taylor series:

$$E(z) = 1 + \sum_{i=1}^{\infty} c_i g_i(z) \rightarrow E_J(z) = 1 + \sum_{i=1}^{J} c_i g_i(z) \;\; \forall J \leq N$$

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$$\sum_{J=1}^{N} P(M_J) = 1$$

Weighted probability density

$$P[E(z)] = \sum_{J=1}^{N} P(E(z)|M_J)P(M_J)$$

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$$P[E(z)] = \sum_{J=1}^{N} P(E(z)|M_J)P(M_J)$$

$$P[E(z)] = \frac{\sum_{J=1}^{N} P(E(z)|M_J)B_{J*}}{\sum_{J=1}^{N} B_{J*}}$$

where $\frac{P(M_J)}{P(M_*)}$ can be identified with the Bayes ratio B_{J*}, i.e. the ratio of evidences

$$B_{J*} = \frac{\mathcal{E}_J}{\mathcal{E}_*} = \frac{\int \mathcal{L}(\mathcal{D}|\vec{c}_J)\pi(\vec{c}_J)d\vec{c}_J}{\int \mathcal{L}(\mathcal{D}|\vec{c}_*)\pi(\vec{c}_*)d\vec{c}_*}$$

Analytical expression for the evidence $\mathcal{E}$

$$\mathcal{E} = \frac{1}{(2\pi)^{N/2}\sqrt{|C|}}\sqrt{\frac{|D|}{|P|}}e^{-\frac{1}{2}(\chi^2_{\rm min}+\bar{l}_i\bar{l}_j F_{ij}+\bar{p}_i\bar{p}_j P^{-1}_{ij}-\bar{d}_i\bar{d}_j D^{-1}_{ij})}$$

where

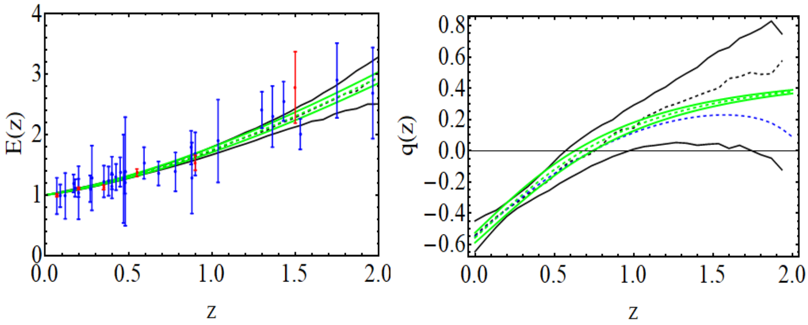
$$F_{ij} = G^i_\mu C^{-1}_{\mu\beta} G^j_\beta \qquad G^i_\mu \equiv g_i(z_\mu)$$

$$\bar{l}_i = y_\mu C^{-1}_{\mu\beta} G^j_\beta F^{-1}_{ij}$$

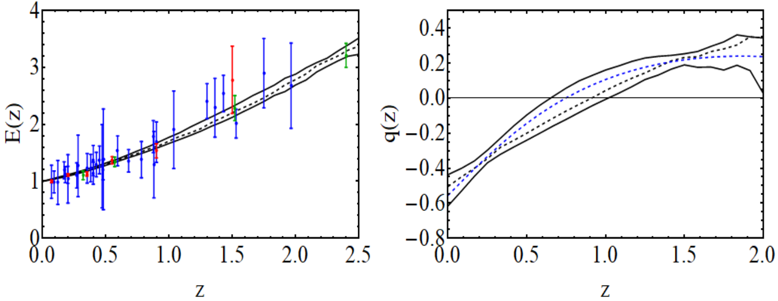
$$D^{-1}_{ij} = F_{ij} + P^{-1}_{ij}$$

$$\bar{d}_k = D_{ki}(F_{ij}\bar{l}_j + P^{-1}_{ij}\bar{p}_j)$$

III. Results

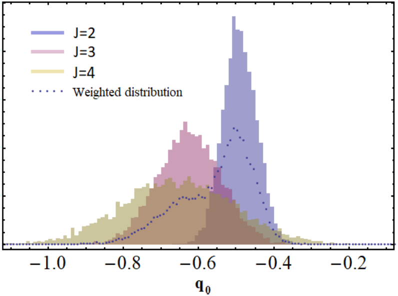


Likelihood: Pantheon+MCT
Prior: CCH



Likelihood: Pantheon+MCT
Prior: CCH+BAO

J	$q_{0,J}$	$z_{t,J}$	$j_{0,J}$	$\mathcal{E}_J$	$w_i \equiv \frac{\mathcal{E}_J}{\sum_i \mathcal{E}_i}$
2	$-0.50^{+0.05}_{-0.04}$	$0.91^{+0.09}_{-0.06}$	$0.59^{+0.13}_{-0.12}$	318.57	0.52
3	$-0.64^{+0.09}_{-0.07}$	$0.58^{+0.17}_{-0.07}$	$1.48^{+0.25}_{-0.51}$	274.48	0.45
4	$-0.62^{+0.12}_{-0.17}$	$0.60^{+0.15}_{-0.13}$	$1.5^{+1.3}_{-1.0}$	19.74	0.03



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$$q_0 = -0.51^{+0.08}_{-0.10}$$
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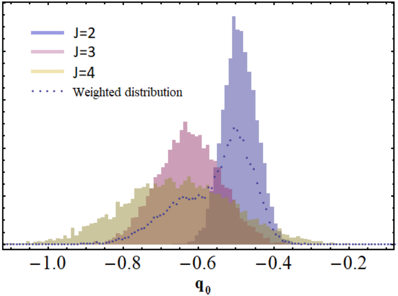

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$$B = \frac{\sum_J \mathcal{E}_J(q_0 \neq 0)}{\sum_J \mathcal{E}_J(q_0 = 0)} = 1137 \rightarrow \ln B = 7.04$$

Working with a family of expansions of $q(z)$.

$$q(z) = q_0 + \frac{q_1 z}{1+z} + \frac{q_2 z^2}{(1+z)^2} + ...$$

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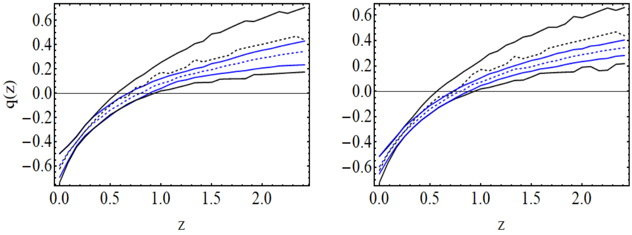
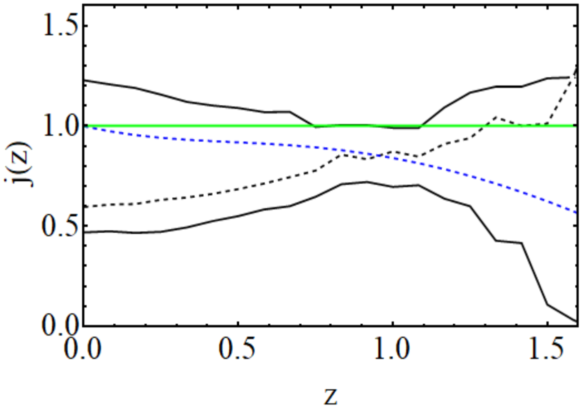


Prior	Likelihood	q_0	z_t
SnIa	CCH	$-0.62^{+0.13}_{-0.11}$	$0.71^{+0.24}_{-0.14}$
SnIa	CCH+BAOs	-0.60 ± 0.10	$0.80^{+0.09}_{-0.12}$
CCH	SnIa	$-0.62^{+0.11}_{-0.10}$	$0.74^{+0.21}_{-0.17}$
CCH+BAOs	SnIa	$-0.60^{+0.08}_{-0.06}$	$0.81^{+0.08}_{-0.09}$


1.5
1.0
0.5
0.0
j(z)
0.0
0.5
1.0
1.5
z

IV. Conclusions

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- The jerk is still quite unconstrained by the data and it is unable in practice to be employed as a null-test of the ΛCDM model.

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Questions?