Relativistic Effects in N-body Simulations of Cosmic Large-Scale Structure

Julian Adamek

based partially on **Phys. Rev. D100 (2019) 021301(R)** with
Chris Clarkson, Louis Coates, Ruth Durrer and Martin Kunz

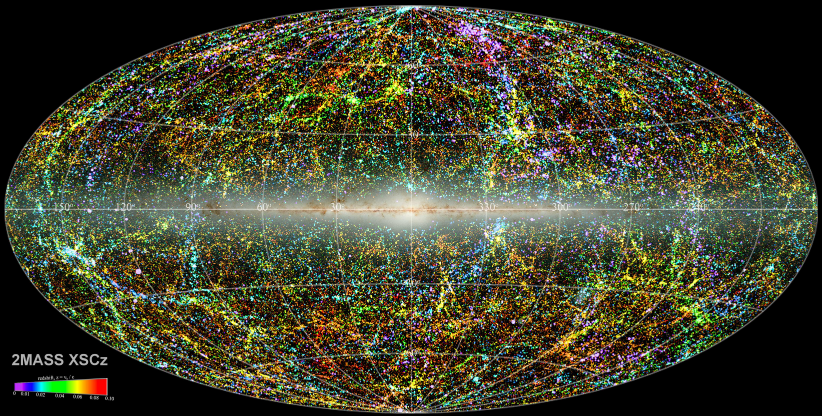
and on **arXiv:1905.11721** (**JCAP** in press) with Christian Fidler

COSMO19

RWTH Aachen, 2. September 2019

Queen Mary
University of London

Large Scale Structure



Relativistic Effects

Light propagation. . .

Observed redshift

$$z + 1 = \frac{a_{\mathrm{obs}}}{a_{\mathrm{src}}} \left[1 + \mathbf{v}_{\mathrm{src}} \cdot \mathbf{n} - \mathbf{v}_{\mathrm{obs}} \cdot \mathbf{n} - \psi_{\mathrm{src}} + \psi_{\mathrm{obs}} - \int\limits_{\mathrm{src}}^{\mathrm{obs}} (\phi' + \psi')\, d\chi \right]$$

Observed position

$$\alpha = \int\limits_{\mathrm{src}}^{\mathrm{obs}} \frac{\chi_{\mathrm{src}} - \chi}{\chi_{\mathrm{src}}} \nabla_\perp (\phi + \psi)\, d\chi$$

Magnification

$$\kappa = \int\limits_{\mathrm{src}}^{\mathrm{obs}} \frac{\chi_{\mathrm{src}} - \chi}{2\chi_{\mathrm{src}}} \nabla_\perp^2 (\phi + \psi)\, d\chi$$

Shapiro delay

$$\delta\chi = \int\limits_{\mathrm{src}}^{\mathrm{obs}} (\phi + \psi)\, d\chi$$

. . . at leading order & neglecting B_i and h_{ij}

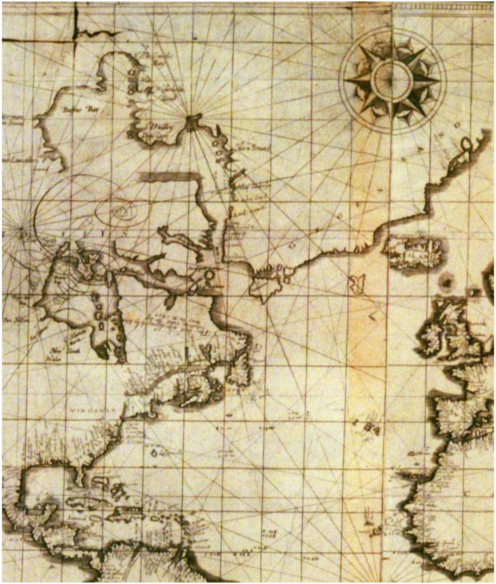
Coordinate Remapping

Coordinates (positions) are not invariant!

N-body simulation
N-body gauge

$$\Downarrow$$

Light propagation
Poisson gauge



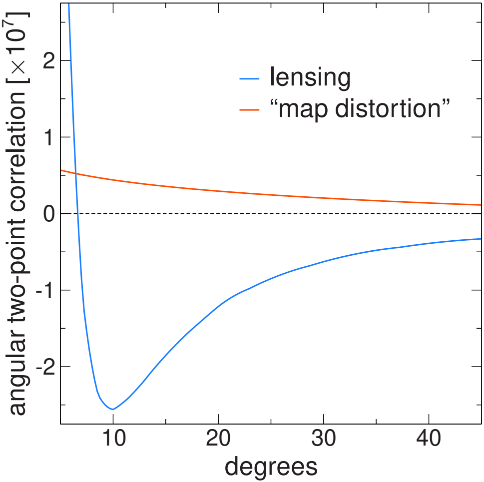
Coordinate Remapping

Coordinates (positions) are not invariant!

N-body simulation
N-body gauge

$\Downarrow$

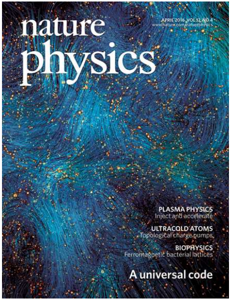
Light propagation
Poisson gauge



Adamek & Fidler, arXiv:1905.11721 (JCAP in press)

A Brief Overview of *gevolution*

gevolution, a general relativistic particle-mesh N-body code
Adamek, Daverio, Durrer & Kunz, Nature Phys. **12** (2016) 346–349

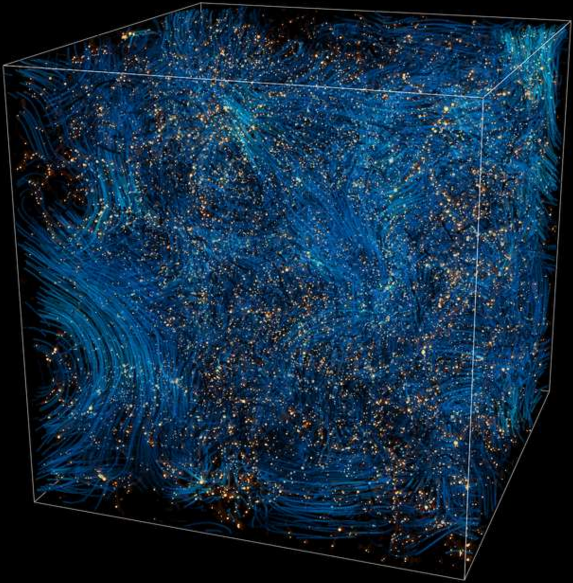


- based on weak-field expansion in Poisson gauge

$$ds^2 = a^2(\tau)\big[-e^{2\psi}d\tau^2 + e^{-2\phi}\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j - 2B_i dx^i d\tau\big]$$

- for any given T^μ_ν computes the six metric d.o.f. (ϕ, ψ, B_i, h_{ij})
- N-body particle ensemble evolved using relativistic geodesic equation

https://github.com/gevolution-code/gevolution-1.2.git



Features

Most recent public release: version 1.2

- multiple particle species (CDM, baryons, neutrinos)
- initial condition generation “on the fly”
- auto- and cross-power spectra
- linear perturbations in the radiation field (*CLASS* interface)
- Newtonian mode compatible with radiation perturbations (using N-body gauge)
- massive neutrinos can be treated as linear perturbations and/or as particles
- particle & metric light cones for ray tracing and post-processing
- linear dark energy fluids (w-c_s-parametrisation)

Ray Tracing

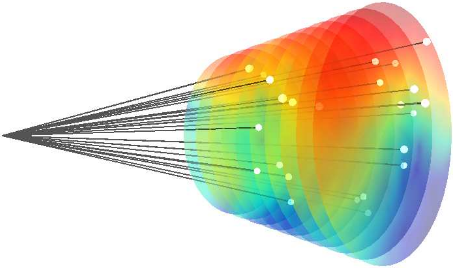
Instead of keeping *snapshots* $= \{\text{data} \mid \tau = \tau_{\text{snap}}\}$, we store a *thick light cone* $= \{\text{data} \mid \tau - \tau_o + r \in [-\Delta\tau, \Delta\tau]\}$, where $\Delta\tau$ is chosen such that the *perturbed light cone* $\subset$ *thick light cone*.

In a post-processing step, we integrate backwards in time (without approximation):

null geodesic equation

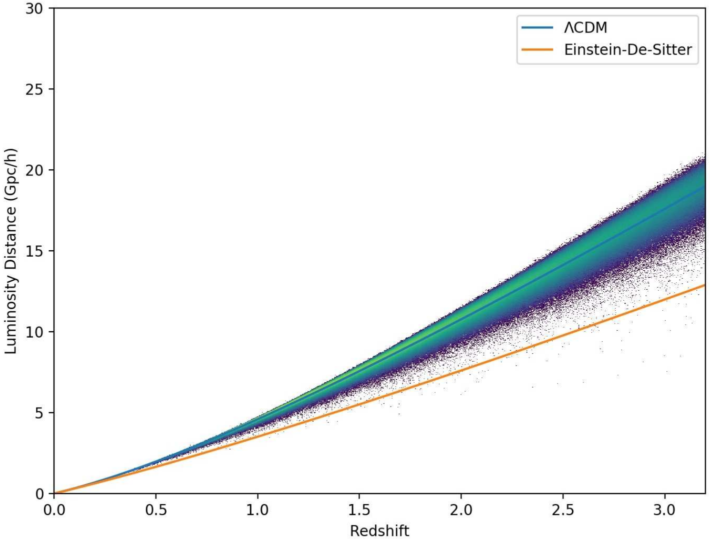
$\Downarrow$

observed angles & redshifts



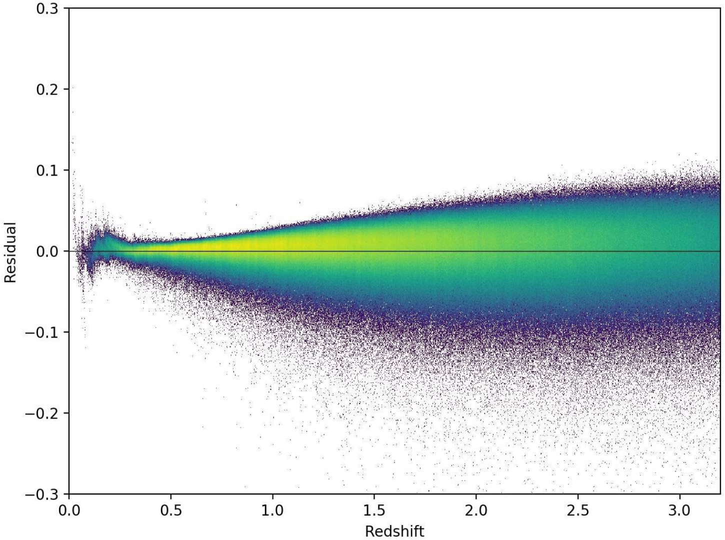
This allows us to construct the statistics of observed sources.

Ray Tracing



Adamek, Clarkson, Coates, Durrer & Kunz, Phys. Rev. **D100** (2019) 021301(R)

Ray Tracing



Adamek, Clarkson, Coates, Durrer & Kunz, Phys. Rev. **D100** (2019) 021301(R)

Summary

“Big Data” cosmological surveys have unprecedented statistical power

- perturbations of spacetime geometry are signal, not noise!

One can use a unified relativistic treatment to predict large-scale structure observables

- N-body simulations can be fitted with a relativistic spacetime
- ray tracing allows the inclusion of projection effects without approximation

Canonical Momentum

One-particle action $\Rightarrow$ canonical momentum

$$\mathcal{S} = -m \int \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau \qquad \Rightarrow \qquad \mathbf{q} = \frac{\partial \mathcal{L}}{\partial \mathbf{v}}$$

Geodesic equation

$$\frac{dq_i}{d\tau} = -\frac{\partial}{\partial x^i} \left(e^\psi \sqrt{\mathbf{q}^2 e^{2\phi} - q^j q^k h_{jk} + m^2 a^2} + q^j B_j \right)$$

$$\frac{dx^i}{d\tau} = \frac{\partial}{\partial q_i} \left(e^\psi \sqrt{\mathbf{q}^2 e^{2\phi} - q^j q^k h_{jk} + m^2 a^2} + q^j B_j \right)$$

Stress-energy tensor

$$T^0_0 = -\delta^{(3)}(\mathbf{x} - \mathbf{x}_{(n)}) \frac{e^{3\phi}}{a^4} \left(\sqrt{\mathbf{q}^2 e^{2\phi} - q^i q^j h_{ij} + m^2 a^2} + q^i B_i \right)$$

Einstein's Equations

$$-\frac{a^2}{2}G^0_0 = \frac{3}{2}e^{-2\psi}\left(\mathcal{H}-\phi'\right)^2 + e^{2\phi}\left[\Delta\phi - \frac{1}{2}\left(\nabla\phi\right)^2\right]$$

$$\frac{a^2}{2}G^0_i = e^{-\psi}\nabla_i\left[e^{-\psi}\left(\mathcal{H}-\phi'\right)\right] - \frac{1}{4}\Delta B_i$$

$$a^2\left(G^i_j - \frac{1}{3}\delta^i_j G^k_k\right) =$$
$$\left(\delta^{ik}\delta^l_j - \frac{1}{3}\delta^i_j\delta^{kl}\right)\Big[e^{\phi+\psi}\nabla_k\nabla_l e^{\phi-\psi} - 2e^{2\phi}\left(\nabla_k\psi\right)\left(\nabla_l\psi\right) +$$
$$B'_{(k,l)} + 2\mathcal{H}B_{(k,l)} + \frac{1}{2}h''_{kl} + \mathcal{H}h'_{kl} - \frac{1}{2}\Delta h_{kl}\Big]$$

Here I dropped quadratic and higher-order terms only with B_i or h_{ij}.

For computational efficiency the exponentials can be expanded (weak-field expansion).

Power Spectra