Weak singularities in large-scale structure: identification and workaround

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MARIE CURIE ACTIONS

We know that...

- at early times, cold dark matter (CDM) is in the **single-stream regime** (that comes with a single-valued velocity)

- collisionless nature of CDM leads to crossing of trajectories, called **shell-crossing** (where the density $\rightarrow \infty$)

Outline for today

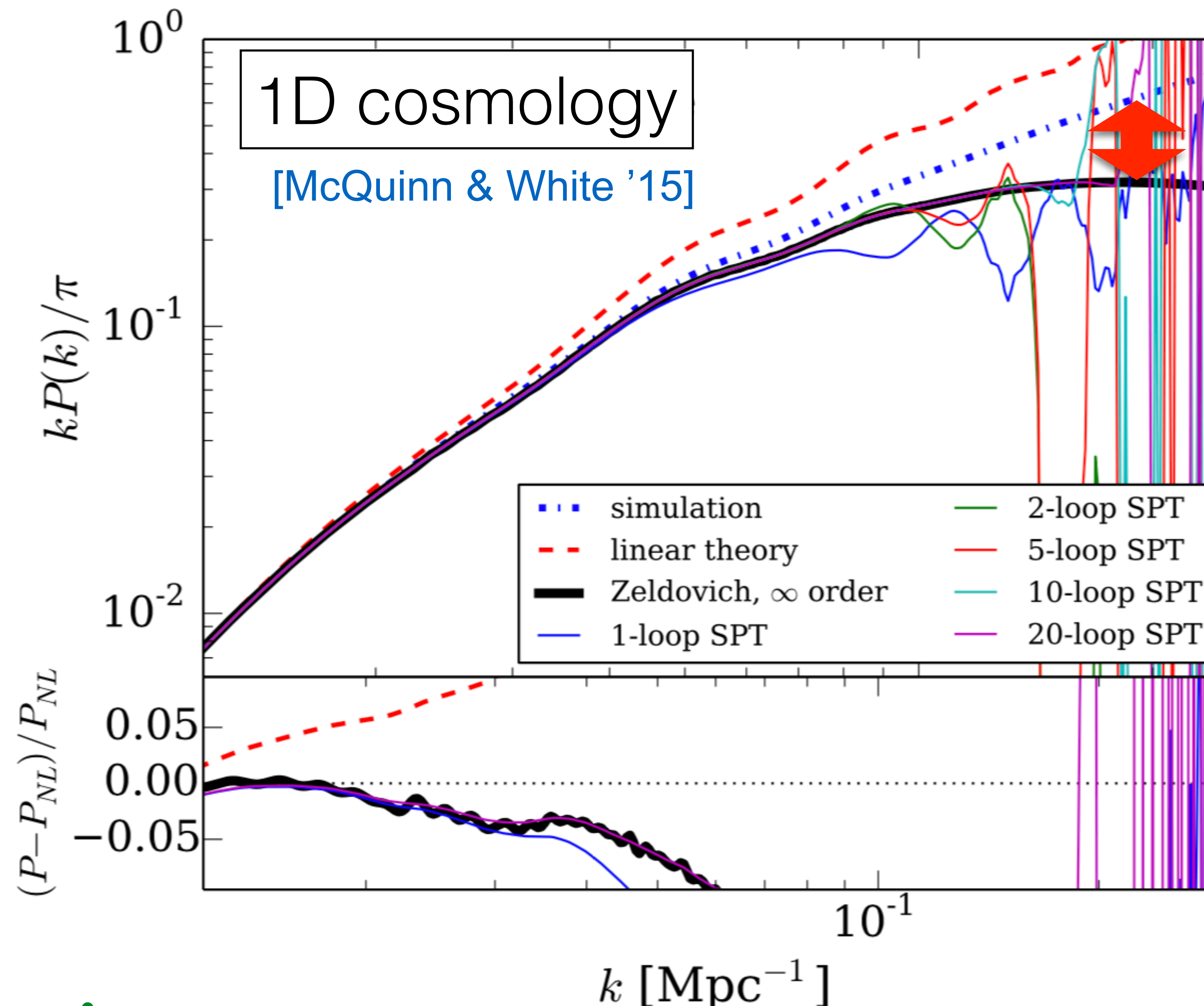
- ✦ use **novel analytical method** to follow CDM, *through shell-crossings*, into the **multi-stream regime** (where the velocity dispersion is non-zero)

- ✦ in the multi-stream regime, particle trajectories exhibit **weakly singular behaviour** (e.g. edges in the particle acceleration)

- ✦ **confirmed** by high-resolution N-body simulations

Motivation

Shell-crossing / multi-streaming effects are key **theoretical uncertainties** for the matter power spectrum



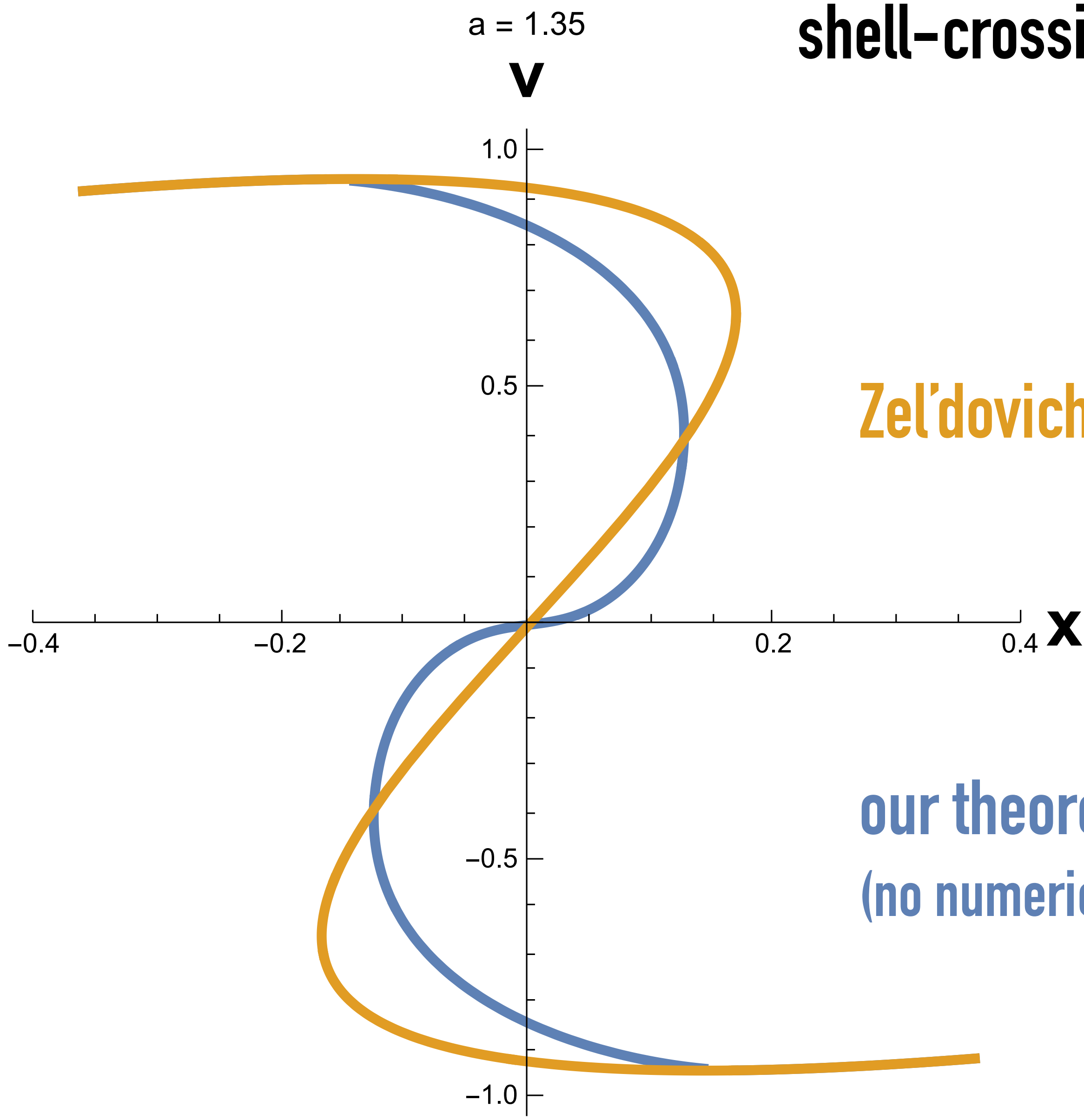
Analytical insight could assist in…

- closing the gap between theory and numerics
- make numerical simulations more efficient (including fastPM, COLA)
- calculate counter / UV terms for effective theories

Sneak Preview: phase-space evolution in 1D

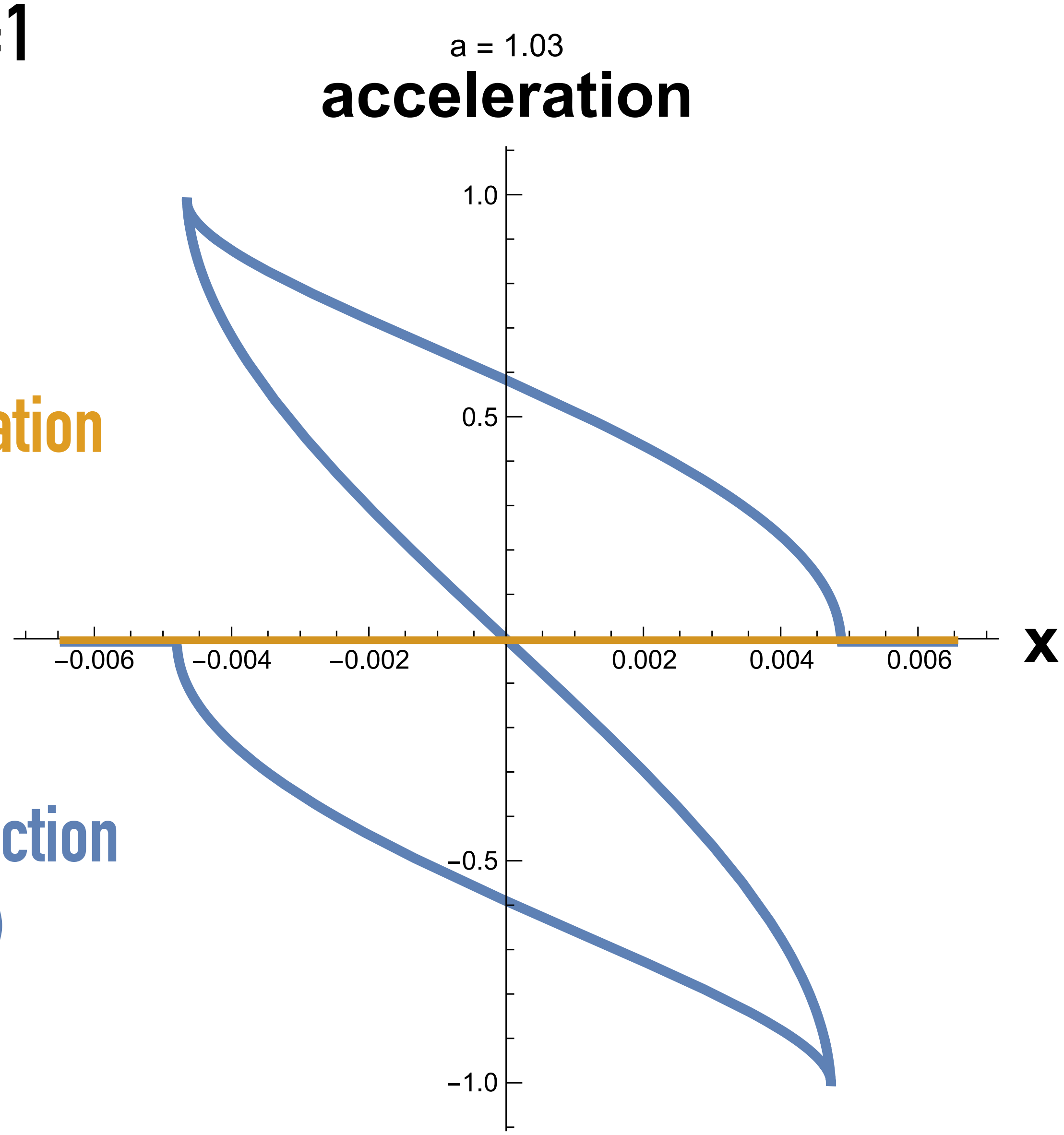
[CR, Hahn & Frisch, in prep.]

shell-crossing is at a=1



Zel'dovich approximation

our theoretical prediction
(no numerics required!!)



acceleration is locally not differentiable -> weak singularities in Vlasov-Poisson

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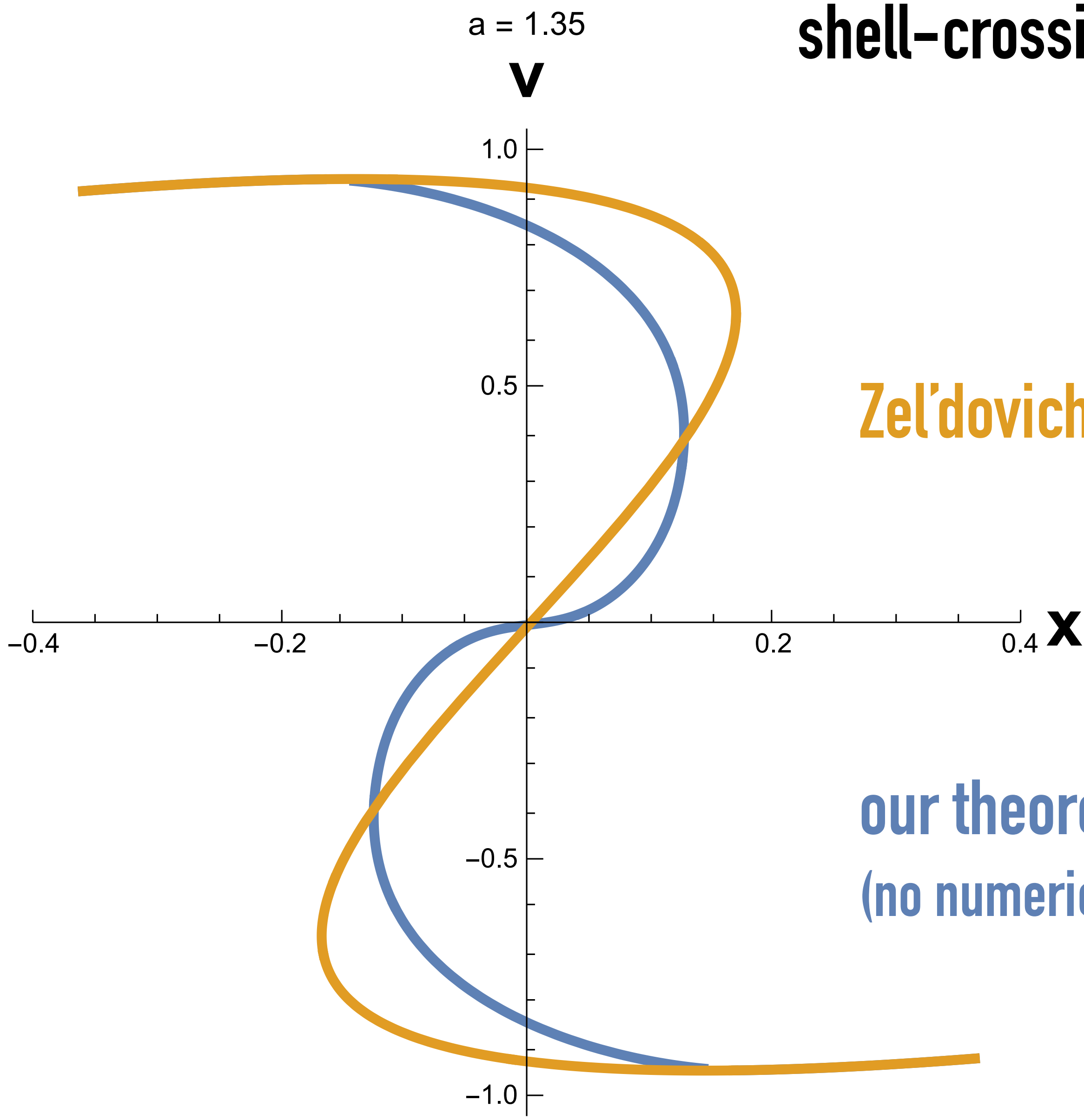

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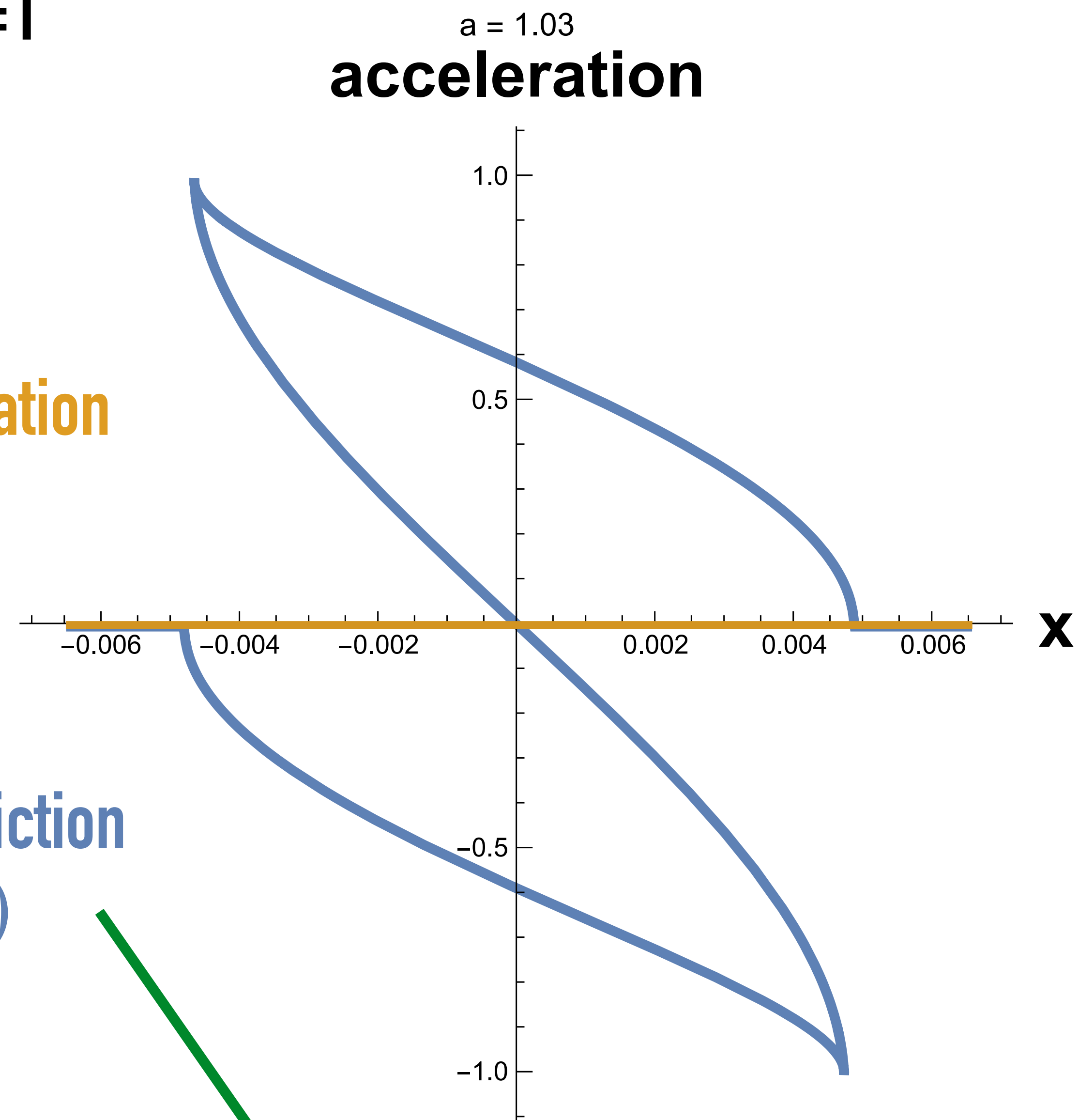


agrees with N-body simulations to very high precision!

acceleration is locally not differentiable -> weak singularities in Vlasov-Poisson

Our theoretical model

central object: the **Lagrangian map** $\boldsymbol{q} \mapsto \boldsymbol{x}(\boldsymbol{q}, a) = \boldsymbol{q} + \boldsymbol{\xi}(\boldsymbol{q}, a)$

initial position of CDM particles

current position of CDM particles

displacement field

- contains all the dynamical information of the physical system at all times
- in contrast to the density: does not blow up at shell-crossing!

General strategy:

1. solve for the CDM trajectories until the first shell-crossing
   by Lagrangian perturbation theory (LPT)
2. provide boundary conditions at shell-crossing
   for this we need shell-crossing solutions to sufficient accuracy
3. solve refined multi-stream equations
   see later

Exact analytical solutions in the single-stream regime for

(= representable by converging Taylor series in LPT)

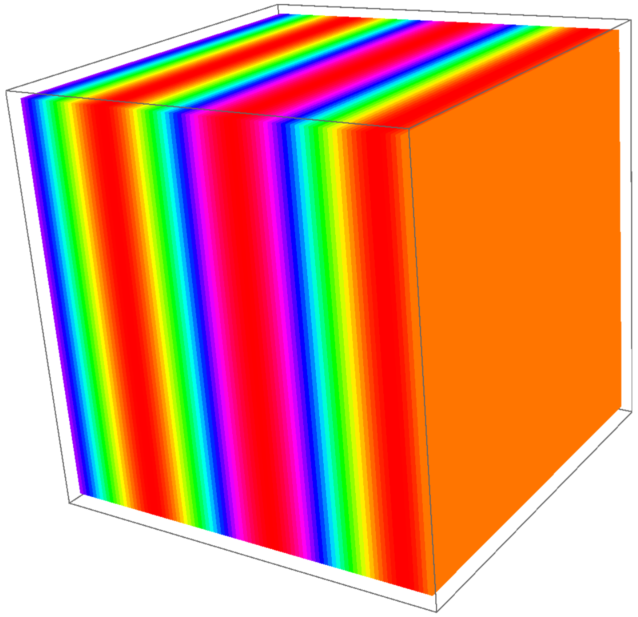
- 1D collapse
- quasi-1D collapse

} **embedded in 3D**

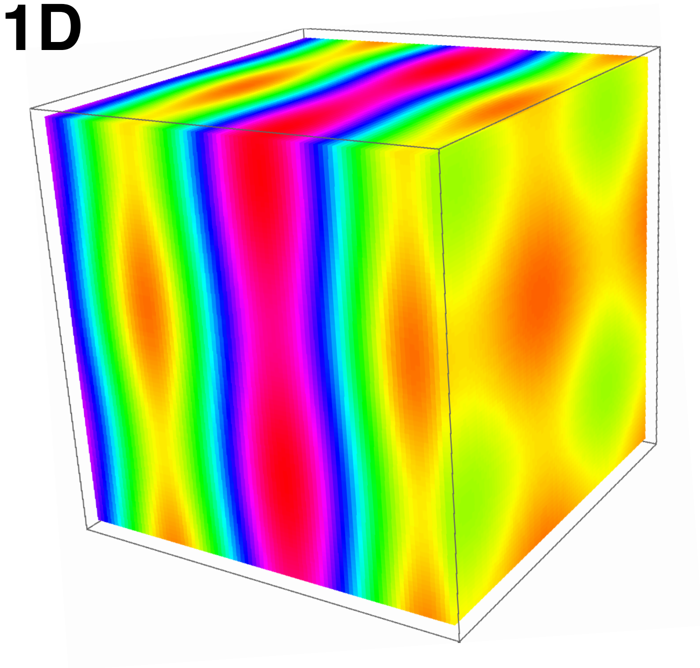
[Novikov '69, Zel'dovich '69]

[CR & Frisch '17]

1D



quasi-1D



Single-stream regime

Exact analytical solutions in the single-stream regime for

(= representable by converging Taylor series in LPT)

- 1D collapse [Novikov '69, Zel'dovich '69]
- quasi-1D collapse [CR & Frisch '17]

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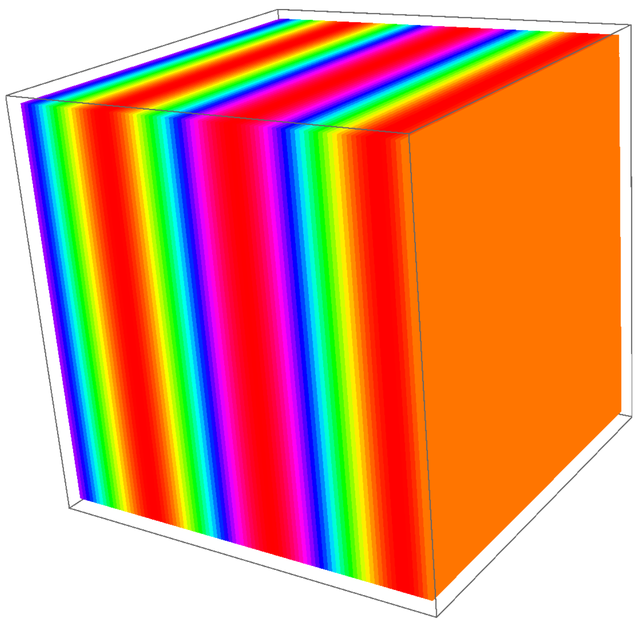
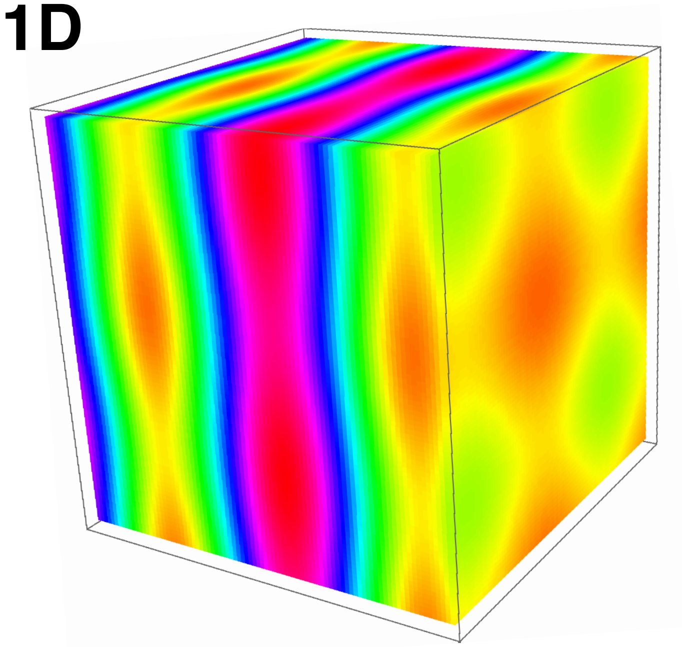
1D



quasi-1D

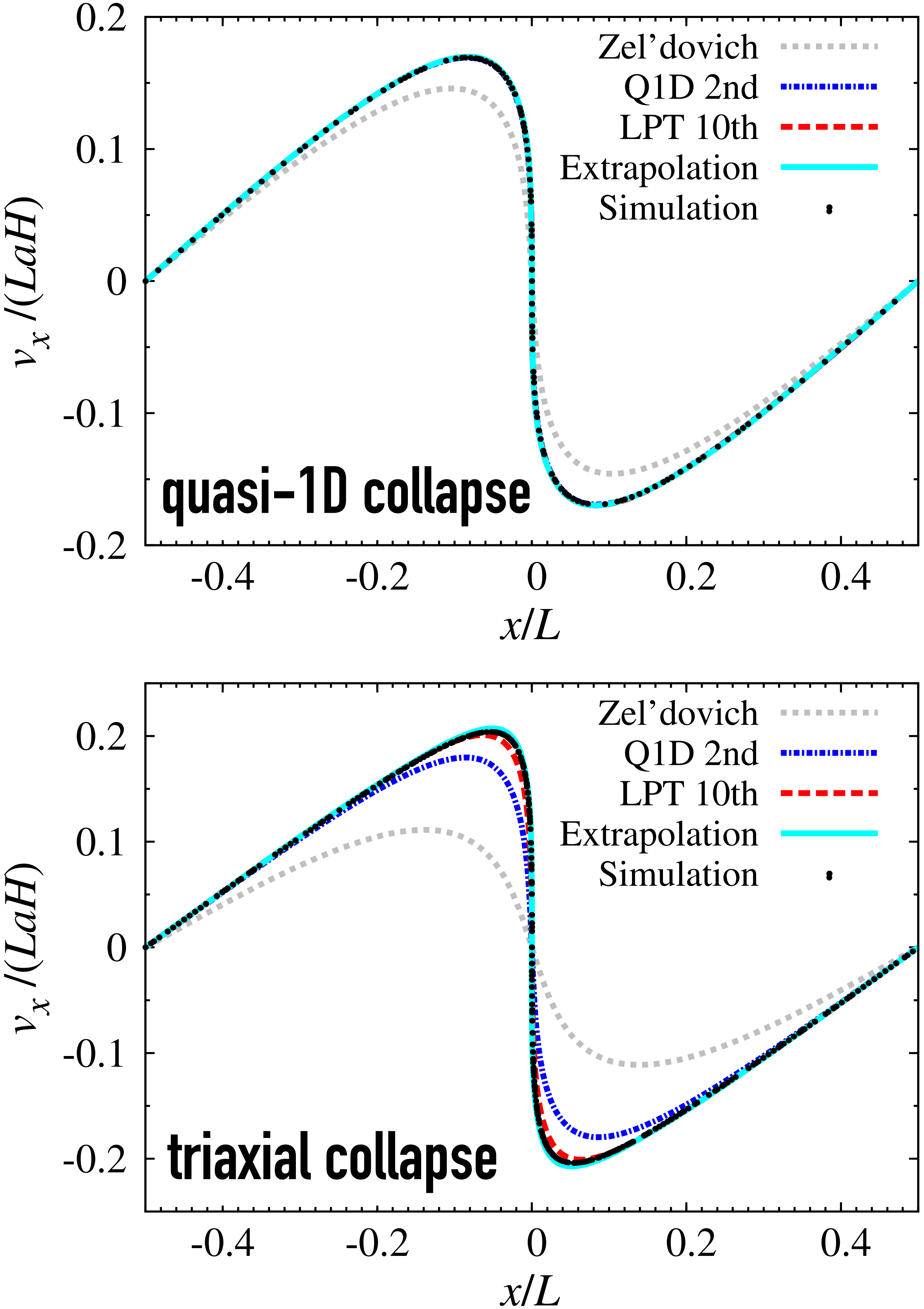


- spherical top hat collapse [excluding shell-crossing] [Peebles '67]
- quasi-spherical collapse (perturbed top hat) [CR '19]

… and for cosmological initial conditions, **LPT is certainly converging**, most likely even until shell-crossing [Zheligovsky & Frisch '14; CR, Villone & Frisch '15]

LPT solutions at shell-crossing in 3D

[Saga, Taruya & Colombi, PRL 2018]

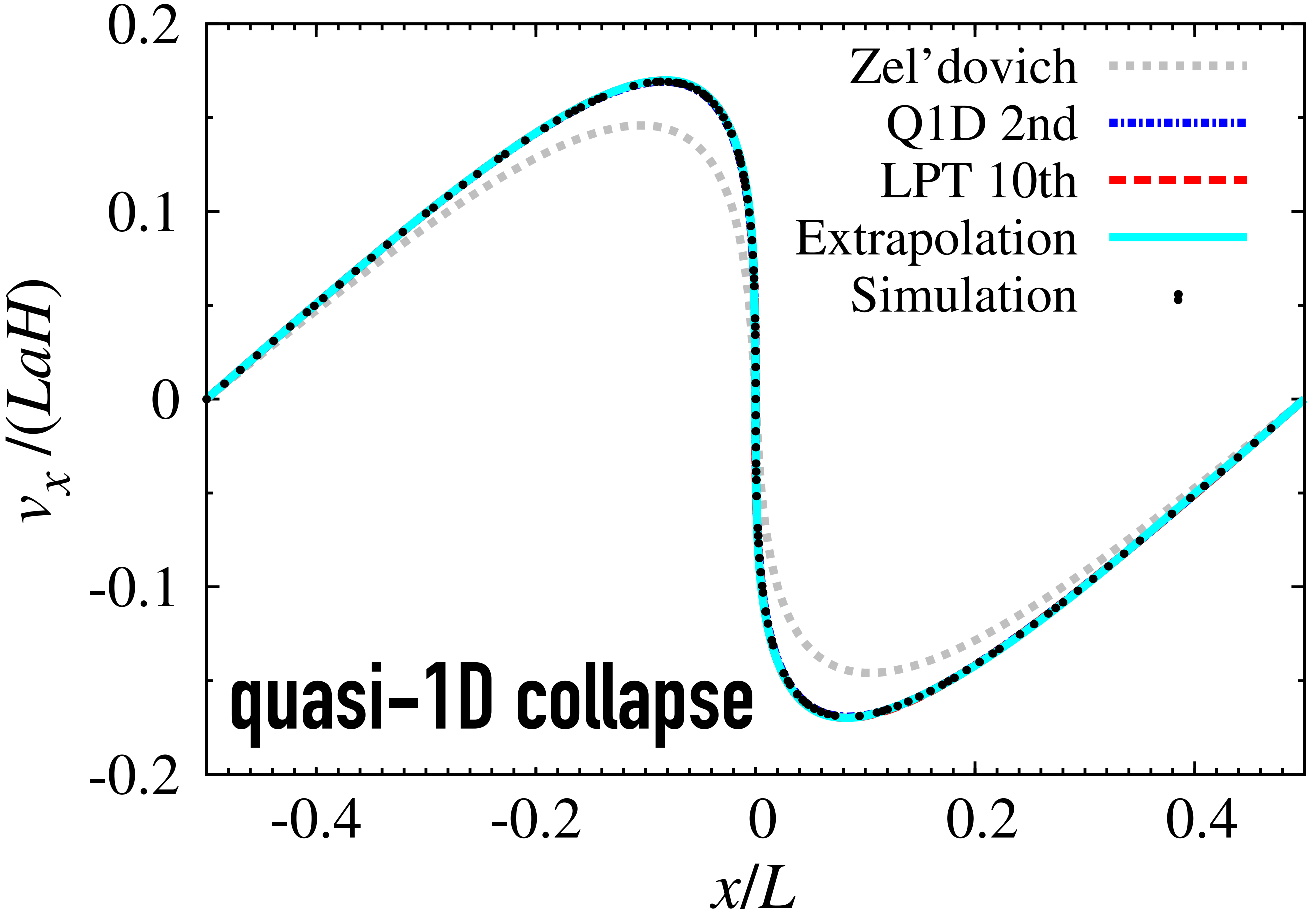


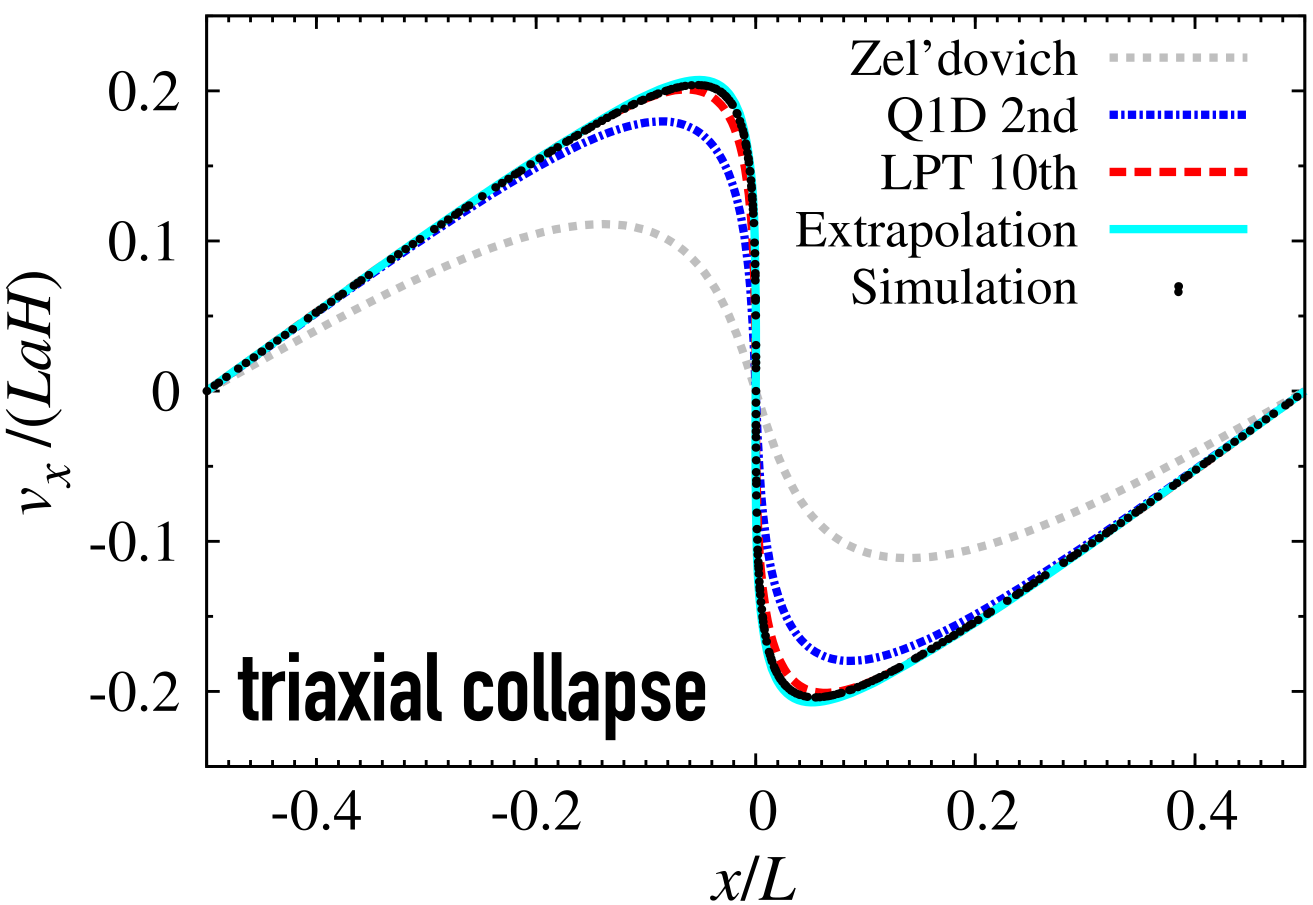
initial conditions merely

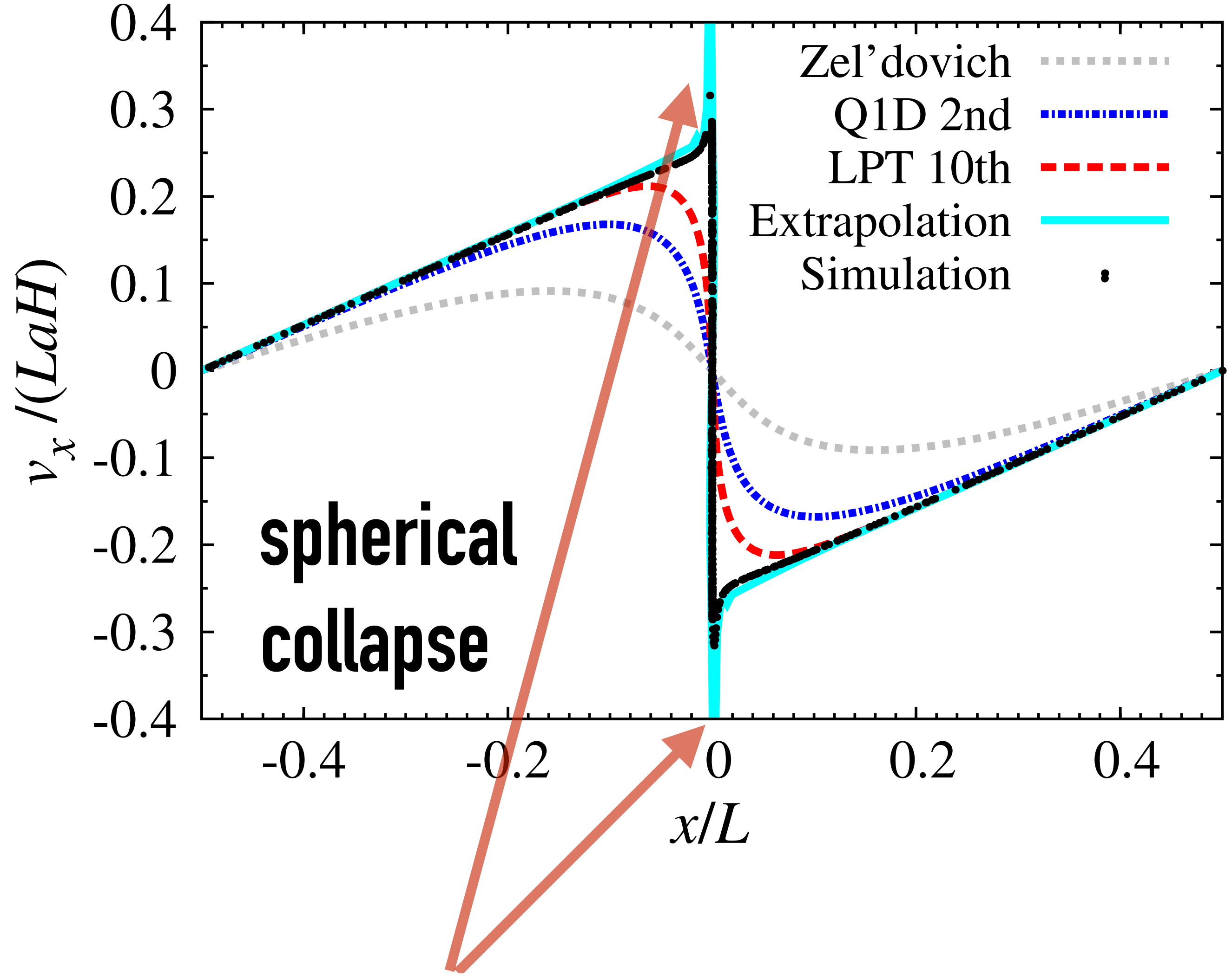
affect the speed of convergence!

LPT solutions at shell-crossing in 3D

[Saga, Taruya & Colombi, PRL 2018]







singular velocity at shell-crossing,
hence avoid simplistic spherical collapse
(quasi spherical is free of singularities)

[CR '19]

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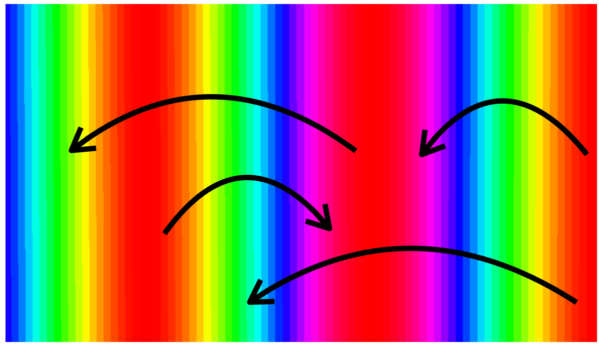
momentum conservation: $\ddot{\boldsymbol{x}}(\boldsymbol{q},a) \propto -\nabla_x \varphi_{\rm g}(\boldsymbol{x}(\boldsymbol{q},a))$

acceleration $\propto$ gravitational force

here ignoring Hubble
$\left(\nabla_x^2 \varphi_{\rm g} \propto \delta\right)$

- force computation is non-trivial in multi-stream regions:



mass reshuffling in 1D

- to determine the multi-stream solutions we approximate

$$\ddot{\boldsymbol{x}}(\boldsymbol{q},a) \propto -\nabla_x \varphi_{\rm g}(\boldsymbol{x}_{\rm sc}(\boldsymbol{q},a))$$

where $\boldsymbol{x}_{\rm sc}$ is the solution that is valid until shell-crossing

delivers very accurate solution $x(q,a)$ shortly after shell-crossing

(… and the accuracy can be increased with perturbative refinements)

Results in 1D

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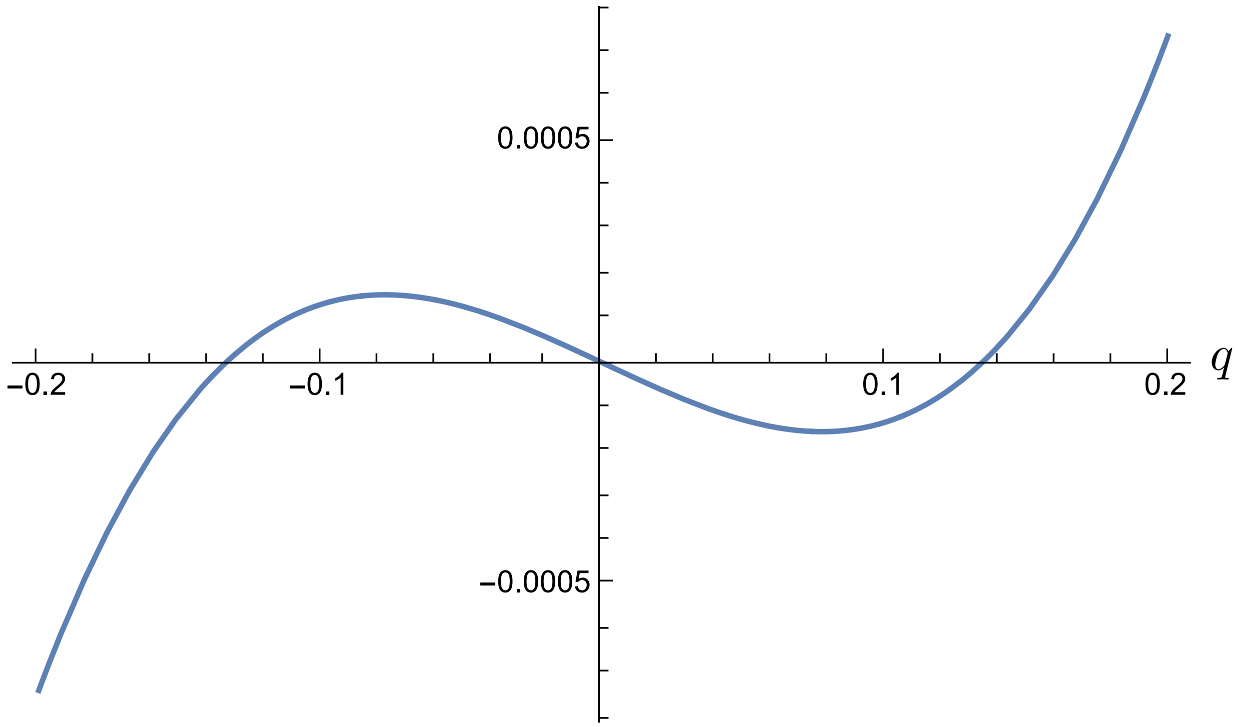
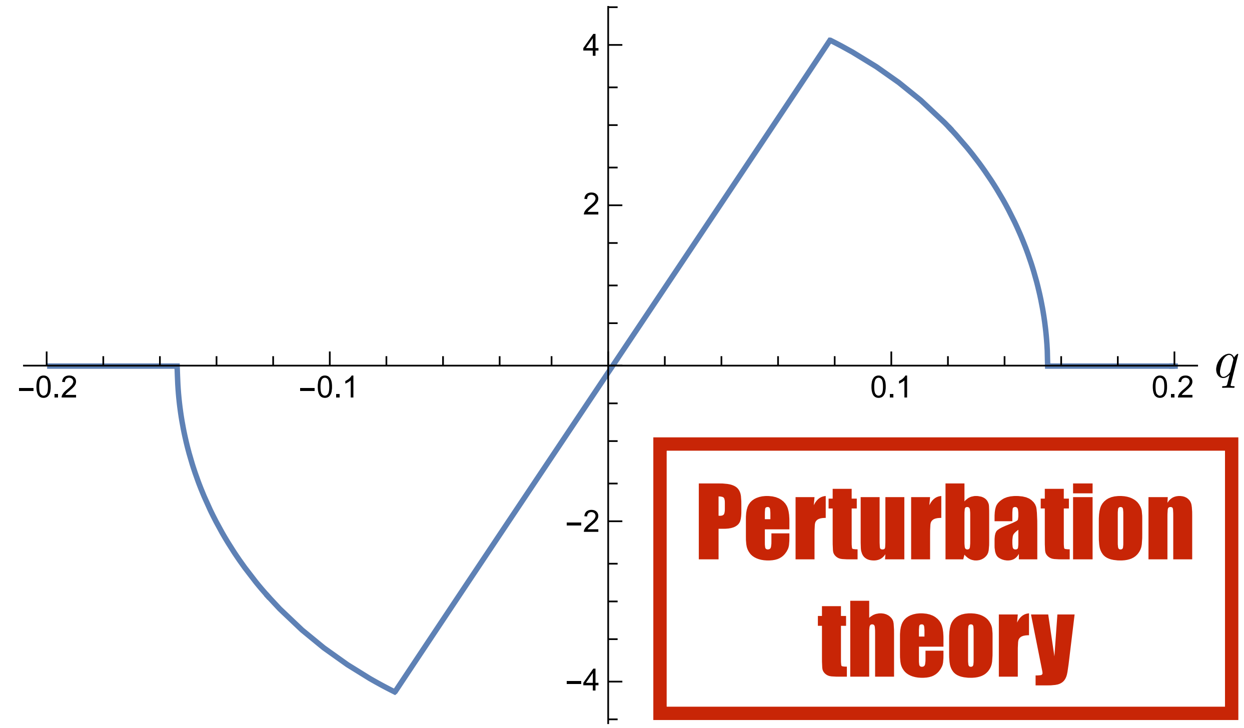
[CR, Hahn & Frisch, in prep.]

post-collapse particle trajectory $x(q)$



post-collapse acceleration $\ddot{x}(q)$



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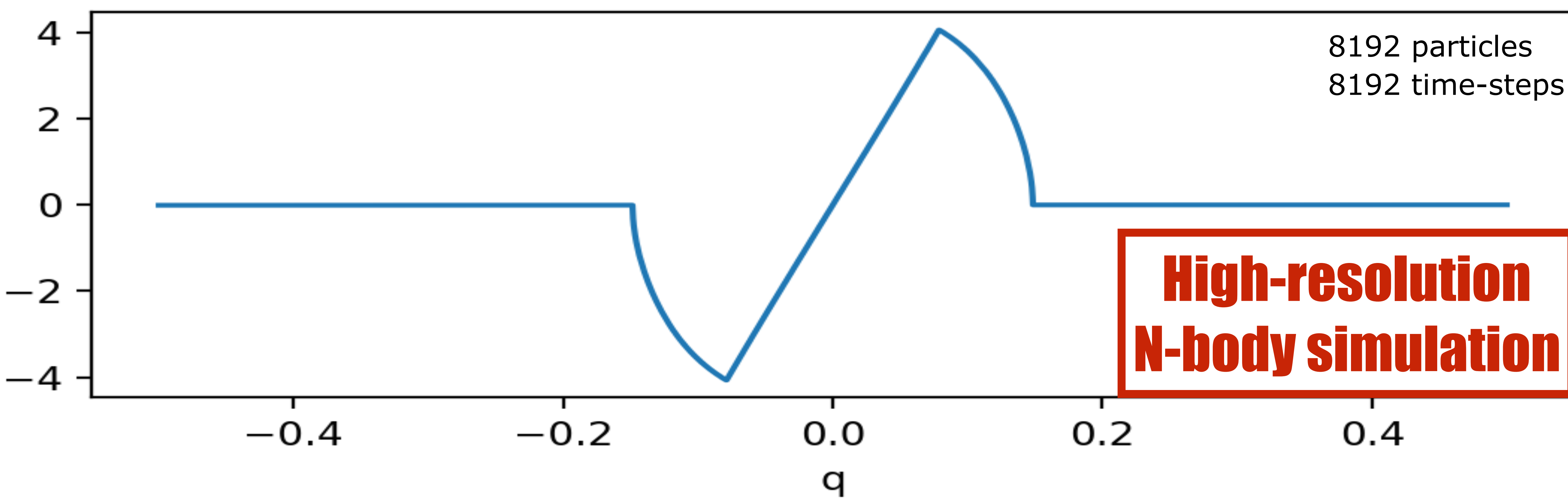

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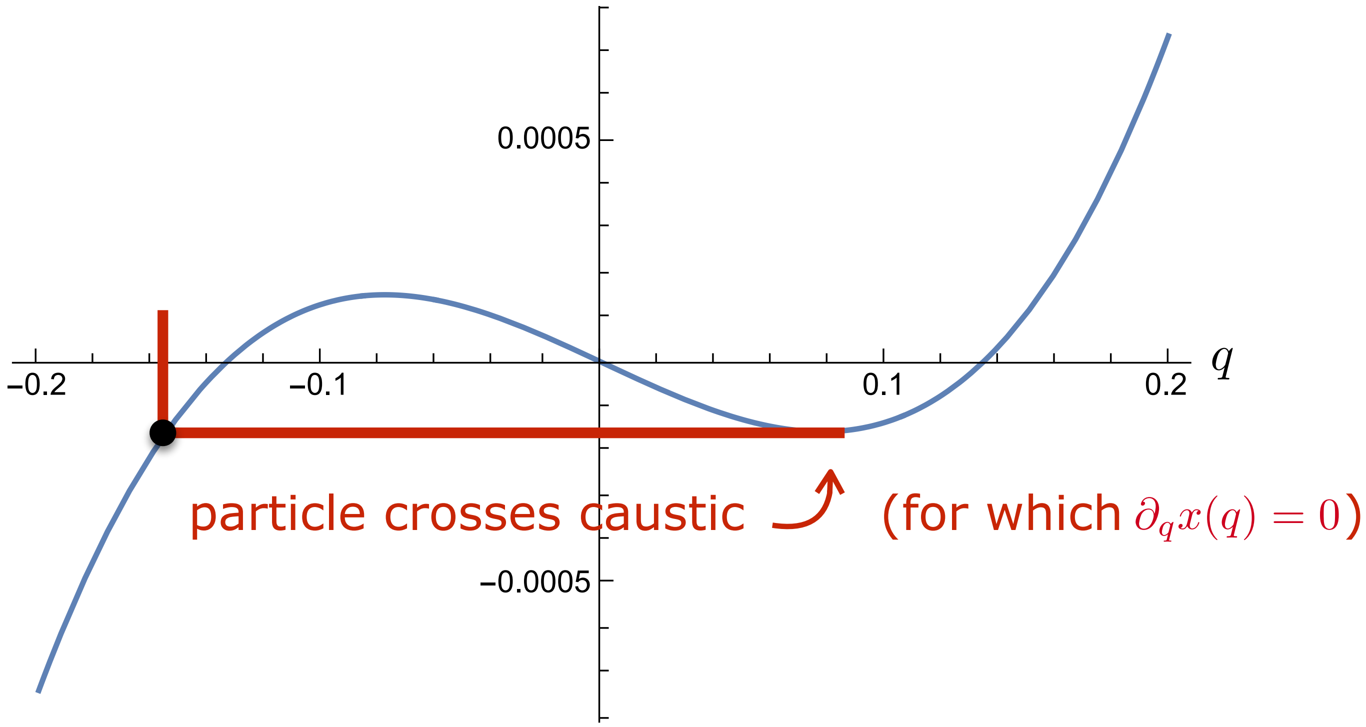

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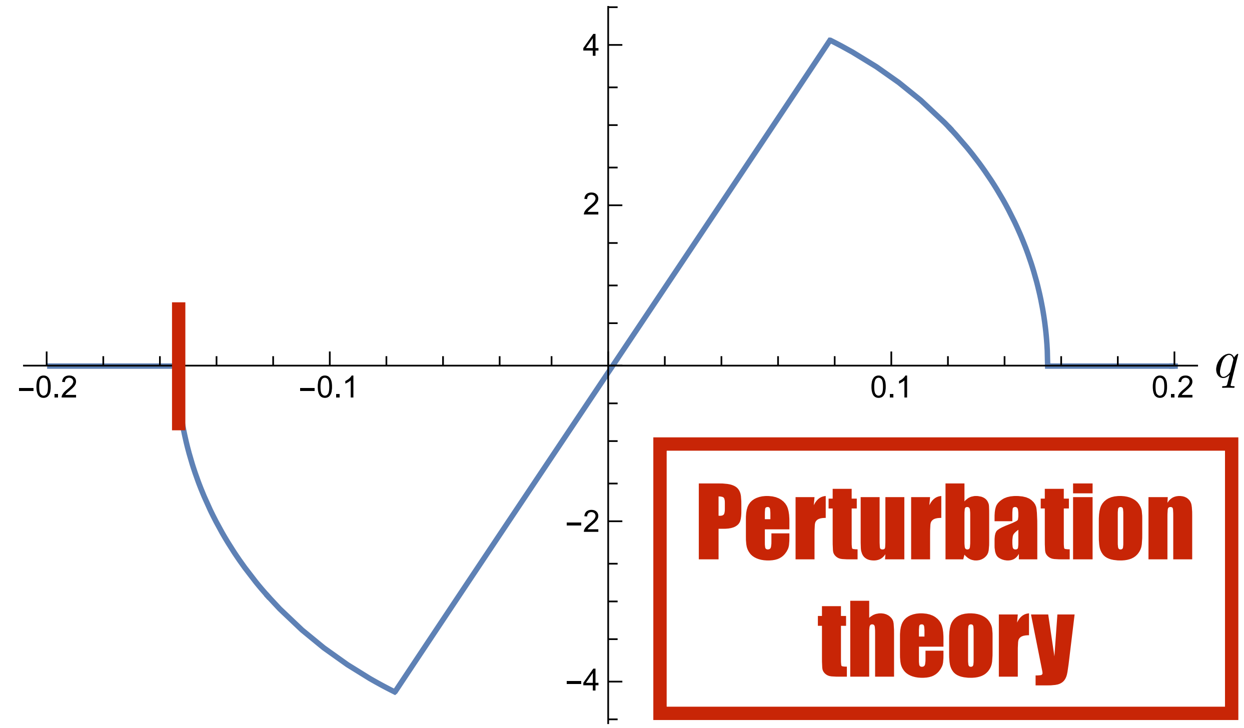

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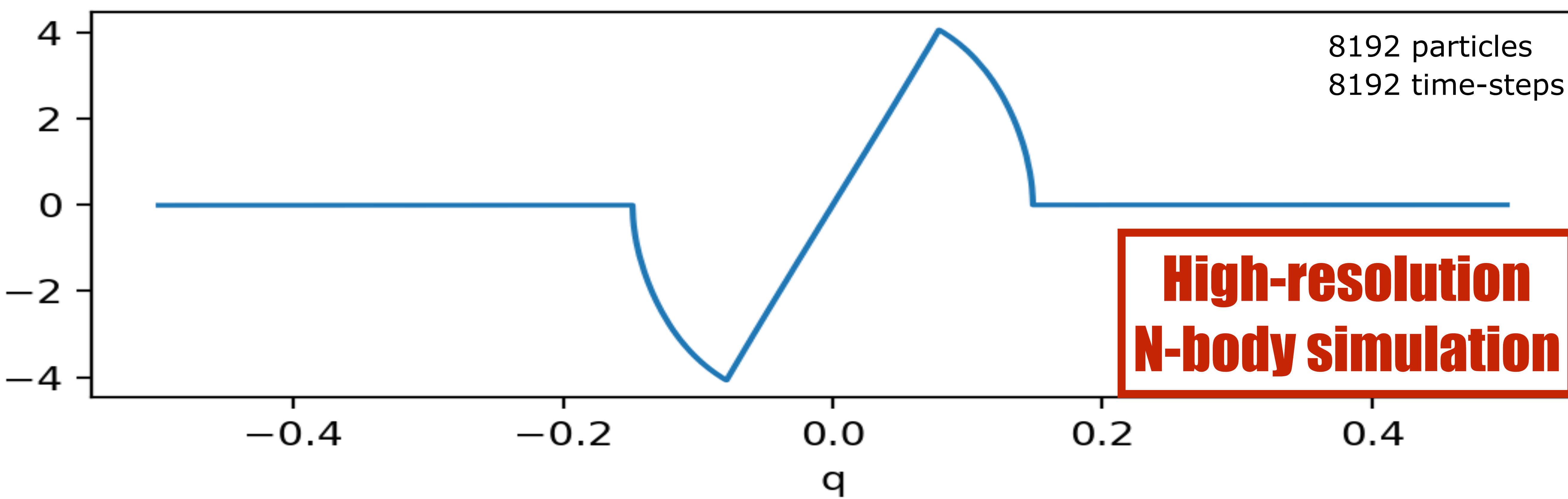

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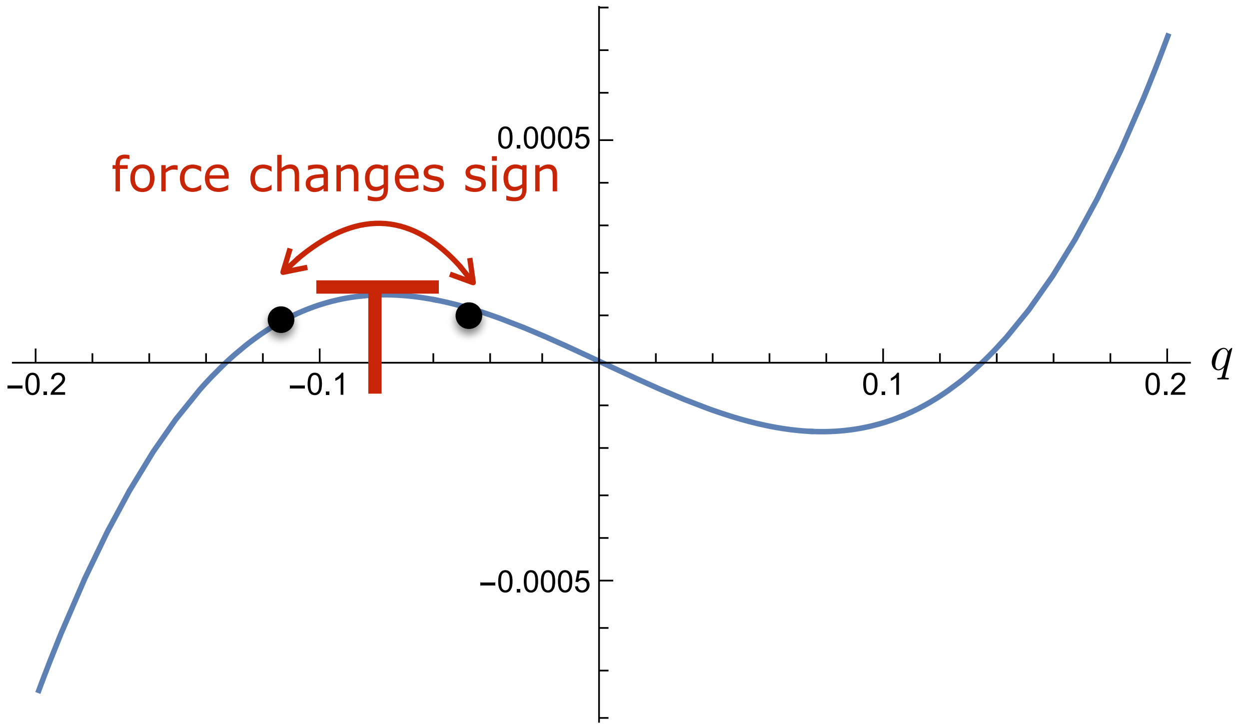

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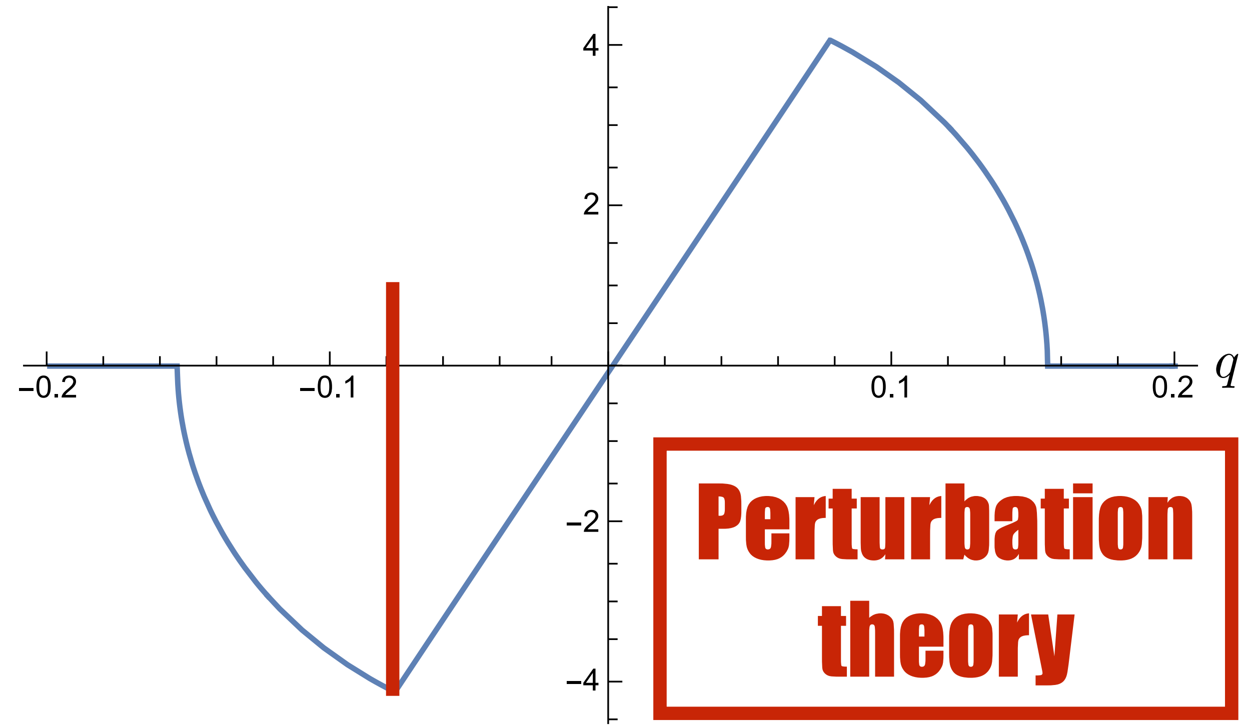

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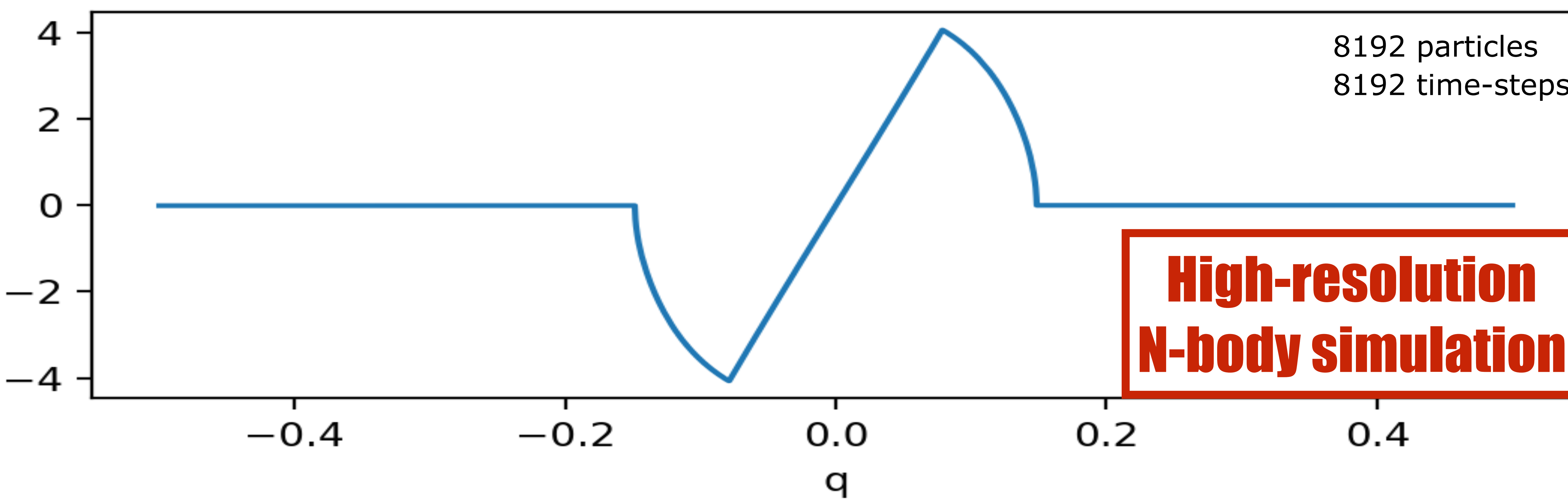

Conclusions & Outlook

- weak singularities in the multi-stream regime of structure formation, due to the perfect coldness of CDM
- we have significantly closed the gap between theory and numerics
- still a lot of work to do beyond 1D
- **singular features** may be **regulated** by
  - **deviating from perfect coldness**:
    never single-stream, theoretical modelling complicated, requires full-fledged Vlasov-Poisson
  - **employing semiclassical (Schrödinger-like) descriptions**
    $\hbar > 0$ acts as a softening scale that regulates singularities

    [Kopp, Vattis & Skordis '18; Mocz et al '18; Uhlemann, CR, Gosenca & Hahn '19]

XIII Tonale Winter School on Cosmology

9th – 13th December 2019, Passo del Tonale, Italy

Registration by 1st October 2019

indico.physi.uni-Heidelberg.de/event/103/

Neutrino Cosmology
Marco Drewes, UCLouvain

Non-Linear Perturbation Theory
Massimo Pietroni, INFN

Testing Dark Energy with observations
Vanina Ruhlmann-Kleider, CEA

CMB polarization and spectral distortions
Mathieu Remazeilles, U. Manchester

The number of participants is limited; travel grants available.

#WSCtonale

Organizers: L. Amendola, S. Casas, M. Irfan, M. Maturi, C. Rampf, J. Rubio, E. Villa.

Convergence for generic ICs

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- Except for singularities in the spherical collapse (which are removed in the quasi-spherical collapse), **LPT is converging, probably** even until shell-crossing

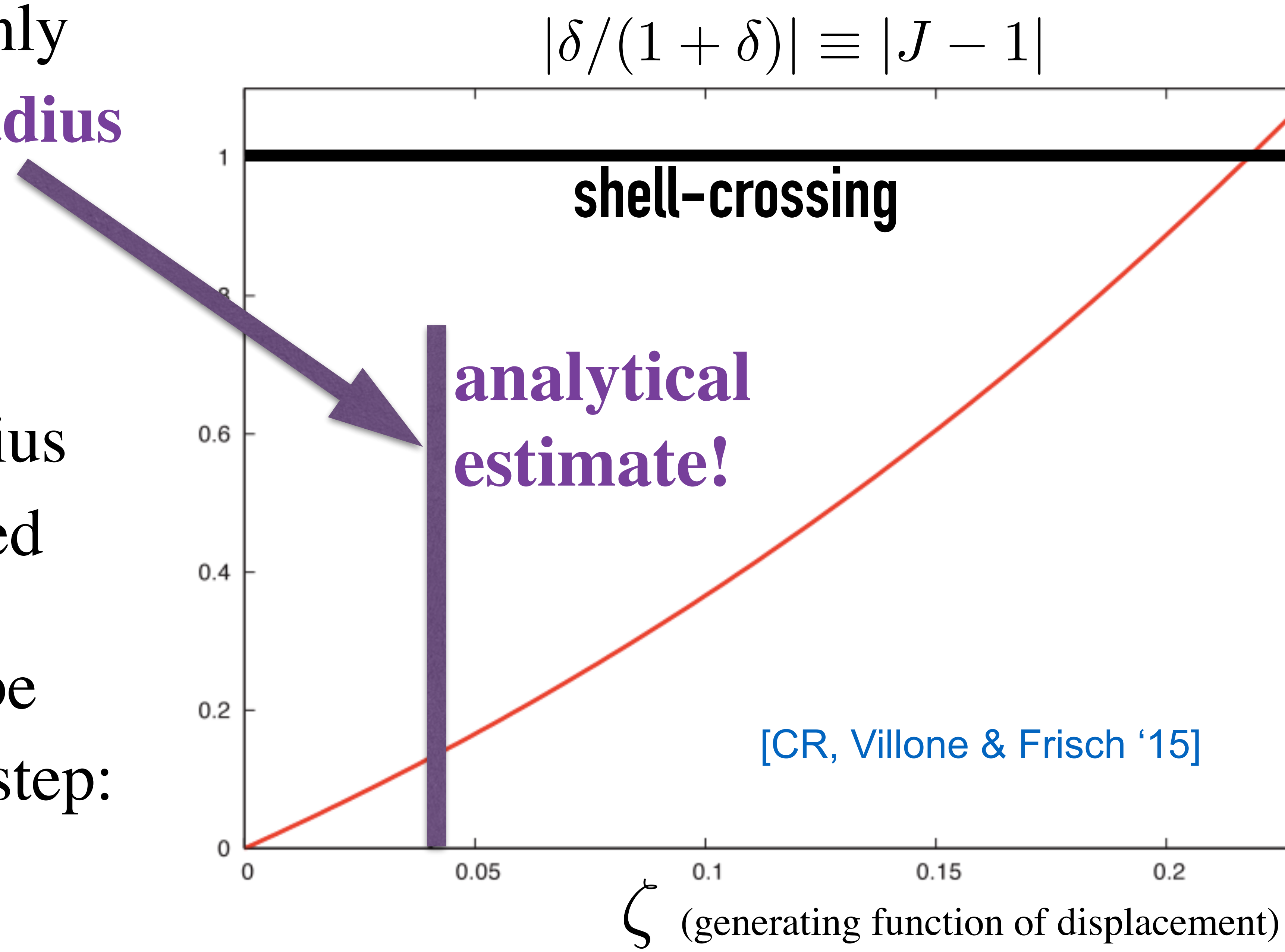
- For **cosmological ICs**, only a **lower bound on the radius** of convergence is known

  [Zheligovsky & Frisch '14]

- Numerical studies of radius of convergence are needed

- If shell-crossing cannot be reached in a single time-step: analytic continuation!

  [both in preparation]



From the Lagrangian scalar equation, one gets $(\nabla^{\mathrm{L}} \equiv \nabla_{\boldsymbol{q}})$

$$\nabla^{\mathrm{L}} \cdot \boldsymbol{\xi}^{(n)} = \nabla^{\mathrm{L}} \cdot \boldsymbol{v}^{(\mathrm{init})} \delta_1^n + \sum_{0<s<n} \frac{s^2 + (s-n)^2 + (n-3)/2}{2n^2 + n - 3} \left(\xi_{i,j}^{(n-s)} \xi_{j,i}^{(s)} - \xi_{i,i}^{(n-s)} \xi_{j,j}^{(s)} \right)$$
$$- \frac{1}{6} \sum_{s_1+s_2+s_3=n} \frac{s_1^2 + s_2^2 + s_3^2 + (n-3)/2}{n^2 + (n-3)/2} \varepsilon_{ikl} \varepsilon_{jmn} \xi_{i,j}^{(n_1)} \xi_{k,m}^{(n_2)} \xi_{l,n}^{(n_3)}$$

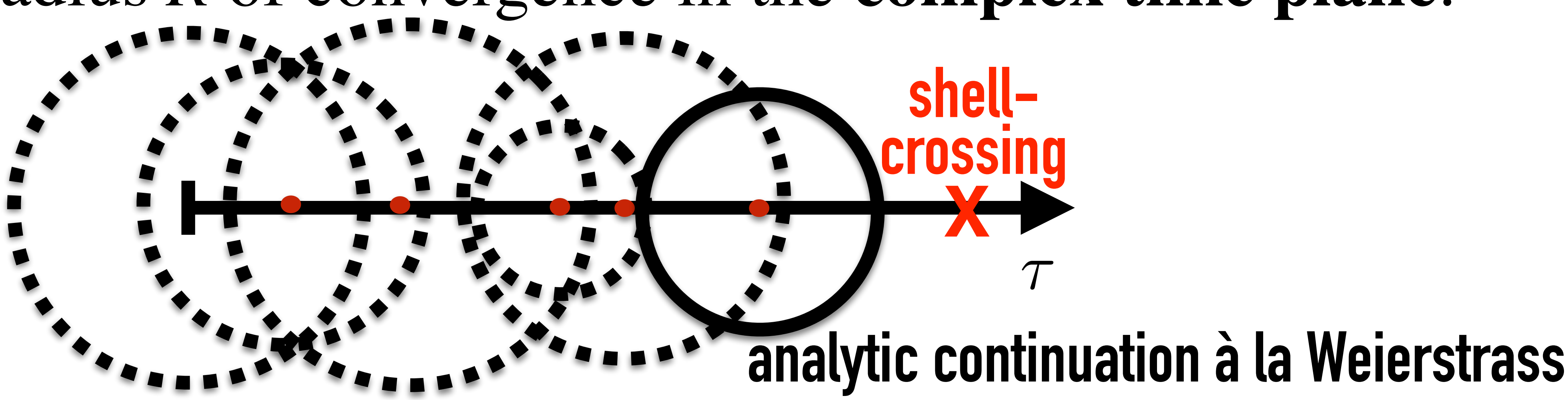
And from the vector equation:

$$\nabla^{\mathrm{L}} \times \boldsymbol{\xi}^{(n)} = \sum_{0<s<n} \frac{n-2s}{2n} \nabla^{\mathrm{L}} \xi_k^{(s)} \times \nabla^{\mathrm{L}} \xi_k^{(n-s)}$$

Summing up, the usual Helmholtz decomposition then gives

$$\boldsymbol{\xi}(\boldsymbol{q}, \tau) = \sum_{n=1}^{\infty} \boldsymbol{\xi}^{(n)}(\boldsymbol{q})\, \tau^n \qquad \Rightarrow \qquad \boldsymbol{x} = \boldsymbol{q} + \boldsymbol{\xi}$$

Typically, the Lagrangian map is analytic but not entire in time; it has a finite radius *R* of convergence in the **complex time plane**:



[CR, Villone & Frisch, 1504.00032: bounds on radius of convergence in ΛCDM]

Multi-stream computation

- Until shell-crossing, the gravitational force is given by ZA (in 1D).
  To leading order, this also holds shortly after (due to momentum conservation)

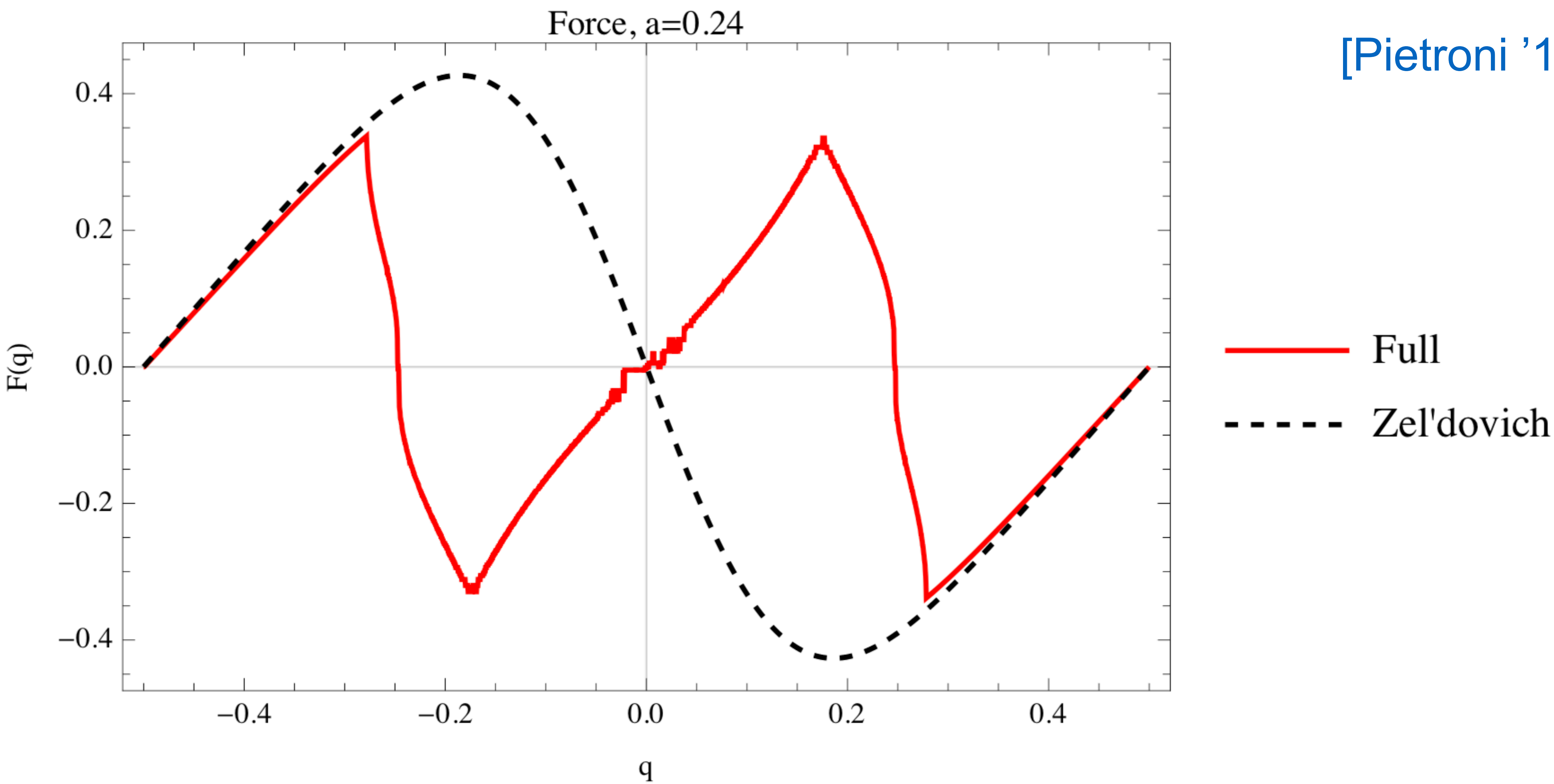
$$\ddot{\boldsymbol{x}}_{\rm PZA}(\boldsymbol{q},a) \propto -\nabla_x \varphi_{\rm g}(\boldsymbol{x}_{\rm ZA}(\boldsymbol{q},a)) \tag{1}$$

- To determine the force, Taruya & Colombi '17 use a (non-local) Green's function approach; some integrals need to be approximated

- To exploit the nonlinear power of LPT, we use the local expression

$$\begin{aligned}\delta(\boldsymbol{x}(\boldsymbol{q},a)) &= \int \delta_{\rm D}^{(3)}\left[\boldsymbol{x}(\boldsymbol{q},a) - \boldsymbol{x}(\boldsymbol{q}',a)\right] {\rm d}^3q' - 1 \\ &= \int \sum_{k=1}^{n} \frac{\delta_{\rm D}^{(3)}\left[\boldsymbol{q}' - \boldsymbol{q}_n\right]}{|\det[\nabla_q \boldsymbol{x}(\boldsymbol{q}_n,a)]|} {\rm d}^3q' - 1\end{aligned}$$

- Analytical solutions for (1) are no simple power laws [see also: Pietroni '18]



[Pietroni ’18]

[in the ‘full’ result, the ZA part is not subtracted out]