On loops in inflation: CMB bounds on field content of the universe

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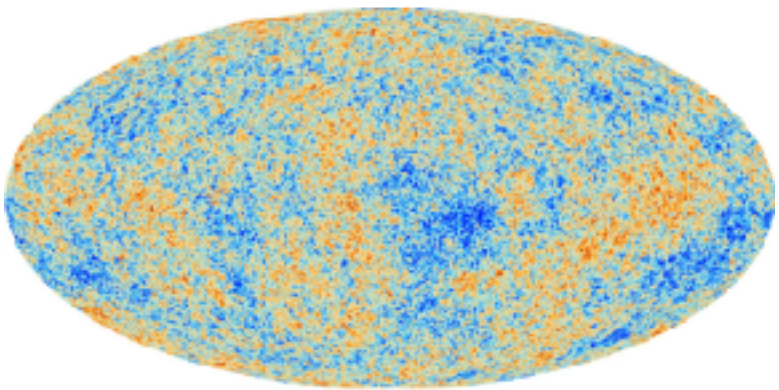
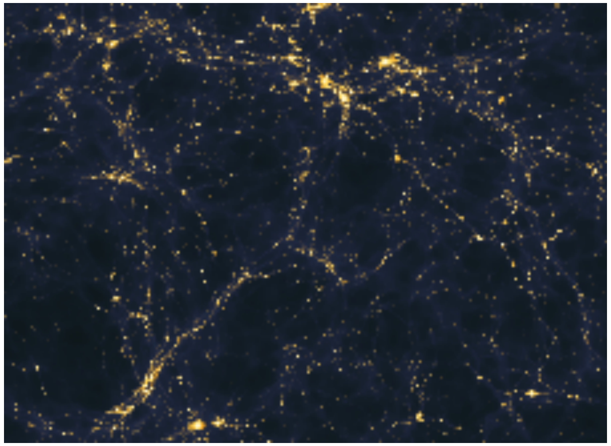
Motivation

- **Inflation** is a natural mechanism for producing a nearly scale invariant spectrum of primordial density fluctuations, consistent with observations.



Cosmic Microwave Radiation Background



Large Scale Structure of the Universe

- Originated as quantum vacuum fluctuations, amplified by the inflationary expansion

$$\mathcal{P}_\zeta(k) = \frac{H^2}{8\pi^2 \epsilon M_{pl}^2} \left(\frac{k}{k_\star} \right)^{n_s - 1}$$

$$\Delta_\zeta \sim 2 \times 10^{-9} \qquad n_s \approx 0.96$$

Motivation

- Amplification of vacuum fluctuations takes place for **any** field in the early universe, whether or not it couples to the inflaton (all fields couple to gravity).

- Some proposals to solve the Hierarchy problem require the existence of a **huge number** of mutuallly non-interacting copies of the SM (hidden fields).

$$\Lambda_G^2 \sim \frac{M_{pl}^2}{N} << M_{pl}^2 \longleftrightarrow N >>> 1$$

"Large extra dimensions": *$N = 10^{32}$* in form of KK gravitons [Dvali 0760.2050]

"*N*naturalness": *$10^4 < N < 10^{16}$* copies of the SM [Arkani-Hamed et al 1607.06821]

Goals

- Analyze the impact of a **large number** N of **hidden fields** on primordial two-point functions of curvature and tensor perturbations.

- Corrections can only come from **loops**. Need to do higher order calculations (not trivial task).

- Is there a distinctive signature of the hidden sector? If so, can we use it to constrain it?

Setup of the calculation

- Action of the theory (gravity + inflaton + hidden sector)

$$S[g,\phi,\chi] = \int d^4x\sqrt{-g}\left\{\frac{M_{pl}^2 R[g]}{2} - \frac{1}{2}g^{ab}\nabla_a\phi\nabla_b\phi - V[\phi]\right\} + \boxed{S[\chi]}$$

- Scalar and tensor perturbations around a quasi-de Sitter background can be worked out from ADM metric in unitary gauge: $\phi(t,x) = \phi_0(t)$,

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$h_{ij}(t,\vec{x}) = a^2(t) e^{2\zeta(t,\vec{x})} \exp\left[\gamma_{ij}(t,\vec{x})\right] \qquad \epsilon = -\frac{\dot{H}}{H^2} << 1$$

Setup of the calculation

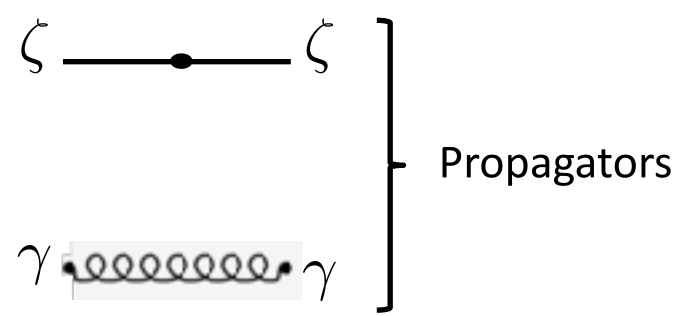
- Solve for the lapse *N* and shift N^i, expand the action in perturbation theory:

$$S[\zeta,\gamma,\chi] = S_{\zeta\zeta} + S_{\gamma\gamma} + S_{\chi\chi} + S_{\zeta\chi\chi} + S_{\gamma\chi\chi} + \dots$$

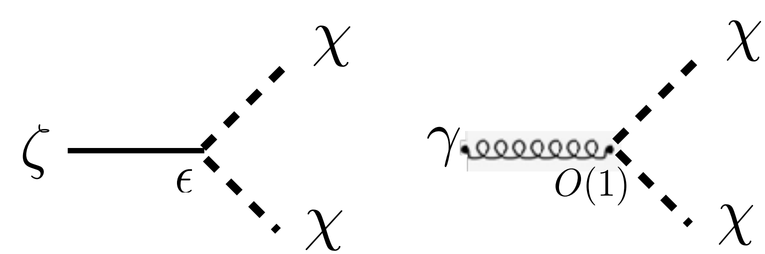
- The problem admits a similar diagrammatic interpretation as in S-matrix calc.:

$$S_{\zeta\zeta} = M_{pl}^2 \int d^4x\, a^3\, \epsilon \left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial\zeta)^2\right]$$

$$S_{\gamma\gamma} = \frac{M_{pl}^2}{8} \int d^4x\, a^3 \left[\dot{\gamma}_{ij}\dot{\gamma}_{ij} - \frac{1}{a^2}\partial_k\gamma_{ij}\partial_k\gamma_{ij}\right]$$



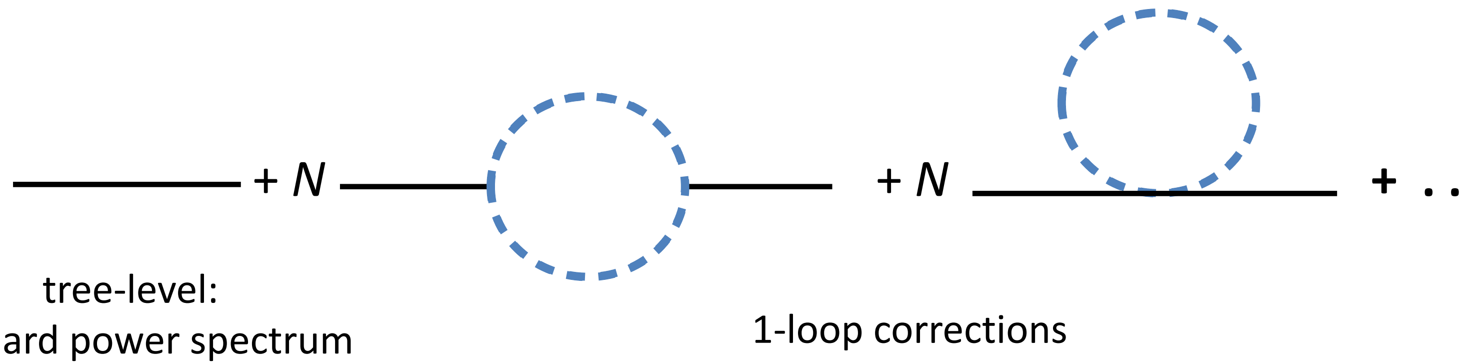
$S_{\zeta\chi\chi}, S_{\gamma\chi\chi}$ = cubic interaction vertices



Setup of the calculation

- Two-point functions get corrections from loops of hidden fields χ

$\langle \zeta \zeta \rangle$ =



- The direct calculation using the traditional in-in formalism is actually quite involved. Is there a better way?

Shortcut: 1-loop effective action

- **'Integrate out'** the hidden sector in the path integral:

$$S[g,\phi,\chi] = \int d^4x\sqrt{-g}\left\{\frac{M_{pl}^2 R[g]}{2} - \frac{1}{2}g^{ab}\nabla_a\phi\nabla_b\phi - V[\phi]\right\} + S[\chi]$$

$$\Downarrow$$

$$\Gamma^1[g,\phi] = \int d^4x\sqrt{-g}\left\{\frac{M_{pl}^2 R[g]}{2} - \frac{1}{2}g^{ab}\nabla_a\phi\nabla_b\phi - V[\phi] - \frac{1}{5760\pi^2}\left(a_s R_{ab}R^{ab}[g] + b_s R^2[g]\right)\right\}$$

- Allows to calculate all **one-loop amplitudes** of the original theory **as tree-level** processes in the effective theory. Valid for any hidden field of spin ***s***.

Tensor power spectrum

- Consider tensor perturbations given by

$$g_{ab}(x)dx^a dx^b = -dt^2 + a^2(t)e^{\gamma_{ij}(t,\vec{x})}dx^i dx^j \ ,$$

- Expand 1-loop effective action to second order in γ_{ij}, leading order in slow-roll parameters and cutoff scale H/M_pl:

$$\Gamma^1[\gamma] = \underbrace{\int d^4x\, a^3 \frac{M_{pl}^2}{8}\left[\dot{\gamma}_{ij}\dot{\gamma}_{ij} - \frac{1}{a^2}\partial_k\gamma_{ij}\partial_k\gamma_{ij}\right]}_{\text{Usual graviton action}} \underbrace{\left[1 - N\frac{R(a_s(1+\epsilon/2)+4b_s)}{5760\pi^2 M_{pl}^2}\right]}_{\text{How integrated d.o.f. correct the dynamics of tensor perturbations}} + O\left(\gamma^3, \epsilon^2, \frac{H^2}{M_{pl}^2}\right)$$

Usual graviton action

How **integrated d.o.f. correct the dynamics** of tensor perturbations

Tensor power spectrum

- Define Mukhanov-Sasaki variable:

$$\gamma_{ij} = \frac{v_{ij}}{z} \qquad z^2 = M_{pl}^2 \frac{a^2}{4}\left[1 - \frac{NR(a_s(1-\epsilon/2)+4b_s)}{5760\pi^2 M_{pl}^2}\right]$$

- Derive equations of motion:

$$v''_{\vec{k}} + \left(k^2 - \frac{z''}{z}\right) v_{\vec{k}} = 0\,, \qquad \frac{z''}{z} = \frac{2+3\epsilon_{eff}}{\eta^2} \qquad \epsilon_{eff} = \epsilon\left[1 + R\frac{N(a_s+4b_s)}{5760\pi^2 M_{pl}^2}\right]$$

- Calculate the (tree-level) power spectrum:

$$P_\gamma(k) \sim \left(\frac{k}{k_*}\right)^{-2\epsilon_{eff}} = \left(\frac{k}{k_*}\right)^{-2\epsilon}\left[1 - \epsilon R\frac{N(a_s+4b_s)}{2880\pi^2 M_{pl}^2}\log\frac{k}{k_\star} + O(\epsilon^2)\right]\,.$$

Full agreement for *s = 0* with DR, Durrer, Patil (2018)

A comment on loops in inflation

- The nature of the logarithmic running led to some debate in the literature:

Weinberg (2005):

$$P_\zeta(k) \sim \frac{H^2}{M_{pl}^2} \log \frac{k}{\mu}$$

Senatore-Zaldarriaga (2009):
("some terms were omitted in dim-reg")

$$\frac{H_*^2}{M_{pl}^2} (\log \frac{k}{\mu} + \log \frac{H_*}{k}) = \frac{H_*^2}{M_{pl}^2} \log \frac{H_*}{\mu}$$

DR-Durrer-Patil (2018): $H \equiv H_k$
("correlation functions do run, but more weakly")

$$\log \frac{H_k}{\mu} = -\epsilon \log \frac{k}{k_*}$$

Physical implications: bounds from observations

- Main feature of hidden sector is to introduce a **running** on the spectrum, clearly distinguishable from the ordinary contribution.

- Spectral tilt of the tensor spectrum receives linear corrections in slow-roll that are cleanly observable. For light scalar hidden fields:

$$\left.\begin{array}{l} n_T = -2\epsilon(1 + ...) \\ \\ r_* = 16\epsilon_* \end{array}\right\} \qquad N \sim \frac{2560}{3}\frac{\Delta_\zeta^{-1}}{r_*^2}(n_T + \frac{r_*}{8}) \sim \frac{10^{11}}{r_*^2}(n_T + \frac{r_*}{8})$$

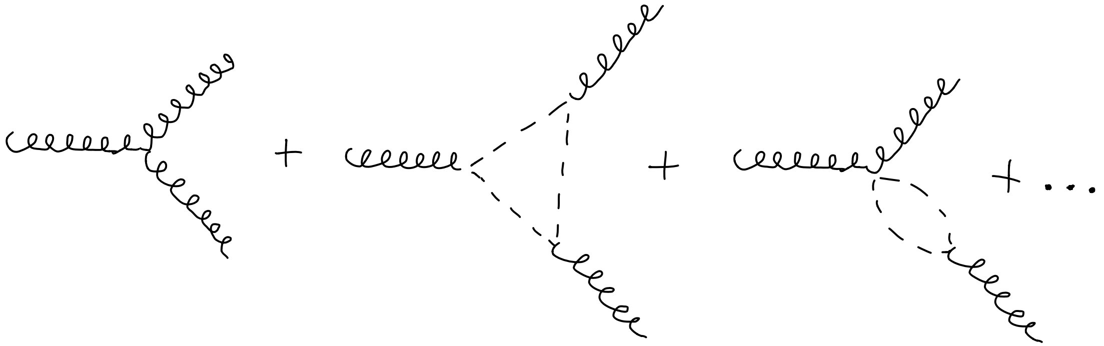
- Experimental bounds on the consistency relation could help constrain the hidden sector.

Conclusions and future prospects

- Bounds on primordial spectrum from observations could in principle constrain the hidden field content of the universe. Not so hidden after all.

- Next goal: calculate curvature power spectrum with 1-loop effective action. Not expecting competitive bounds, however (extra suppression factors).

- Useful method to calculate *n*-point functions for any hidden field (spin). Tensor bispectrum may give additional interesting bounds through non-observation of NG.



Thank you for your attention!