



# Orbital Inflation

Yvette Welling (DESY)

Based on work with **Ana Achúcarro, Dong-Gang Wang, Gonzalo Palma,**  
Oksana Iarygina, Ed Copeland

arXiv: 1901.03657 (Achúcarro, Copeland, Iarygina, Palma, Wang, YW)  
1907.02020 (Achúcarro, YW)  
1907.02951  
1908.06956 (Achúcarro, Palma, Wang, YW)

# The success of single-field inflation

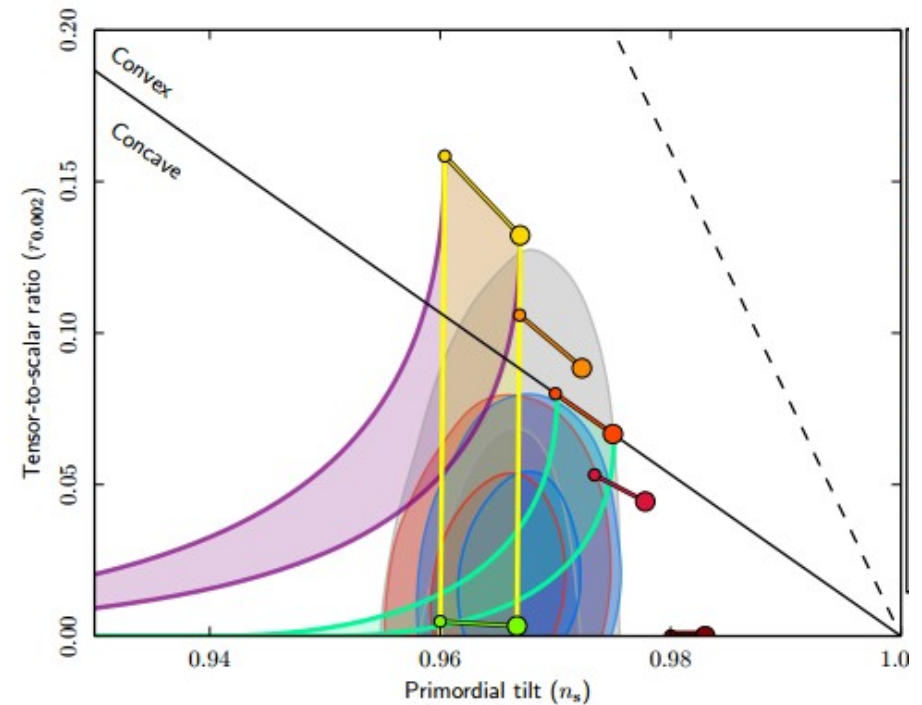
Solves horizon & flatness problem

Provides seeds for structure formation

(Guth, Linde, Starobinsky, Mukhanov)

## Consistent with CMB data

- Small but **non-zero spectral tilt**
- Small **tensor-to-scalar ratio**
- Small **primordial non-Gaussianity**
- Small **isocurvature perturbations**

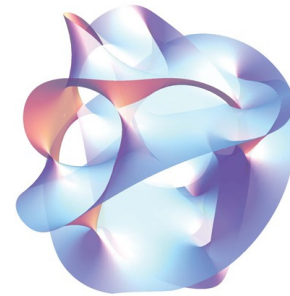


(Planck collaboration, 2018)

# Theoretical challenges

String embeddings of inflation typically contain **multiple scalar fields** living on a curved field space

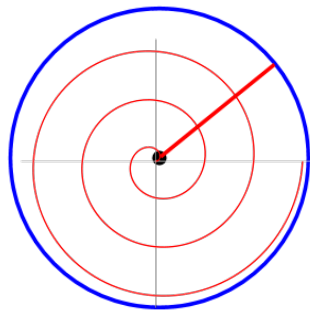
→ they may **interact** with the inflaton (stabilize them all?)



Recent “swampland conjectures” suggest that inflation takes place in a small steep patch of field space (Ooguri & Vafa, 2007 Ooguri, Palti, Shiu & Vafa 2018)

→ **curved trajectories**

( Achúcarro & Palma, 2018) See Achúcarro’s talk!  
+ posters by Wang, Chiovloni



$$|\Delta\phi| \leq \mathcal{O}(1)M_p$$

# A simple multi-field framework

It's desirable to have a **simple framework** that can deal with multi-field inflation with curved trajectories & curved field spaces

To address questions like

- (1) What **symmetries** may protect the inflationary dynamics? (See also Achucarro's and Finelli's talk + poster by Wang)
- (2) What's the role of the **field space geometry**? (See also talks by Achucarro, Sfakianakis, Turzynski + posters by Wang, Pinol, Christodoulis, Chiovoloni, Fumagelli)
- (3) What are the **signatures of new physics**? (See also Pimentel's talk)

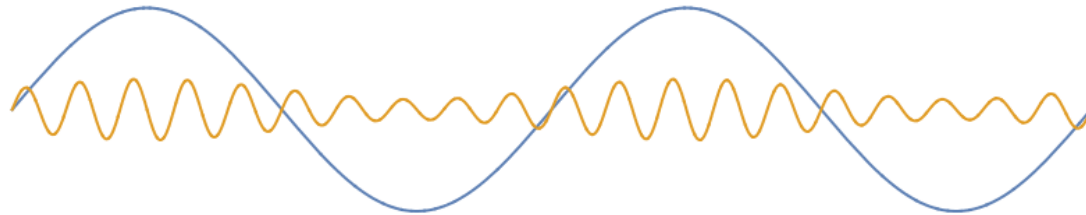
# A lightning review of multi-field effects

- (1) Primordial non-Gaussianities
- (2) Isocurvature perturbations

# Primordial non-Gaussianities

Consider e.g. extra fields during inflation

Interactions = mode coupling

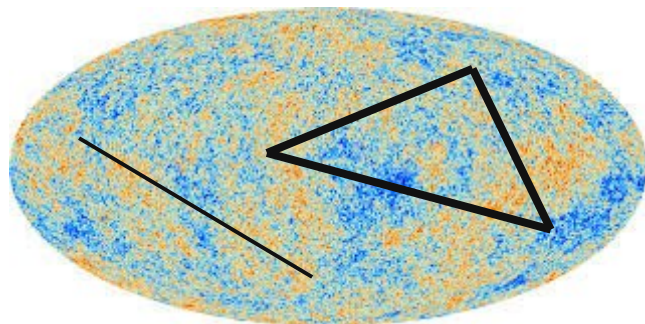


A long wave modulates the amplitude of a short wave

Correlation between different modes

$$\langle \mathcal{R}(k_1)\mathcal{R}(k_2)\mathcal{R}(k_3) \rangle \neq 0$$

non-zero **bispectrum** (non-Gaussian statistics)

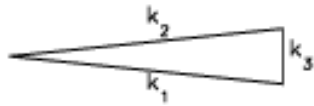


Bispectrum = FT three-point correlation

# Why do we care about primordial non-Gaussianities?

PNG is a smoking gun for single field

All canonical single field models of inflation can be ruled out by detecting a violation of the **single field consistency relation**

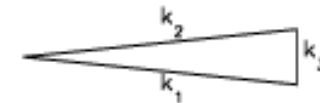
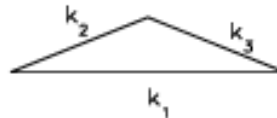
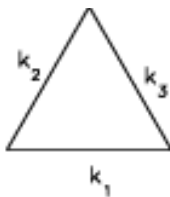


$$f_{\text{NL}} = \frac{5}{12}(1 - n_s)$$

(Maldacena, 2002  
Creminelli, Zaldarriaga, 2004)

PNG is a probe of fundamental physics

Derivative interactions / strong interactions with heavy fields / multiple light fields / cosmological collider physics / alternatives to inflation / ...

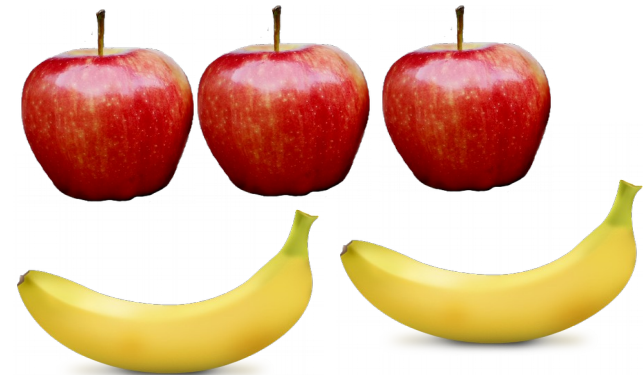
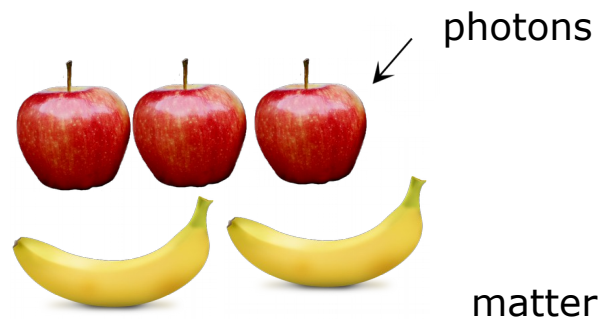


# Isocurvature perturbations

## Curvature = adiabatic perturbations

*Local expansion of homogeneous background (time shift)*

*Composition universe the same, but overall number density varies*



Single field inflation creates adiabatic perturbations that are conserved on SH scales

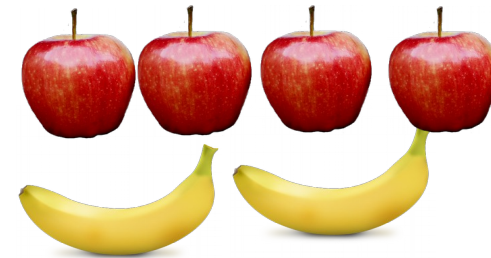
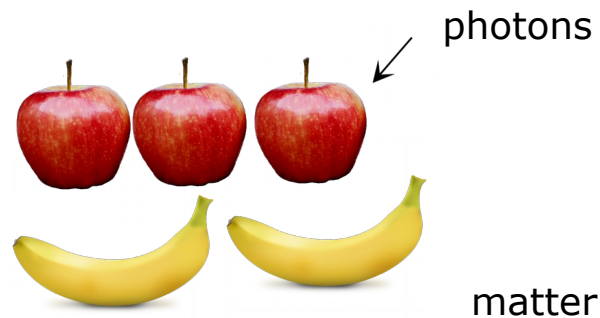
(Weinberg, 2003)

# Isocurvature perturbations

## Isocurvature = non-adiabatic perturbations

*NOT-adiabatic*

Composition universe is *not* the same, *relative ratio of species varies*



Multi-field inflation is generically expected to produce large isocurvature  $pt^*$  and  $f_{NL} \sim O(1)$  primordial non-Gaussianities

(Alvarez et al, 2014)

\*Except when reheating washes it out completely

# Inflation with light coupled scalars

**BAD** because:

- They source curvature perturbations → **large non-Gaussianities**
- Light scalars don't decay → **isocurvature pt**

# Inflation with light coupled scalars

**GOOD** because:

- They **efficiently** source curvature perturbations → **smaller  $n_G$**
- Light scalars don't decay → **dynamically suppressed**

*These systems are as multi-field as can be  
But look a lot like single field !*

# Simplest extension of the single field EFT

$$S^{(2)} = \frac{1}{2} \int d^4x a^3 \left[ 2\epsilon \dot{\mathcal{R}}^2 - 2\epsilon \frac{(\partial_i \mathcal{R})^2}{a^2} \right] \quad (\text{Single field})$$

(Two field)

Curvature perturbation

Isocurvature perturbation

$$S^{(2)} = \frac{1}{2} \int d^4x a^3 M_p^2 \left[ 2\epsilon \left( \dot{\mathcal{R}} - \omega \sigma \right)^2 - 2\epsilon \frac{(\partial_i \mathcal{R})^2}{a^2} + \dot{\sigma}^2 - \mu^2 \sigma^2 - \frac{(\partial_i \sigma)^2}{a^2} \right]$$

Efficiency of  
interaction

Isocurvature mass

# Simplest extension of the single field EFT

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(Two field)



Bottom up simplest: static coefficients

$$S^{(2)} = \frac{1}{2} \int d^4x a^3 M_p^2 \left[ 2\epsilon \left( \dot{\mathcal{R}} - \omega \sigma \right)^2 - 2\epsilon \frac{(\partial_i \mathcal{R})^2}{a^2} + \dot{\sigma}^2 - \mu^2 \sigma^2 - \frac{(\partial_i \sigma)^2}{a^2} \right]$$

Efficient coupling

Small mass

*How to realize this explicitly?*

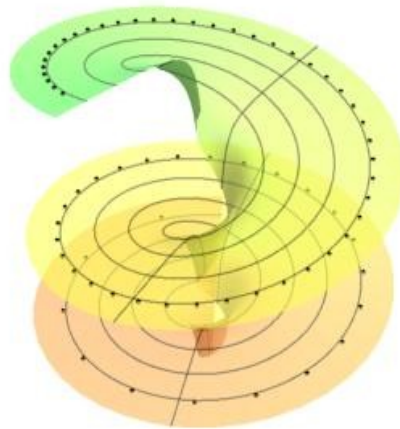
*Phenomenology?*

*What are the symmetries protecting the coefficients?*

# Main obstacle: no potential gradient flow

Inflationary trajectories generically **do not follow the potential gradient flow**

$$\dot{\phi}^a \not\propto -\nabla^a V$$



→ **The potential does not reflect the symmetries of the perturbations**

# Orbital Inflation

(Achúcarro, Copeland, Iarygina, Palma, Wang, YW, 1901.03657)  
(Achúcarro, YW, 1907.02020)

**Idea:** consider inflationary models that attract to the **Hubble gradient flow**

$$\dot{\phi}^a = -2M_p^2 G^{ab} H_{,b}$$

(Generalization of single-field Hamilton-Jacobi,  
Salopek & Bond 1990)

+ align this with an isometry of field space, e.g. 'angular'  $\theta$  direction

$$\sqrt{-g}^{-1} \mathcal{L} = \frac{1}{2} \left[ e^{2\rho/R_0} (\partial\theta)^2 + (\partial\rho)^2 \right] - V(\rho, \theta)$$

$$\dot{\rho} = 0 \quad \text{and} \quad \dot{\theta} \neq 0$$

This gives a set of possible Hubble parameters  $\rightarrow$  family of potentials that admit a **constant coupling  $\omega$**

$$V = 3H^2 - 2G^{ab} H_{,a} H_{,b}$$

# Exact solution

(Achúcarro, Copeland, Iarygina, Palma, Wang, YW, 1901.03657)  
(Achúcarro, YW, 1907.02020)

Remarkably, the background attractor is an **exact solution**

$$\dot{\phi}^a = -2M_p^2 G^{ab} H_{,b}$$

The Hubble parameter also determines the properties of the perturbations

e.g. if  $\dot{\rho} = 0$  and  $\dot{\theta} \neq 0$

$$H(\theta, \rho) = W(\theta)X(\rho)$$

Determines  $\varepsilon, \eta, \dots$

Determines the **couplings** with  
the isocurvature perturbations

# Applications of Orbital Inflation

- (1) Playground for **Quasi-Single-Field Inflation** (Chen, Wang, 2010)
- (2) We can **exactly solve the phenomenology** of Orbital Inflation in the limit of small isocurvature mass and a small radius of curvature
- (3) We gain **new insights on the symmetries** that protect the inflationary dynamics

# Applications of Orbital Inflation

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# An exact model of QSF inflation

(Chen, Wang, 2010)  
(YW, 1907.02951)

Lagrangian

$$\mathcal{L} = \frac{1}{2}\rho^2(\partial\theta)^2 + \frac{1}{2}(\partial\rho)^2 - V(\rho, \theta)$$

Potential

$$V(\theta, \rho) = 3 \left( W^2(\theta) - \frac{2W_\theta^2(\theta)}{3\rho^2} \right) \times \left( 1 + \frac{\lambda}{2}(\rho - \rho_0)^2 + \frac{\alpha}{6}(\rho - \rho_0)^3 + \dots \right)^2$$

Determines  $\epsilon, \eta, \dots$

Determines the couplings with  
the isocurvature perturbations

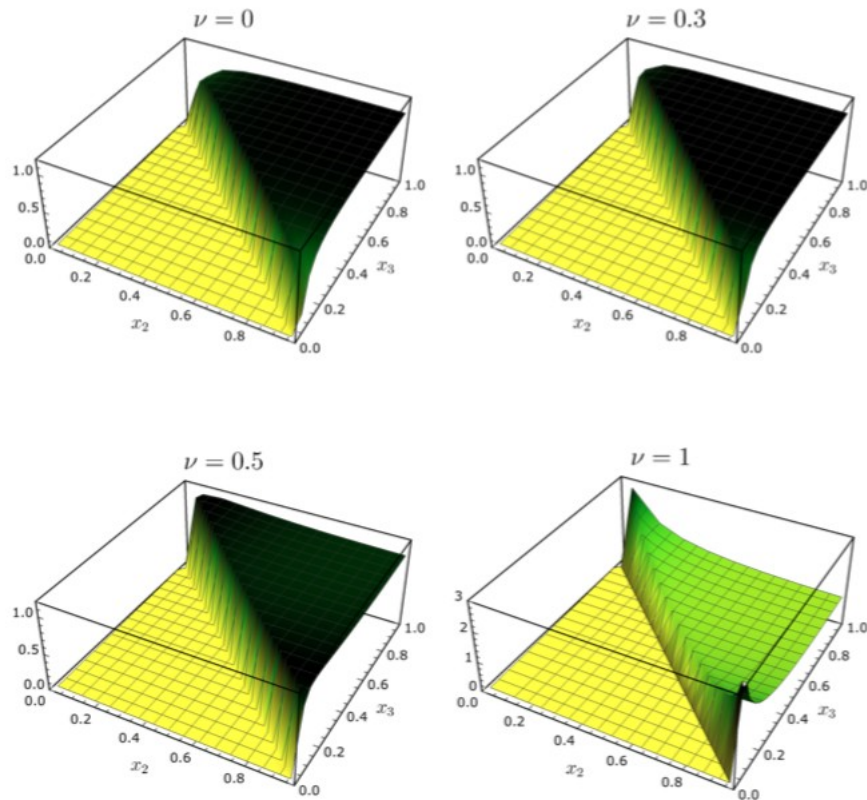
$$\mu^2/H^2 = 6\lambda + \mathcal{O}(\epsilon) \quad \omega/H \sim 1/\rho_0$$

$$V_{\rho\rho\rho} = 6\alpha + \mathcal{O}(\epsilon)$$

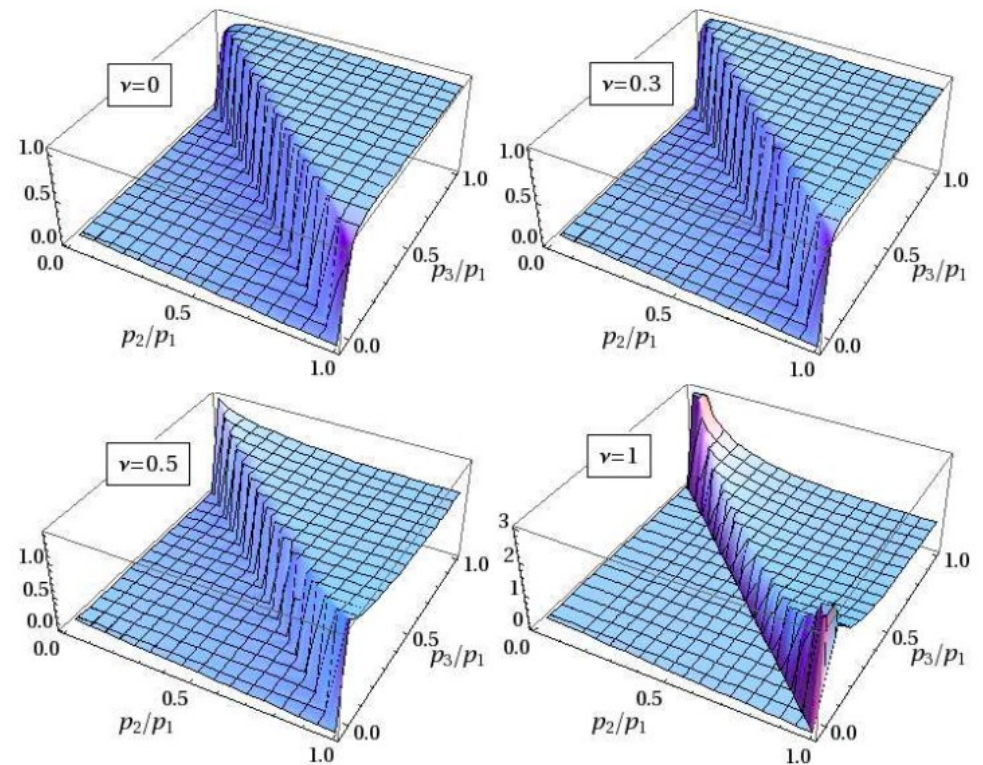
# An exact model of QSF inflation

Numerical verification of the analytical ansatz of **dimensionless bispectrum QSF**

$$\nu \equiv \sqrt{9/4 - \mu^2/H^2}, \quad \alpha = 1000, \quad \rho_0 = 2$$



(YW, 1907.02951)  
Numerical computations  
done with PyTransport



(Chen, Wang, 2010)

# Applications of Orbital Inflation

(1) Playground for Quasi-Single-Field inflation

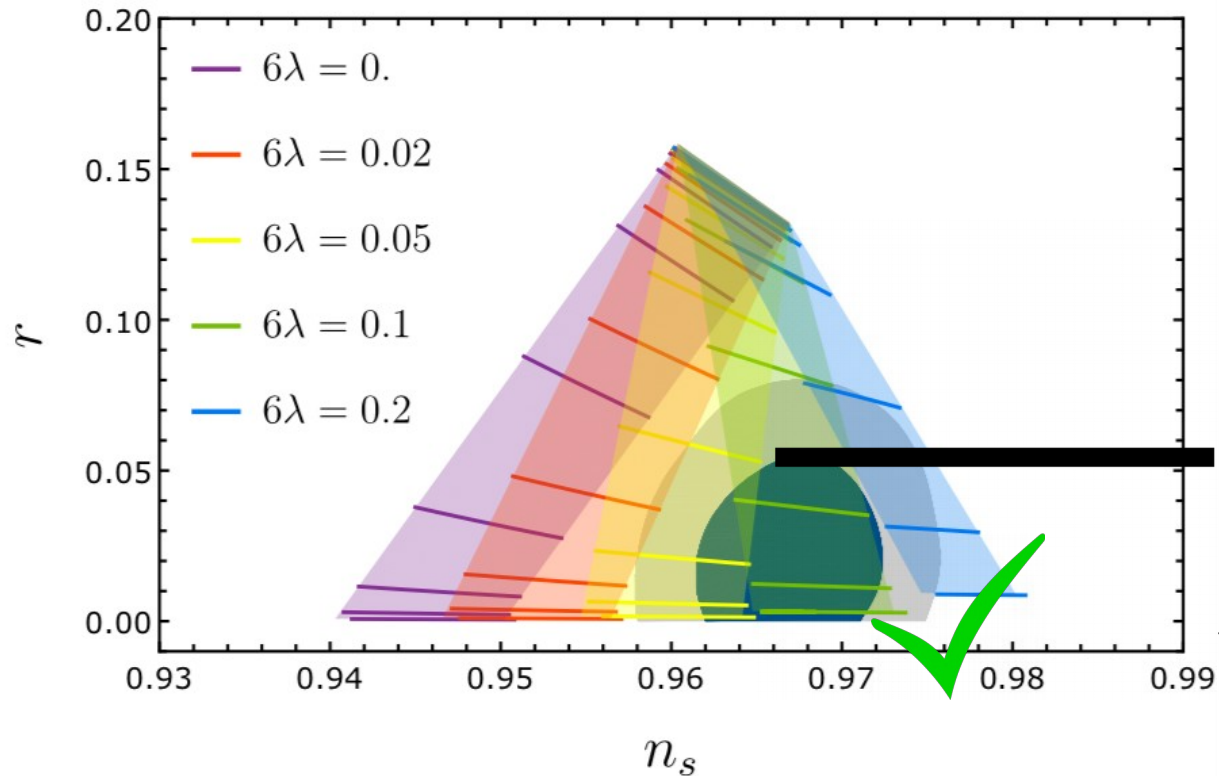
**(2) We can exactly solve the phenomenology of Orbital Inflation in the limit of small isocurvature mass and a small radius of curvature**

(3) We gain new insights on the symmetries that protect the inflationary dynamics

# Phenomenology for small isocurvature mass

(Achúcarro, YW, 1907.02020)

Phenomenology for small  $\mu^2/H^2$

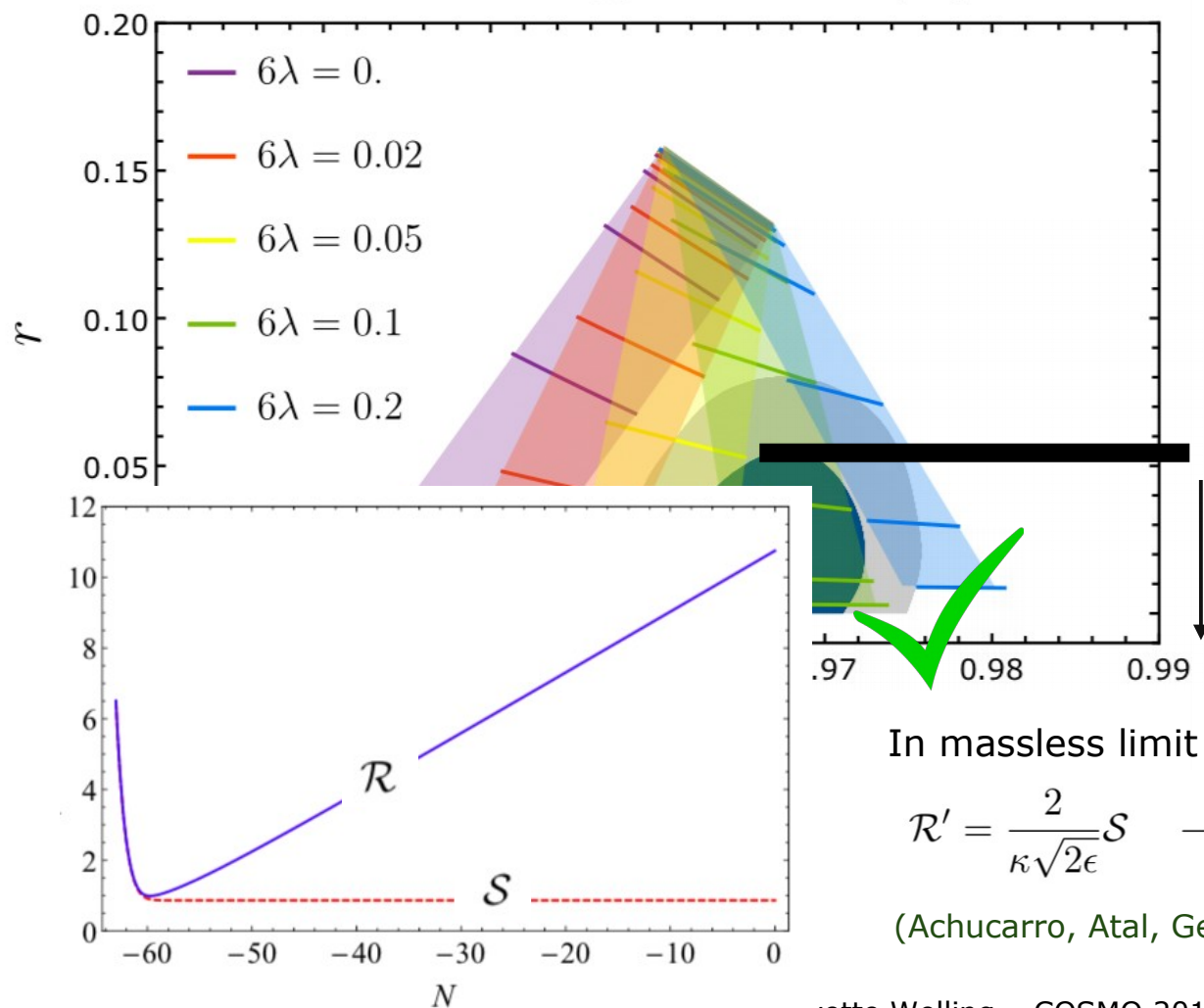


$\varepsilon$ : starting point  
 $\lambda$ : sets mass. Angle in which it vans out in  $(n_s, r)$  plane  
 $\kappa^{-1}$ : sets efficiency interaction. Reduction  $r$

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**“Single field” regime**  
 isocurvature pt suppressed  
 but consistency relation violated!

In massless limit  $\lambda = 0$

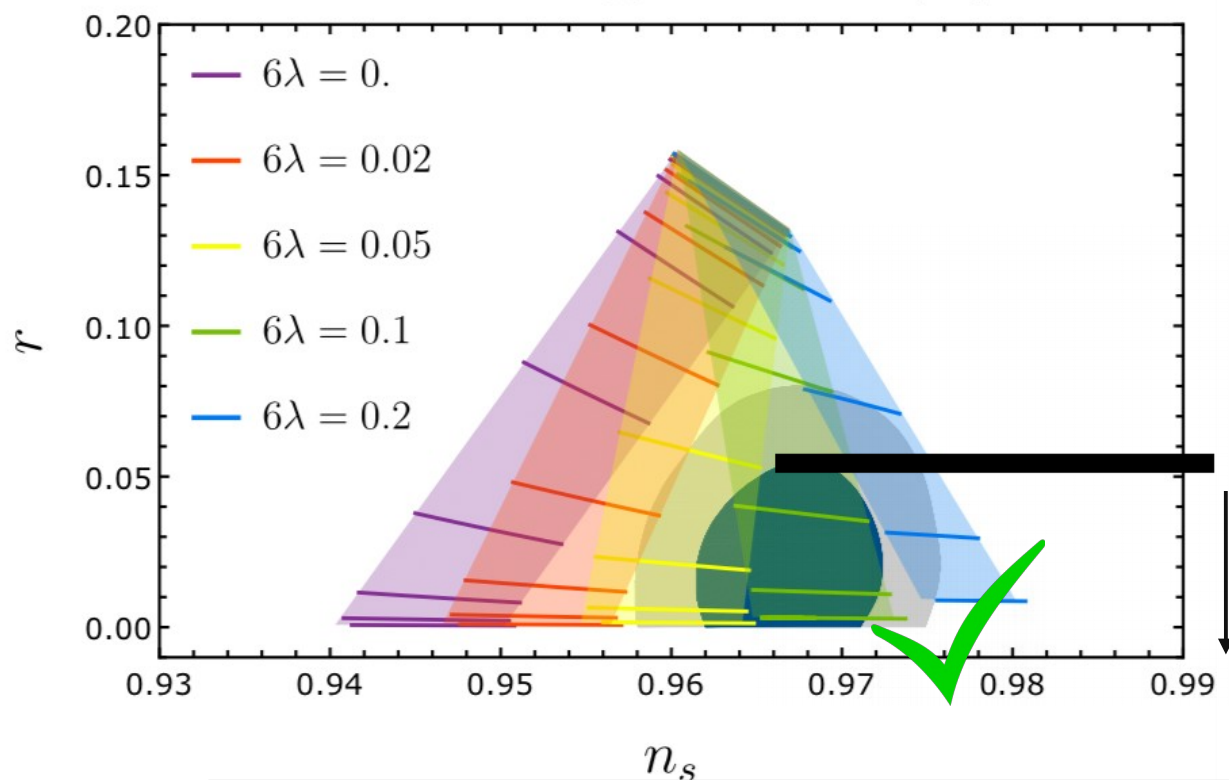
$$\mathcal{R}' = \frac{2}{\kappa\sqrt{2\epsilon}}\mathcal{S} \longrightarrow \mathcal{R}_{\text{end}} \approx \frac{2\Delta N}{\kappa\sqrt{2\epsilon}}\sigma_* \longrightarrow P_{\mathcal{R}} \gg P_{\mathcal{S}}$$

(Achucarro, Atal, Germani, Palma)

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**“Single clock” regime**  
 isocurvature pt suppressed  
 but consistency relation violated!

$$f_{\text{NL}} = -\frac{5}{12} \left( \alpha\kappa + \lambda \frac{10 - \mathbb{R}\kappa^2}{2} \right) + \frac{5}{N_\rho} \frac{2 - \mathbb{R}\kappa^2}{12\kappa}$$

$$N_\rho = \frac{1}{\kappa\lambda} (1 - e^{-2\lambda\Delta N})$$

Small if  $\lambda, \alpha$  are suppressed!

# Applications of Orbital Inflation

(1) Playground for Quasi-Single-Field Inflation

(2) We can exactly solve the phenomenology of Orbital Inflation in the limit of small isocurvature mass and a small radius of curvature

**(3) We gain new insights on the symmetries that protect the inflationary dynamics**

# Scaling similarity $\leftrightarrow$ massless fields

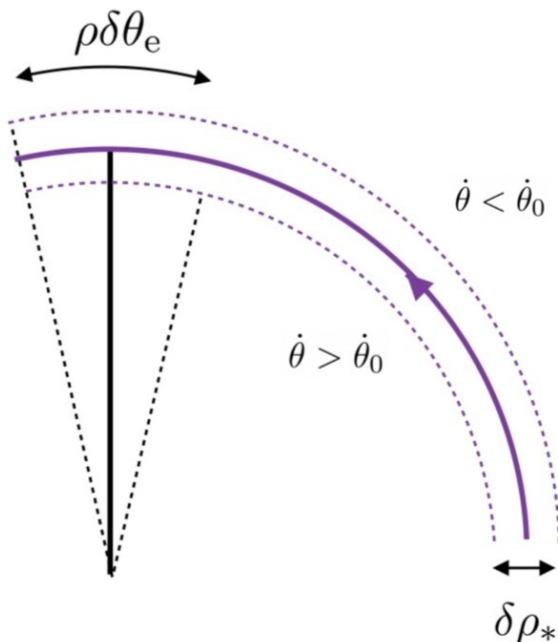
(Achúcarro, Palma, Wang, YW, 1908.06956)

Inspecting the **massless case**  $H = H(\theta)$  in more detail

$$S_\phi = -\frac{1}{2} \int d^4x \sqrt{-g} \left( (\partial\rho)^2 + e^{2\rho/R_0} (\partial\theta)^2 + \Lambda \left( \theta^2 - \frac{2p}{3e^{2\rho/R_0}} \right) \right)$$

It has a **scaling similarity** !

**Relating BG solutions**



$$\rho(x) \rightarrow \rho'(x') = \rho(x) + \Lambda c$$

$$\theta(x) \rightarrow \theta'(x') = e^{-c\Lambda/R_0} \theta(x)$$

$$x^\mu \rightarrow x'^\mu = e^c x^\mu$$

$$S \rightarrow S' = e^{2c} S$$

**Perturbations**

$$\rho(x) = \bar{\rho}(t + \pi(x)) + \mathcal{S}(x)$$

$$\theta(x) = e^{-\mathcal{S}(x)/R_0} \bar{\theta}(t + \pi(x))$$

**Massless isocurvature pt**

# Scaling similarity ↔ massless fields

(Achúcarro, Palma, Wang, YW, 1908.06956)

We can generalize this to the family of potentials

$$S_\phi = -\frac{1}{2} \int d^4x \sqrt{-g} \left( (\partial\rho)^2 + e^{2\rho/R_0} (\partial\theta)^2 + \sum_m c_m \theta^{2n-2m} e^{-2m\rho/R_0} \right)$$

Same **scaling similarity** → **massless** perturbation AND **small self-interactions**

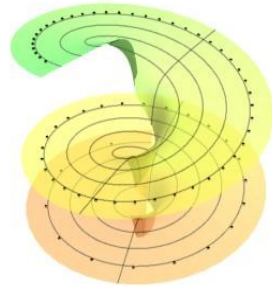
$$f_{\text{NL}} \approx \frac{5}{12} \left( \frac{2\epsilon}{n} + \eta - \frac{2\dot{\rho}}{R_0 H} \right)$$

(If the radial field has a sufficiently small velocity)

In multi-field set-ups  $f_{\text{NL}}$  becomes slow-roll suppressed if:

- The isocurvature pt are responsible for the final curvature pt
- The isocurvature self interactions are small

# Conclusions



(Achúcarro, Palma, Wang, YW, 1908.06956)  
(Achúcarro, Copeland, Iarygina, Palma, Wang, YW, 1901.03657)  
(Achúcarro, YW, 1907.02020)  
(YW, 1907.02951)

**Orbital Inflation:** a class of exact multi-field attractors in curved field spaces and with curved trajectories (that contain QSF)

In the limit of small isocurvature self interactions and a small radius of curvature  
Orbital Inflation **mimics the phenomenology of single-field inflation**

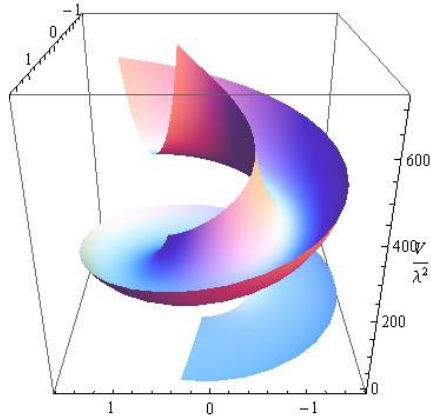
A **scaling similarity** can explain the above properties

Successful inflation doesn't necessarily require to get rid of all light fields!!

**Thank you!**

# Old slides

# Derivative interactions



Example: a potential which forces the inflaton to **turn** at constant radius

$$\mathcal{L} = \frac{1}{2}\rho^2\dot{\theta}^2 + \frac{1}{2}\dot{\rho}^2 - V(\rho, \theta) + \dots$$

Expand around bg: **derivative coupling** proportional to **turn rate**

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\rho_0 + \delta\rho)^2(\dot{\theta}_0 + \delta\dot{\theta})^2 + \frac{1}{2}\delta\dot{\rho}^2 - V(\rho, \theta) + \dots \\ &\supset \underline{2\rho_0\dot{\theta}_0\delta\rho\delta\dot{\theta}}\end{aligned}$$

This happens whenever **background trajectory**  $\neq$  **geodesic**

# Derivative interactions

General quadratic action:

$\mathcal{R}$  : curvature pt

$\sigma$  : isocurvature pt

$$S^{(2)} = \frac{1}{2} \int d^4x a^3 M_p^2 \left[ 2\epsilon \left( \dot{\mathcal{R}} - \omega \sigma \right)^2 - 2\epsilon \frac{(\partial_i \mathcal{R})^2}{a^2} + \dot{\sigma}^2 - \mu^2 \sigma^2 - \frac{(\partial_i \sigma)^2}{a^2} \right]$$

The **coupling constant** is proportional to the **turn rate**  $\omega$

The **effective mass**  $\mu$  is given by  $\mu^2 = V_{NN} + \epsilon M_p^2 H^2 \mathcal{R} + 3\omega^2$