Electroweak bubble wall speed limit

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work with Guy D. Moore

JCAP 1705 (2017) no.05, 025; ibid. 0905 (2009) 009 [arXiv:1703.08215; 0903.4099]

First order electroweak phase transition

can make

- gravitational waves
- baryogenesis
- primordial magnetic fields

requires extended scalar sector

Outline

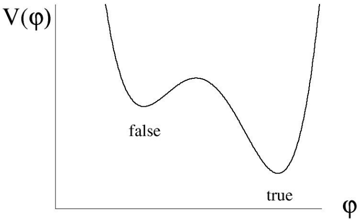
1st order phase transition

bubble wall friction, runaway

transition radiation, speed limit

False vacuum decay

initially metastable, homogenous “false vacuum”



Svetovoy et al. PR D84, 035302(R)

nucleation of true-vacuum bubbles

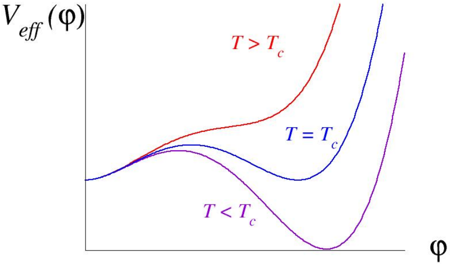
bubbles expand

all energy goes into bubble wall [Coleman 1977]

wall accelerates forever, $v \to c$ “runaway”

First order electroweak phase transition

$V_{\text{eff}} = V + V_T$



plasma affects bubble expansion

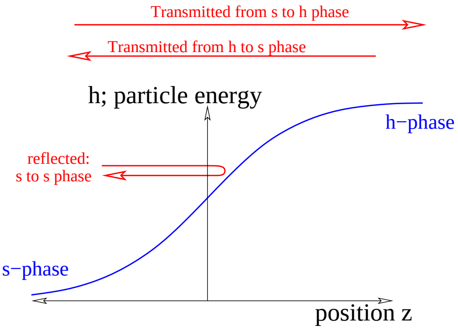
runaway?

Pressure on bubble wall

vacuum pressure $\mathcal{P}_{\rm vac} = \Delta V$ pushes wall into s-phase

Higgs field varies

$\Rightarrow$ particle masses vary



bubble rest frame: energy conserved $\Rightarrow$ $\Delta p_z < 0$ $\Rightarrow$

particles push wall into h-phase

Ultra–relativistic wall

$$\gamma \equiv \frac{1}{\sqrt{1-v^2}} \gg 1$$

all particles hit wall from one side

wall frame: $p_z \sim \gamma T \gg m$ ultra-relativistic

Lorentz-contraction $\Rightarrow$

passing time $\ll$ interaction time (plasma frame)

$$T^{-1}/\gamma \ll (g^4 T)^{-1}$$

$\Rightarrow$ no scattering *inside* the wall

Ultra–relativistic wall

$$\Delta p_z \simeq -\frac{\Delta m^2}{2p^0} \sim \frac{T}{\gamma} \qquad \text{(wall frame)}$$

$$\mathcal{P}_{1\to 1} = \sum_a \Delta m_a^2 \int \frac{d^3 p_{\rm in}}{(2\pi)^3 2p^0}\, f_a^{\rm in}$$

equilibrium in front of the wall $\Rightarrow f_a^{\rm in} = f_a^{\rm in,eq}$

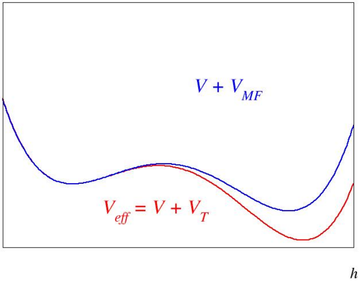
integration measure Lorentz invariant, compute in plasma frame $\Rightarrow$

$$\mathcal{P}_{1\to 1} = \Delta V_{\rm MF} \qquad \text{finite for } \gamma \to \infty$$

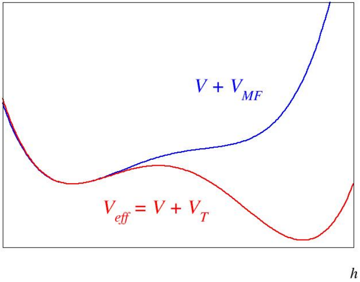
Runaway?

criterion for runaway:

$$\mathcal{P}_{\text{total}} = \Delta V + \Delta V_{\text{MF}} < 0$$



runaway



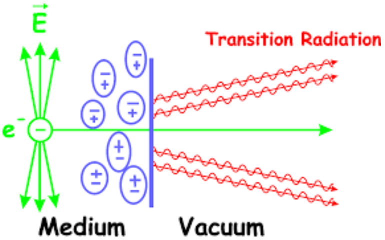
no runaway

DB, G. Moore, JCAP 1705 (2017) no.05, 025

Transition splitting

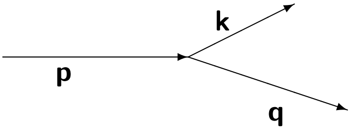
so far: leading order in the coupling

radiative effects?



X1 collaboration, Mainz

Splitting



wall frame: energy conserved

transverse momentum conserved

z-momentum not conserved

splitting increases friction:

some energy goes into transverse momenta, additional masses

$$\mathcal{P}_{1\to 2} = \sum_{abc} \int \frac{d^3p}{(2\pi)^3} f_a \int dP_{a\to bc} \times (p_{z,\mathrm{s}} - k_{z,\mathrm{h}} - q_{z,\mathrm{h}})$$

Longitudinal momenta

assume $\gamma \gg 1$

typical particle energies $k^0 \sim \gamma T$

transverse momenta $k_\perp \sim T$

$$k_z(z) = \sqrt{k_0^2 - m^2(z) - k_\perp^2} \simeq k^0 - \frac{m^2(z) + k_\perp^2}{2k^0}$$

at fixed $k_\perp$ soft emission more important

Splitting-probability amplitude

$$\mathcal{M} \equiv \int dz \; \chi_k^*(z) \chi_q^*(z) V(z) \chi_p(z)$$

$\chi =$ mode functions in z-dependent Higgs background

$V(z)$ vertex factor

WKB approximation:

$$\mathcal{M} = 2ip^0 \left(\frac{V_\mathrm{h}}{A_\mathrm{h}} - \frac{V_\mathrm{s}}{A_\mathrm{s}} \right)$$

$$A \simeq -\frac{k_\perp^2 + m_b^2}{x} \qquad \text{for} \qquad x \equiv \frac{k_0}{p_0} \ll 1$$

Electroweak bubble wall speed limit

dominant: soft transverse, longitudinal W bosons:

$$|V|^2 \sim g^2 \frac{k_\perp^2}{x^2}, \quad g^2 \frac{m_W^2}{x^2}$$

x integral IR divergent, cut off by masses

$$\mathcal{P}_{1\to 2} \sim \gamma g^2 m T^3$$

bubble wall speed limit: $\gamma \lesssim \frac{1}{g^2}$

Conclusions

first order electroweak phase transition interesting for

- gravitational waves

- baryogenesis

- primordial magnetic fields

large bubble wall velocities in Mean Field first order transition

friction dominated by soft W boson emission 'radiation splitting'

maximal velocity $\gamma \sim 1/\alpha$

Backup slides

Splitting probability

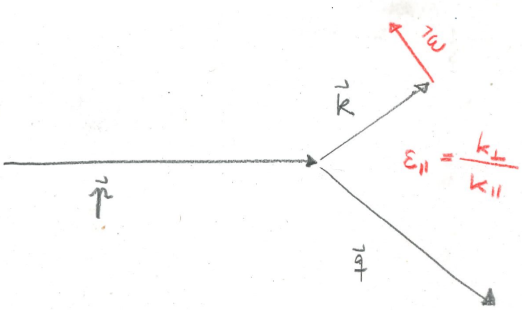
$$\mathcal{P}_{1\to 2} = \sum_{a,b,c} \nu_a \int \frac{d^3p}{(2\pi)^3 4p_0^2} f_p \int \frac{d^2k_\perp}{(2\pi)^2} \int_0^\infty \frac{dk^0}{2\pi 2k^0} \times [1\pm f_k][1\pm f_{p-k}] \frac{k_\perp^2 + m_{b,\mathrm{h}}^2}{2k^0} |\mathcal{M}|^2$$

Vertex factor for $W_{\mathrm{transverse}}$

$p \gg k$:

$$V \sim \varepsilon \cdot J$$

$$J^\mu \simeq p^\mu$$



$$V \sim \frac{k_\perp p_\parallel}{k_\parallel} \sim \frac{k_\perp}{x}$$

Pressure on bubble wall

$$\dot{p}_z = -\frac{1}{2p^0}\frac{dm^2}{dz}$$

force on wall $= -$ force on particles

planar wall:

$$\mathcal{P}_{1\to 1} = \sum_a \int dz \frac{dm_a^2}{dz} \int \frac{d^3p}{(2\pi)^3 2p^0} f_a(\mathbf{p})$$

in general difficult to compute:

non-equilibrium: $f_a \neq f_a^{\mathrm{eq}}$

Boltzmann eqs., Kadanoff-Baym eqs.

Phase equilibrium ($T = T_c$)

$$\mathcal{P}_{1\to1} = \sum_a \int dz \frac{dm_a^2}{dz} \int \frac{d^3p}{(2\pi)^3 2p^0} f_a^{\rm eq}(p^0) = \quad V_T(\mathrm{h}) - V_T(\mathrm{s})$$

V_T = thermal contribution to $V_{\rm eff}$ (1 loop)

Mean Field approximation: neglect mass dependence of **p**-integral

$$\mathcal{P}_{1\to1} \simeq \Delta V_{\rm MF} = \sum_a \Delta m_a^2 \int \frac{d^3p}{(2\pi)^3 2p^0} f_a^{\rm eq}(p^0)$$