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The stochastic gravitational-wave background from stellar-mass binary black holes

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Based on the work described in:
1803.03236, 1904.07757, 1904.07797

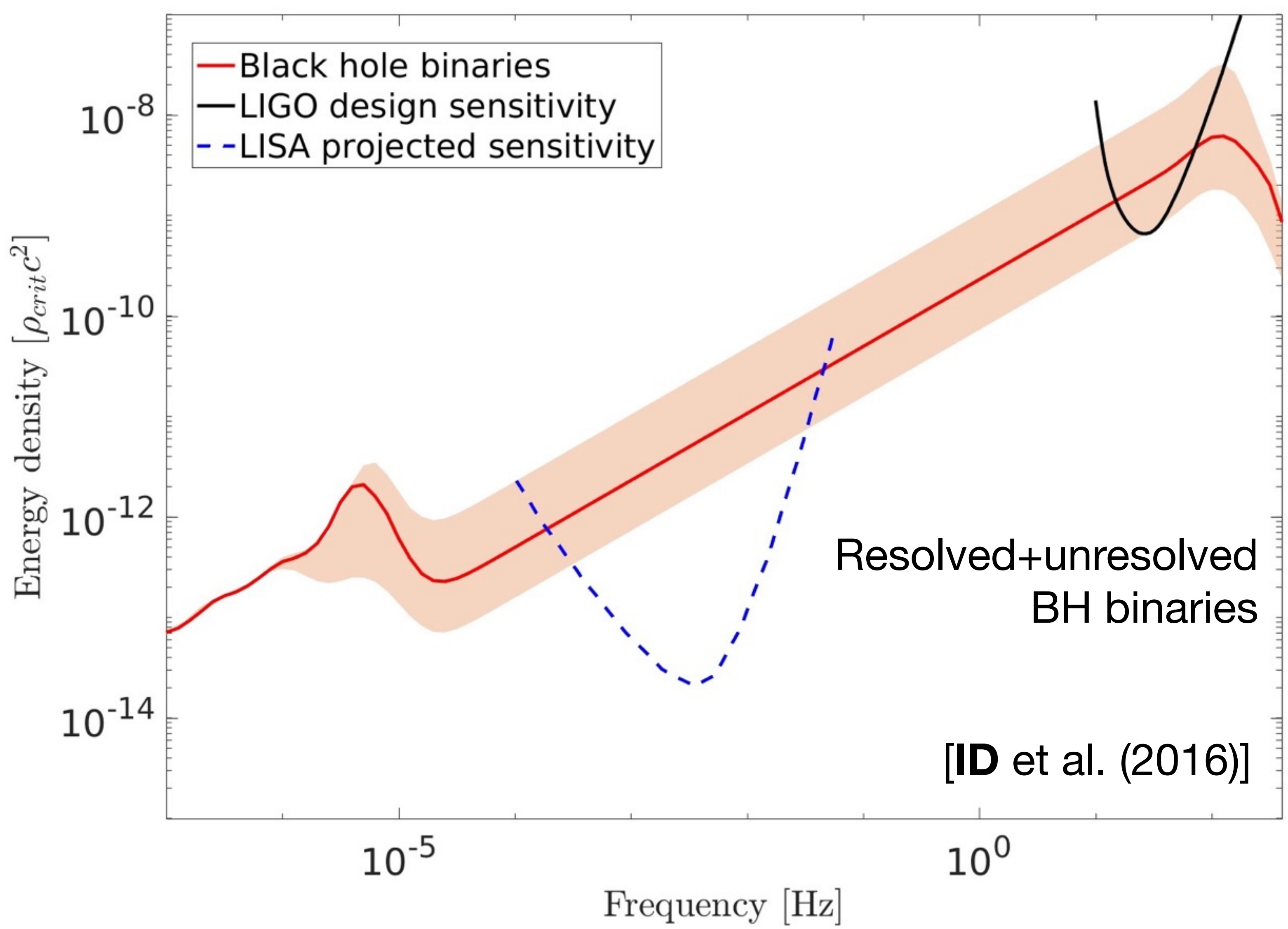
Cosmo'19, Aachen September 5, 2019

Stochastic gravitational-wave background

- Incoherent superposition of unresolved sources creates a stochastic signal
- Assume a background that is Gaussian, unpolarized, spatially homogeneous, isotropic

Energy density of the stochastic background:

$$\Omega_{\rm GW}(f;\theta_k) = \frac{f}{\rho_c H_0}\int_0^{z_{\max}} dz \frac{R_m(z;\theta_k)\frac{dE_{\rm GW}}{df}(f_s;\theta_k)}{(1+z)E(\Omega_M,\Omega_\Lambda,z)}$$



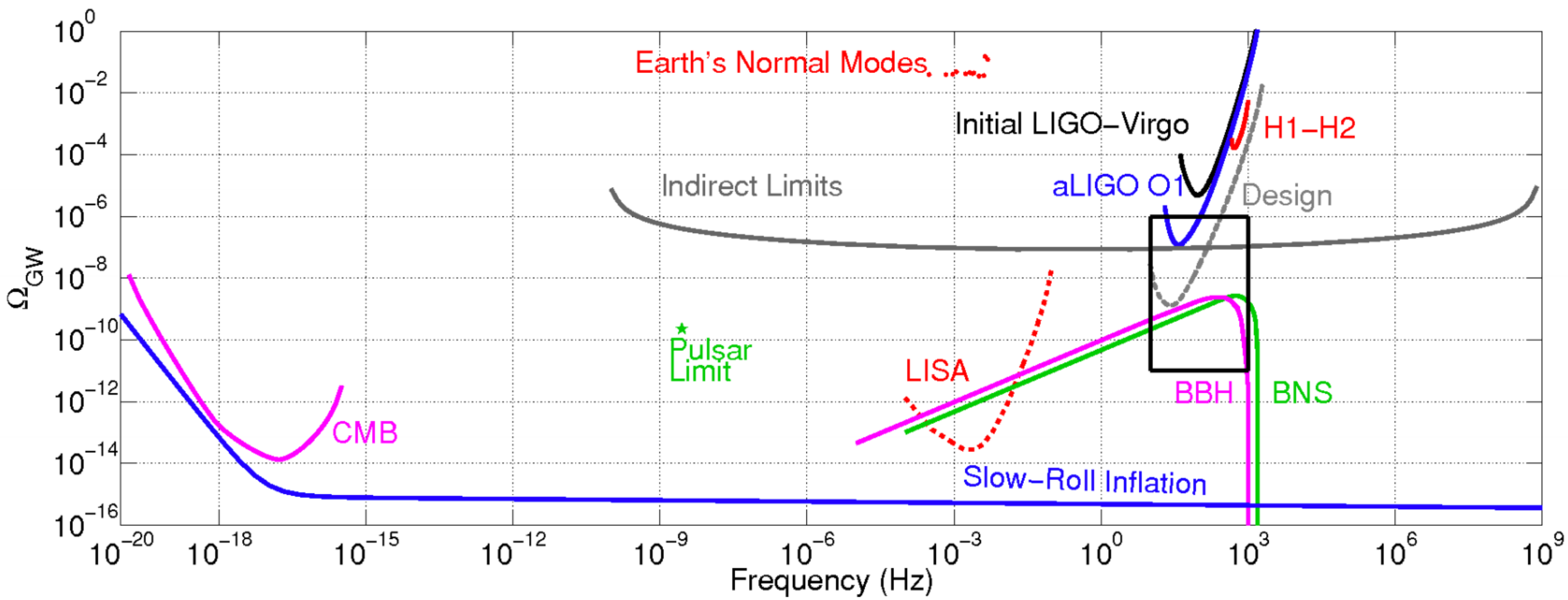
Stochastic gravitational-wave background

Astrophysical and cosmological sources

- ✱ Unresolved binary systems
- ✱ Supernovae
- ✱ Non-axisymmetric neutron stars

- ✱ Inflation
- ✱ Cosmic strings

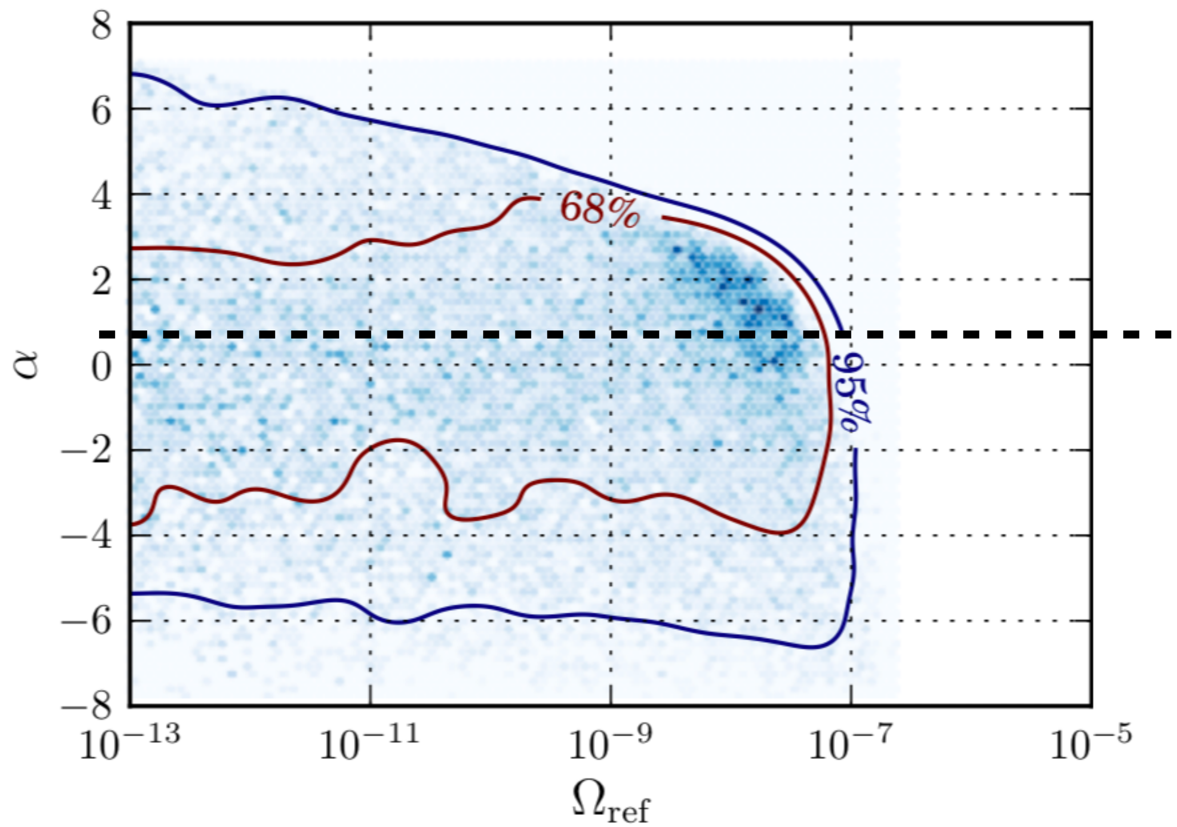
Abbott+ [1612.02029]

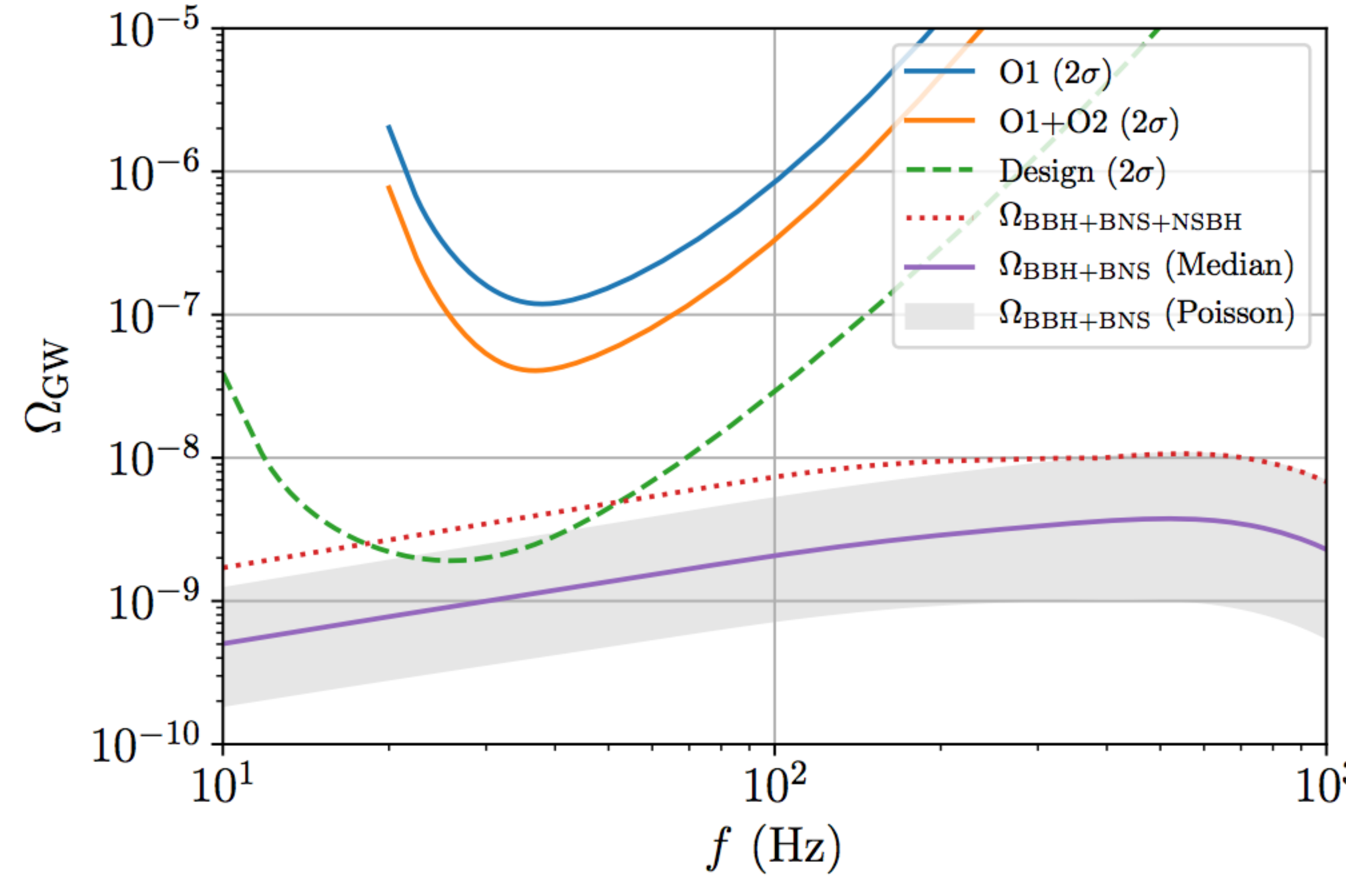


Current upper limits: LIGO O1+O2

$$\Omega_{\rm GW}(f) = \Omega_{\rm ref}\left(\frac{f}{f_{\rm ref}}\right)^{\alpha}$$

LIGO/Virgo [1903.02886]





SNR grows with observation time!

$$S \propto \langle h_1 h_2 \rangle \propto T$$

$$N \propto \langle n_1 n_2 \rangle \propto \sqrt{T}$$

$$\mathrm{SNR} = \frac{3H_0^2}{10\pi^2}\sqrt{2T}\left[\int_0^\infty df \sum_{i=1}^{n}\sum_{j>i}\frac{\gamma_{ij}^2(f)\Omega_{\rm GW}^2(f)}{f^6 P_i(f) P_j(f)}\right]^{1/2}$$

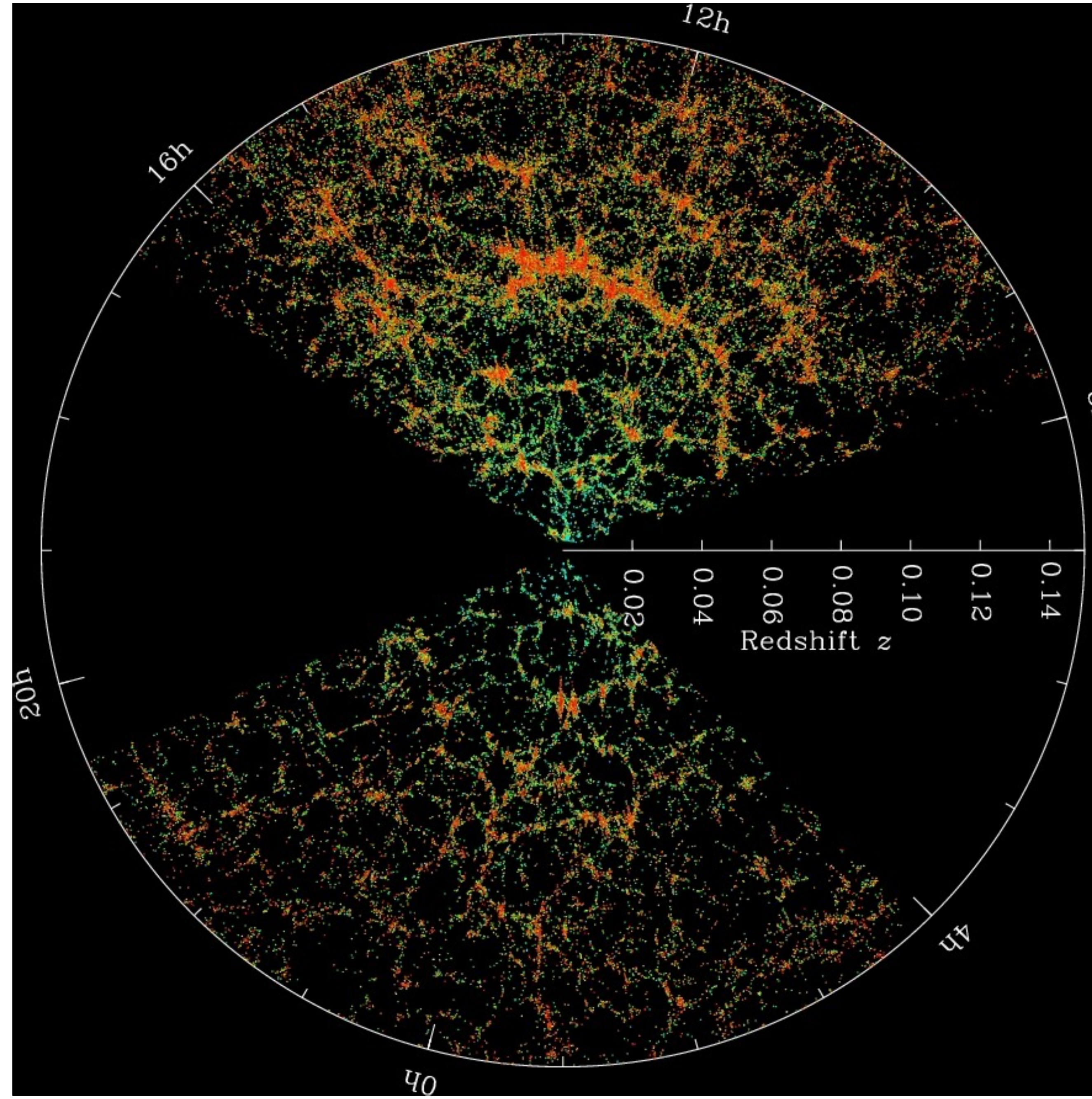
Anisotropic gravitational-wave background

Astrophysical sources are in galaxies

—> GW background is anisotropic

$$\Omega_{\rm gw}(f,\Theta) = \frac{f}{\rho_c}\frac{{\rm d}^3\rho_{GW}}{{\rm d}f{\rm d}^2\Theta}$$

[Analogous with the cosmic infrared background]



SDSS galaxy map

Anisotropic gravitational-wave background

Total energy density

$$\Omega_{GW}(\mathbf{e},f) = \frac{f}{\rho_c}\frac{d^3\rho_{GW}}{d^2\mathbf{e}df}(\mathbf{e},f)$$
$$= \frac{\bar{\Omega}_{GW}(f)}{4\pi} + \delta\Omega_{GW}(\mathbf{e},f)$$

Anisotropic component: [Cusin, Pitrou, Uzan (2017a;b)]

$$\boxed{\begin{aligned}&\delta\Omega_{GW}(\mathbf{e},f) = \\ &= \frac{f}{4\pi\rho_c}\int_{\eta_*}^{\eta_O}\mathrm{d}\eta\,\mathcal{A}(\eta,f)\left[\delta_G + 4\Psi - 2\mathbf{e}\cdot\nabla v + 6\int_{\eta}^{\eta_O}\mathrm{d}\eta'\,\dot{\Psi}\right] \\ &+ \frac{f}{4\pi\rho_c}\int_{\eta_*}^{\eta_O}\mathrm{d}\eta\,\mathcal{B}(\eta,f)\left[\mathbf{e}\cdot\nabla v - \Psi - 2\int_{\eta}^{\eta_O}d\eta'\,\dot{\Psi}\right],\end{aligned}}$$

Limber approximation: [Cusin, **ID**, Pitrou, Uzan (2019)]

$$\boxed{C_\ell^{\mathrm{Limber}} \simeq \left(\ell+\tfrac{1}{2}\right)^{-1} \times \int \mathrm{d}\log k\, k P_{\mathrm{Gal}}(k)\left|\partial_\eta\left(\frac{\bar{\Omega}_{\mathrm{GW}}}{4\pi}\right)\right|^2}$$

$$\delta\Omega_{GW}(\mathbf{e},f) = \sum_{\ell m} a_{\ell m}(f) Y_{\ell m}(\mathbf{e})$$

$$C_\ell = \frac{1}{2\ell+1}\sum_m \langle a_{\ell m} a^*_{\ell m}\rangle$$

$$k\,\Delta\eta = \ell + \frac{1}{2}$$

Conformal time: η.

Astrophysical model

[Cusin, **ID**, Pitrou, Uzan (2018; 2019a;b)]

- Dark matter halo mass function [Tinker et al. 2008]
- Populate halos with galaxies (halo mass - stellar mass relation) [Behroozi et al. 2013]
- Star formation rate
- Observational stellar-mass - metallicity relation [Ma et al. 2016]
- Black hole formation from massive stars [Fryer et al. 2012]
- Merger rate normalised to obtain the observed LIGO/Virgo merger rate at low redshift

Galaxy luminosity

$$\mathcal{L}_{gal}(t) = \int \dot{R}(\theta) \frac{dE}{df} d\theta$$

Total weighted luminosity

$$\mathcal{A} \propto \int \frac{dn}{dM} \mathcal{L}_{gal} dM$$

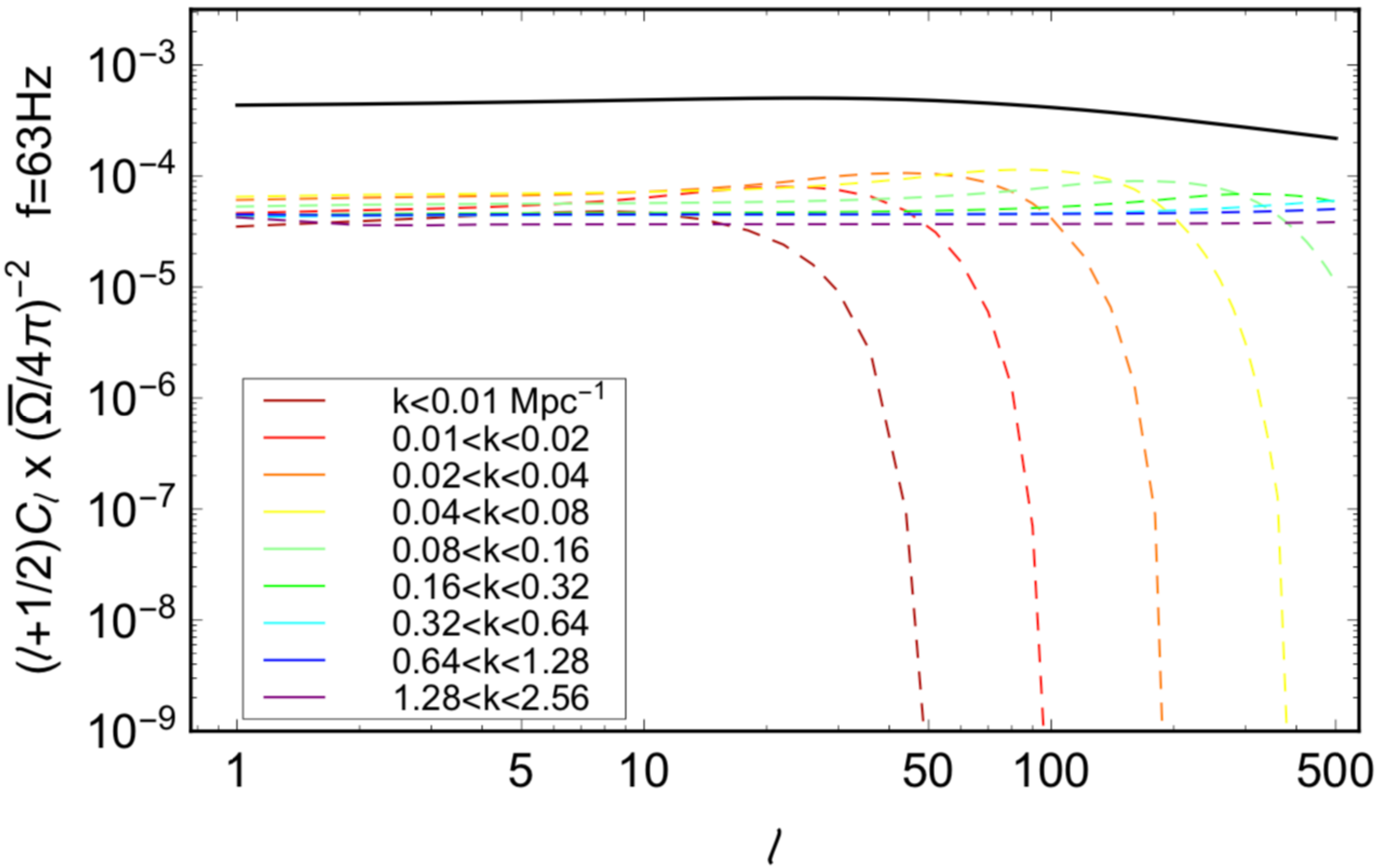
$$\mathcal{B} \propto \int \frac{dn}{dM} \frac{d\mathcal{L}_{gal}}{d\ln f} dM$$

Scale - multipole correspondence

No one-to-one correspondence between wavenumber and multipole

A given wavenumber contributes to all multipoles up to a maximum value:

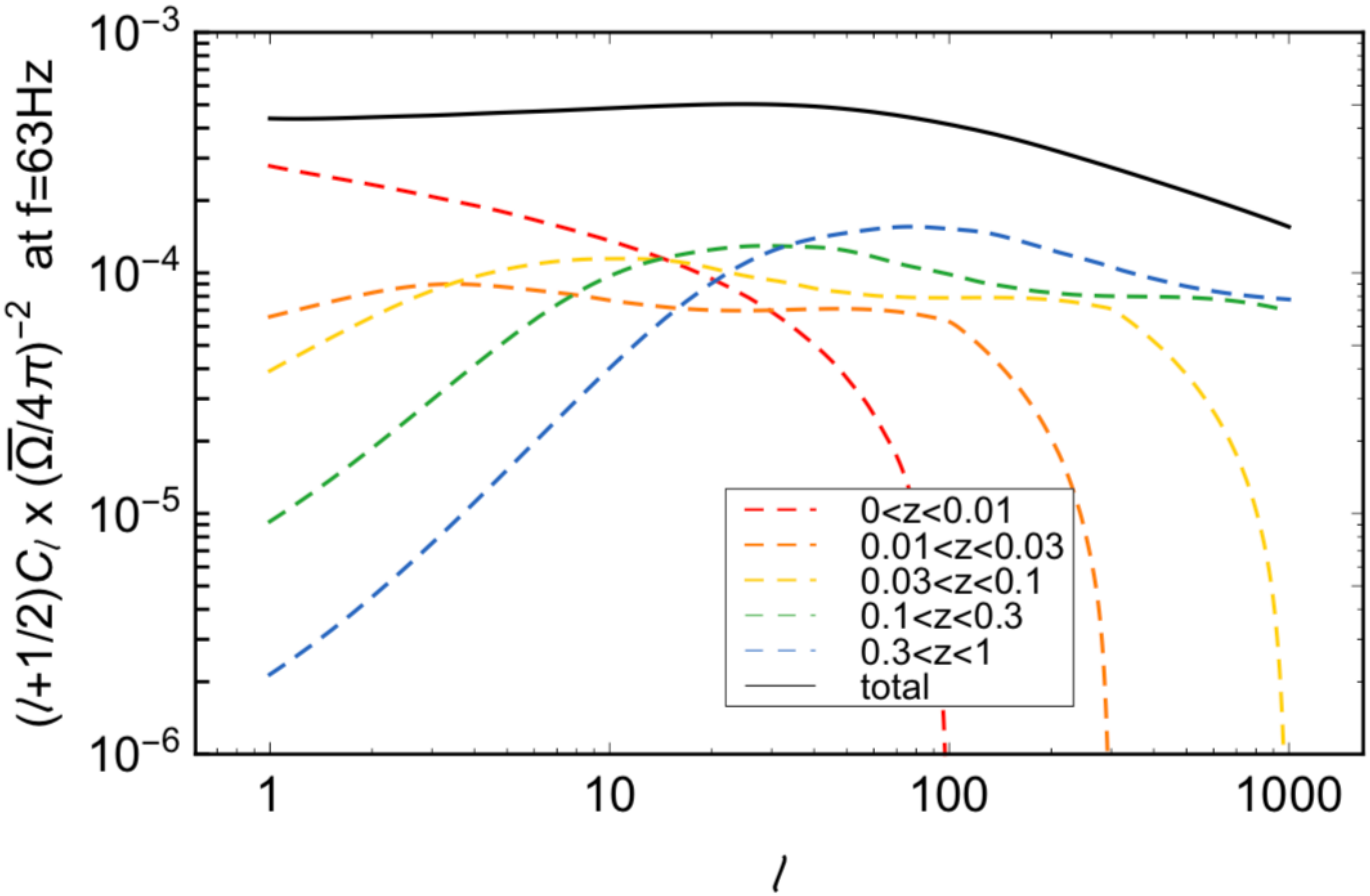
$$\ell_{\max} \equiv k[\eta_0 - \eta(z_{\max})]$$



Angular power spectrum for different bins of k

Redshift - multipole correspondence

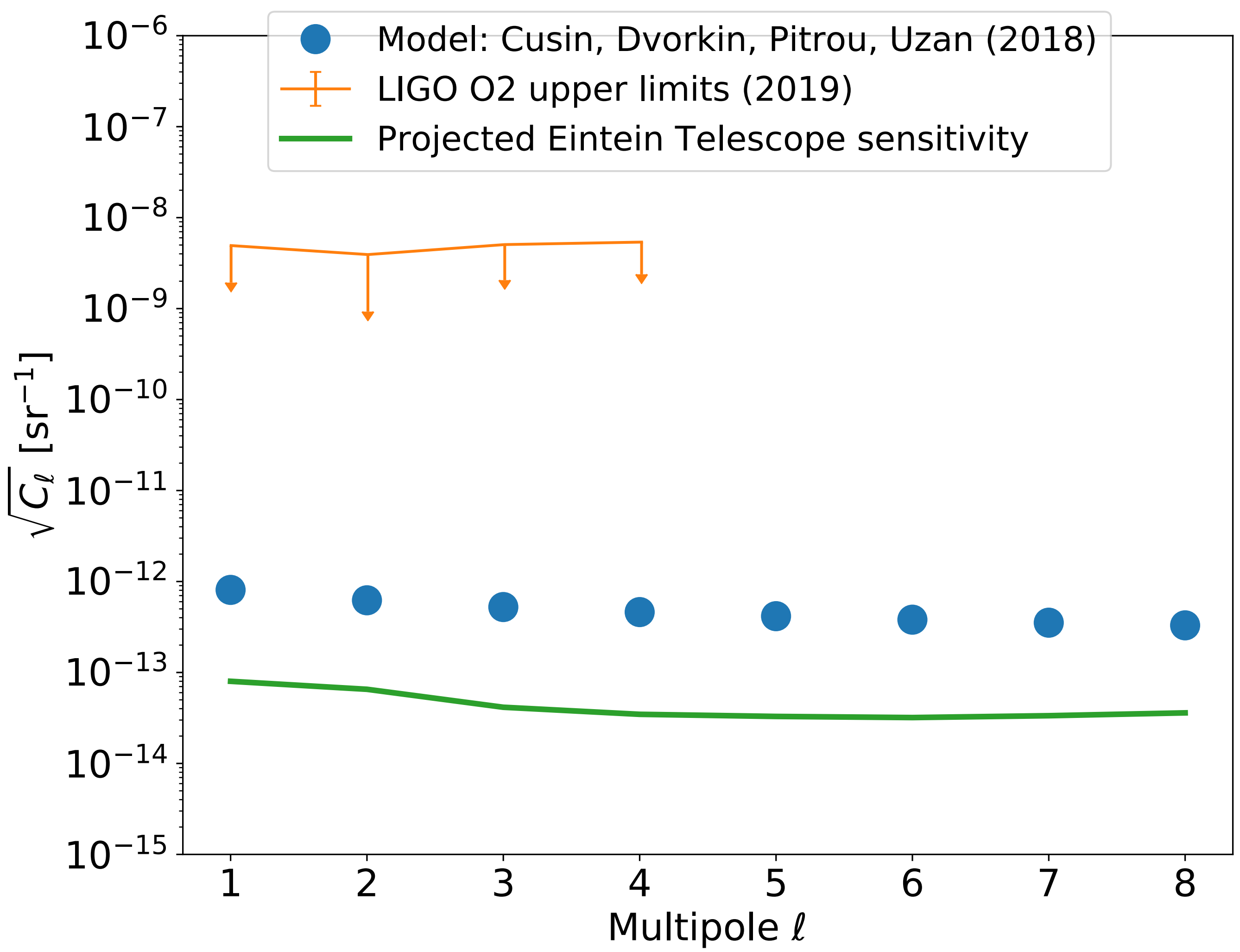
Lowest redshifts (shorter comoving distances) contribute more to larger scales:



Anisotropic gravitational-wave background: predictions and upper limits

The background from stellar-mass binary BHs has a small anisotropic component

- Can we detect it?
- What can we learn from it?



Cusin, **ID**, Pitrou, Uzan [1803.03236, 1811.03582, 1904.07757, 1904.07797]

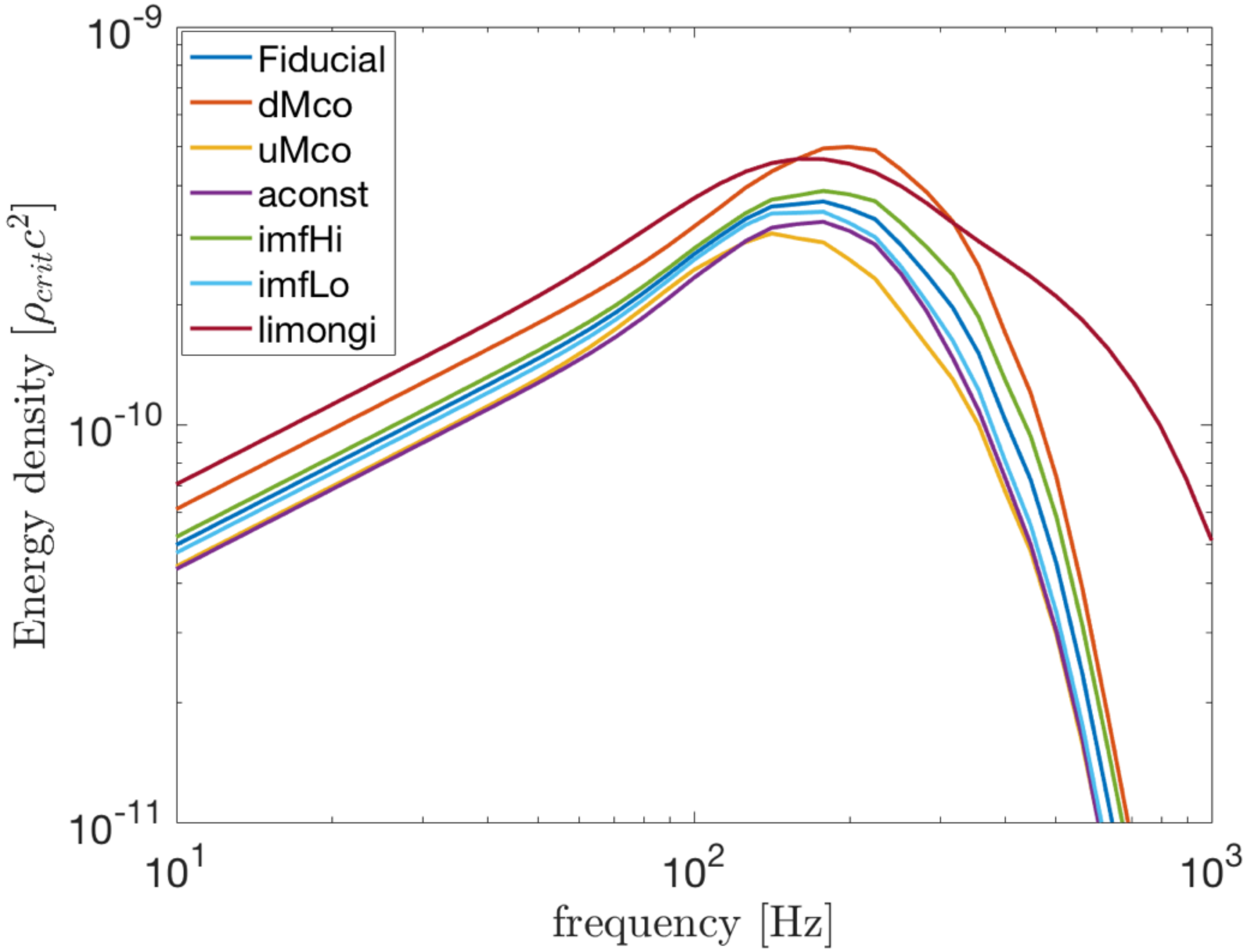
See also: Jenkins+ [1806.01718, 1810.13435]

Astrophysical dependencies

Does the stochastic background depend on the astrophysical model?

Varying: BH formation model, stellar IMF, delay times between binary formation and merger…

All models normalised to obtain the same number of LIGO/Virgo detections



Do the anisotropies depend on the astrophysical model?

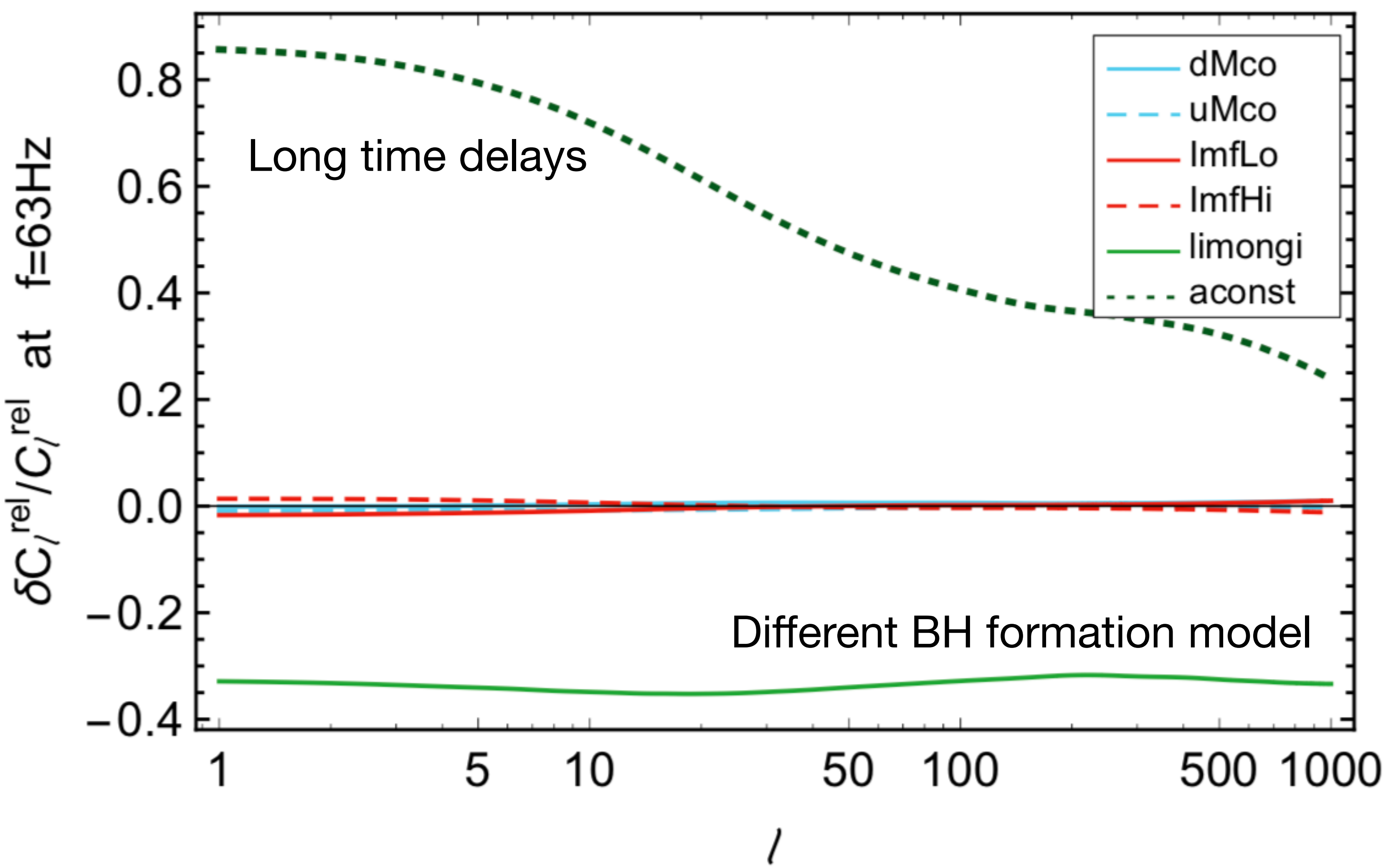
Astrophysical dependencies

$$C_\ell^{\rm rel} \equiv C_\ell \frac{(4\pi)^2}{\bar{\Omega}_{\rm GW}^2}$$

$$\frac{\delta C_\ell}{C_\ell} = \frac{C_\ell^{\rm mod} - C_\ell^{\rm ref}}{C_\ell^{\rm ref}}$$

dMco, uMco : cutoff on BH mass (PISN…)

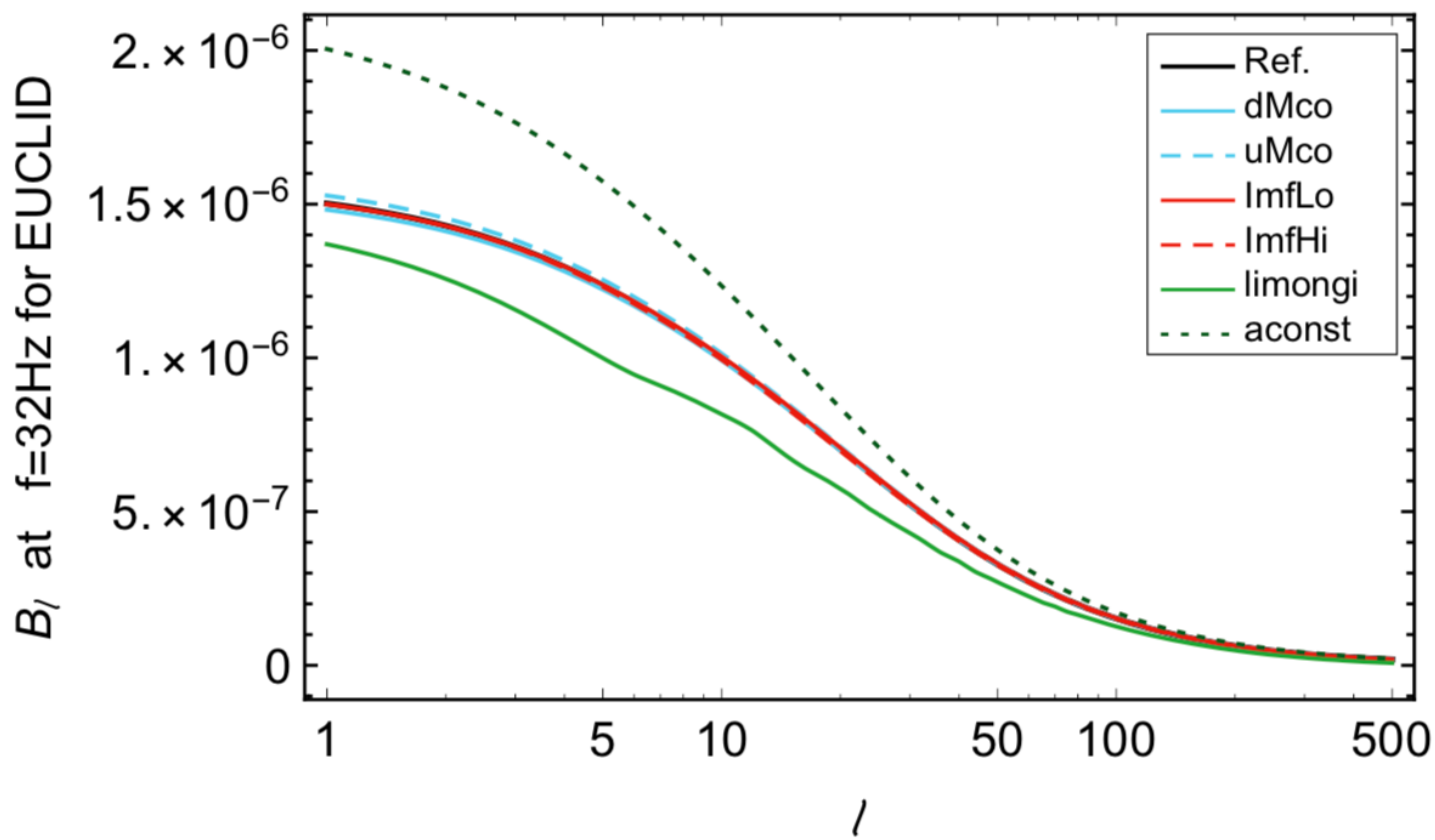
IMFLo, IMFHi : stellar initial mass function slope



Cross-correlation with weak lensing convergence

$$B_\ell(f) = \frac{2}{\pi} \int \mathrm{d}k\, k^2 \delta\Omega_\ell^*(k,f)\, \kappa_\ell(k)$$

Weak lensing convergence



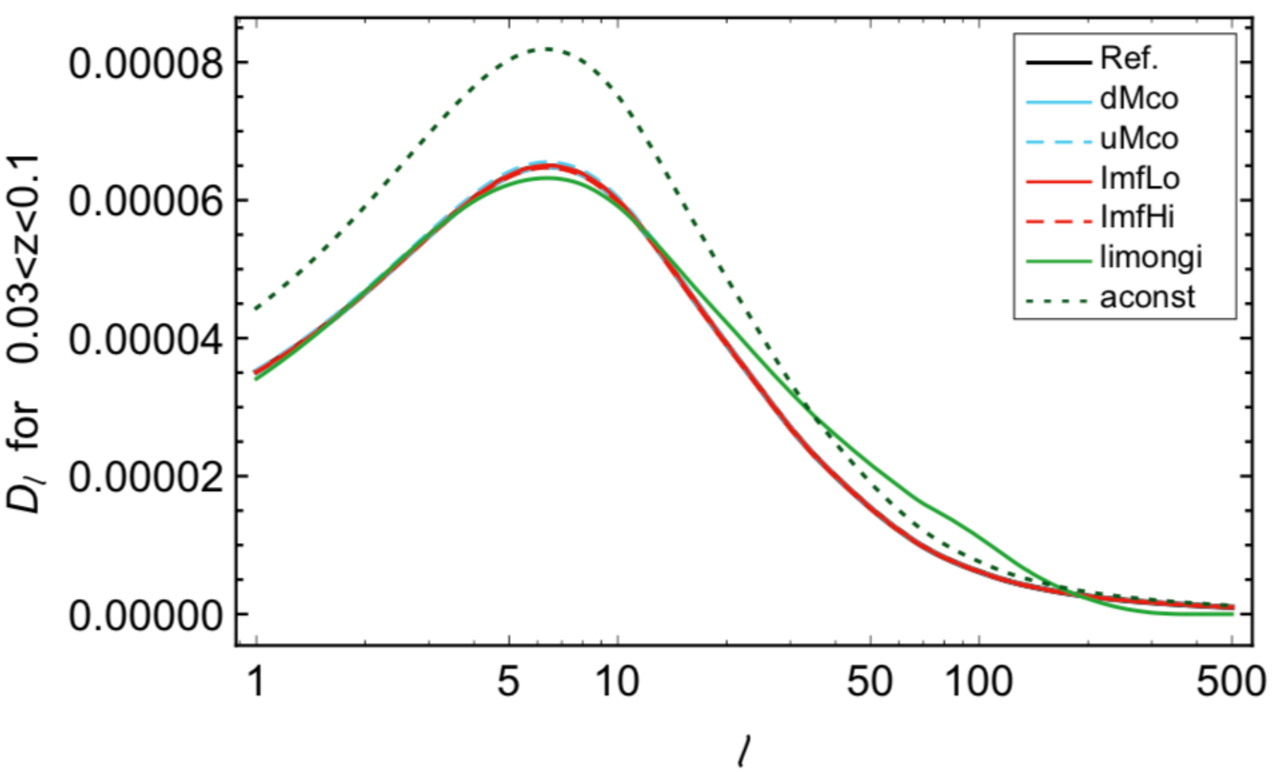
Cross-correlation with galaxy number counts

$$D_\ell(f,z) = \frac{2}{\pi}\int \mathrm{d}k\, k^2 \delta\Omega_\ell^*(k,f)\,\Delta_\ell(k,z)$$

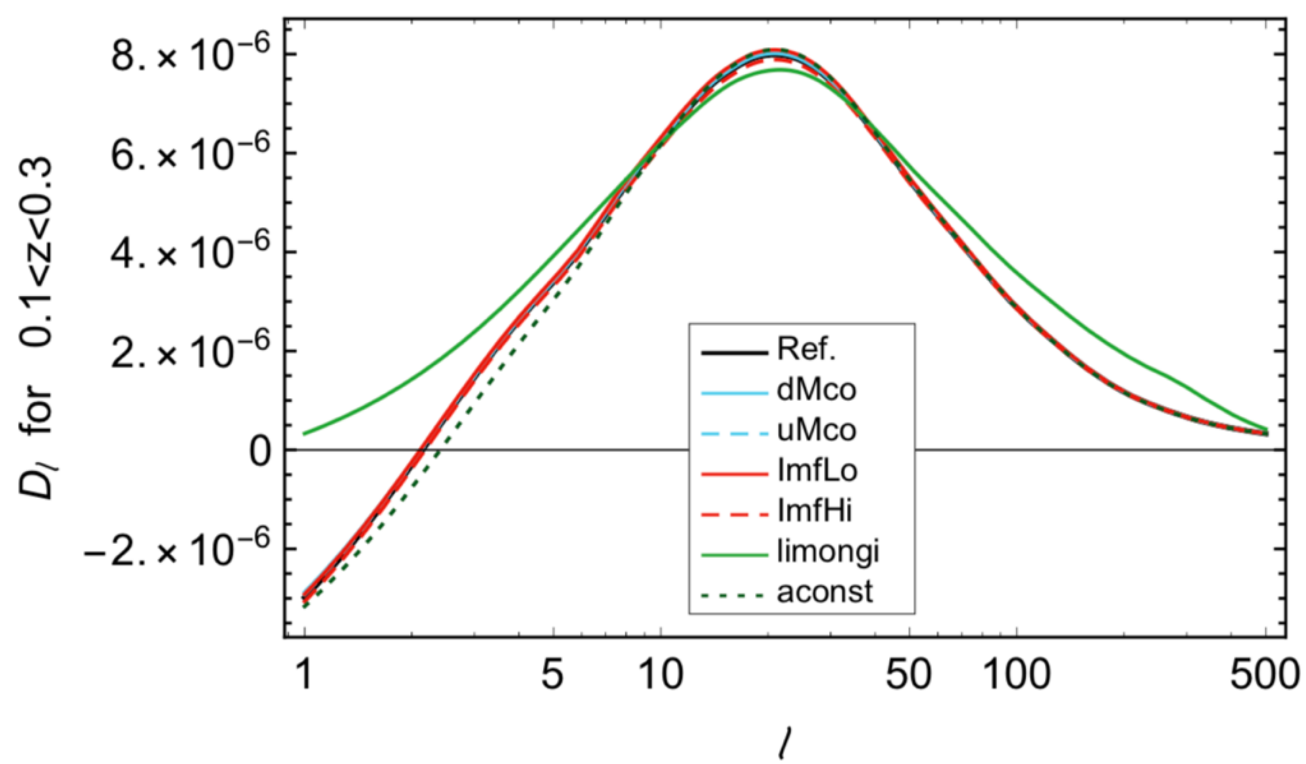
Galaxy number count overdensity

Bonvin & Durrer [1105.5280]

0.03<z<0.1



0.1<z<0.3



Peak multipole is related to the galaxy redshift range

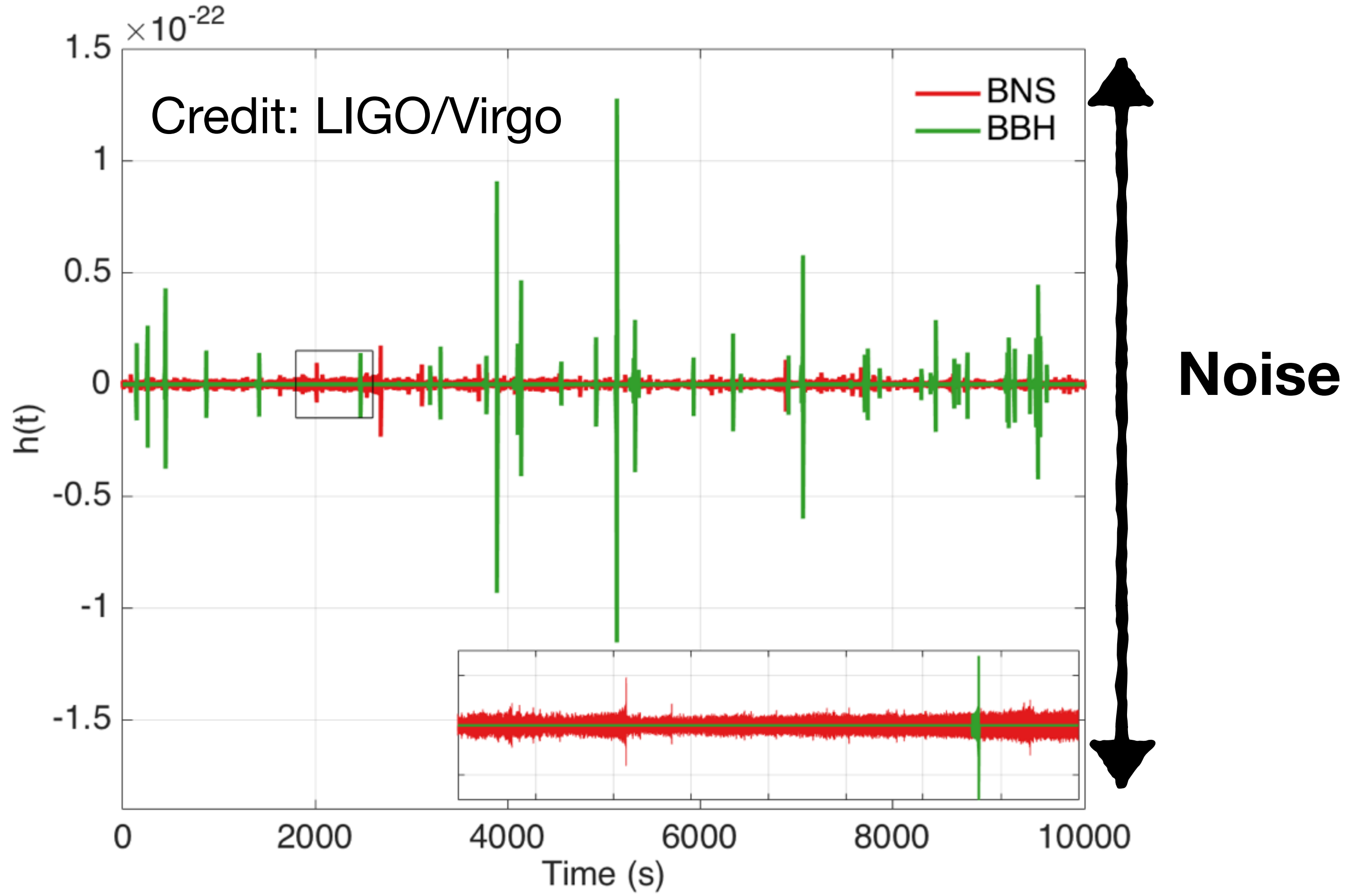
Conclusions

* The astrophysical stochastic gravitational-wave background is expected to have an anisotropic component, due to the inhomogeneous distribution of galaxies
* This anisotropic component depends both on the cosmology (formation and growth of large-scale structure…) and small-scale astrophysics (BH formation model…) and can give complementary information to the isotropic background
* Detection is difficult - signal may be dominated by shot noise!

Additional slides

Detection methods

Signal is stochastic and buried in noise!



Signal from each detector: $s(t) = n(t) + h(t)$

Cross-correlating outputs from two detectors and hoping noise is uncorrelated with the signal and between detectors:

$$\begin{aligned}\langle s_1(t)\; s_2(t)\rangle &= \langle (n_1(t)+h(t))\,(n_2(t)+h(t))\rangle \\ &= \langle n_1(t)\; n_2(t)\rangle + \langle n_1(t)\; h(t)\rangle + \langle h(t)\; n_2(t)\rangle + \langle h(t)\; h(t)\rangle \\ &\approx \langle h(t)\; h(t)\rangle \;\;,\end{aligned}$$

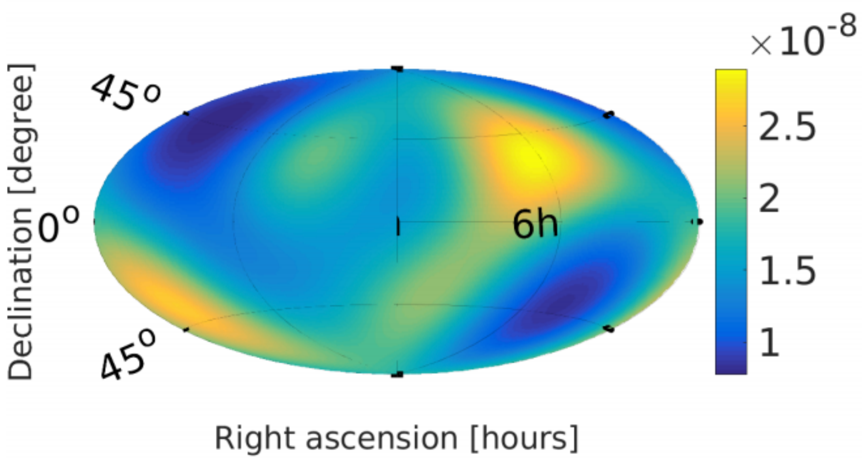
Anisotropic gravitational-wave background

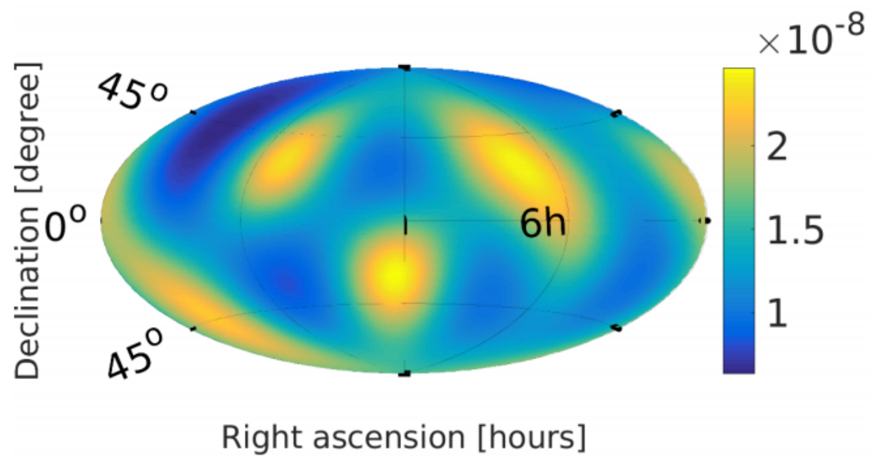
Energy density in gravitational waves at each point in the sky: $$\Omega(f,\Theta) = \Omega_\alpha(\Theta)\left(\frac{f}{f_{\rm ref}}\right)^\alpha$$

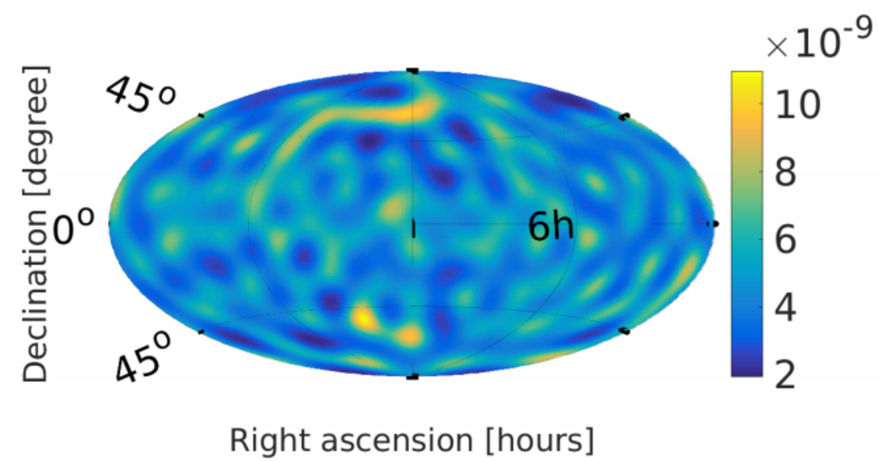
Resolution: $$\theta = \frac{c}{2df} \approx \frac{50\ \mathrm{Hz}}{f_\alpha}$$

[Flat energy density spectrum] | [Compact binary coalescences] | [Flat strain power spectrum density]

α=0 | α=2/3 | α=3







Upper limits (95% CL)

LIGO/Virgo [1903.08844]

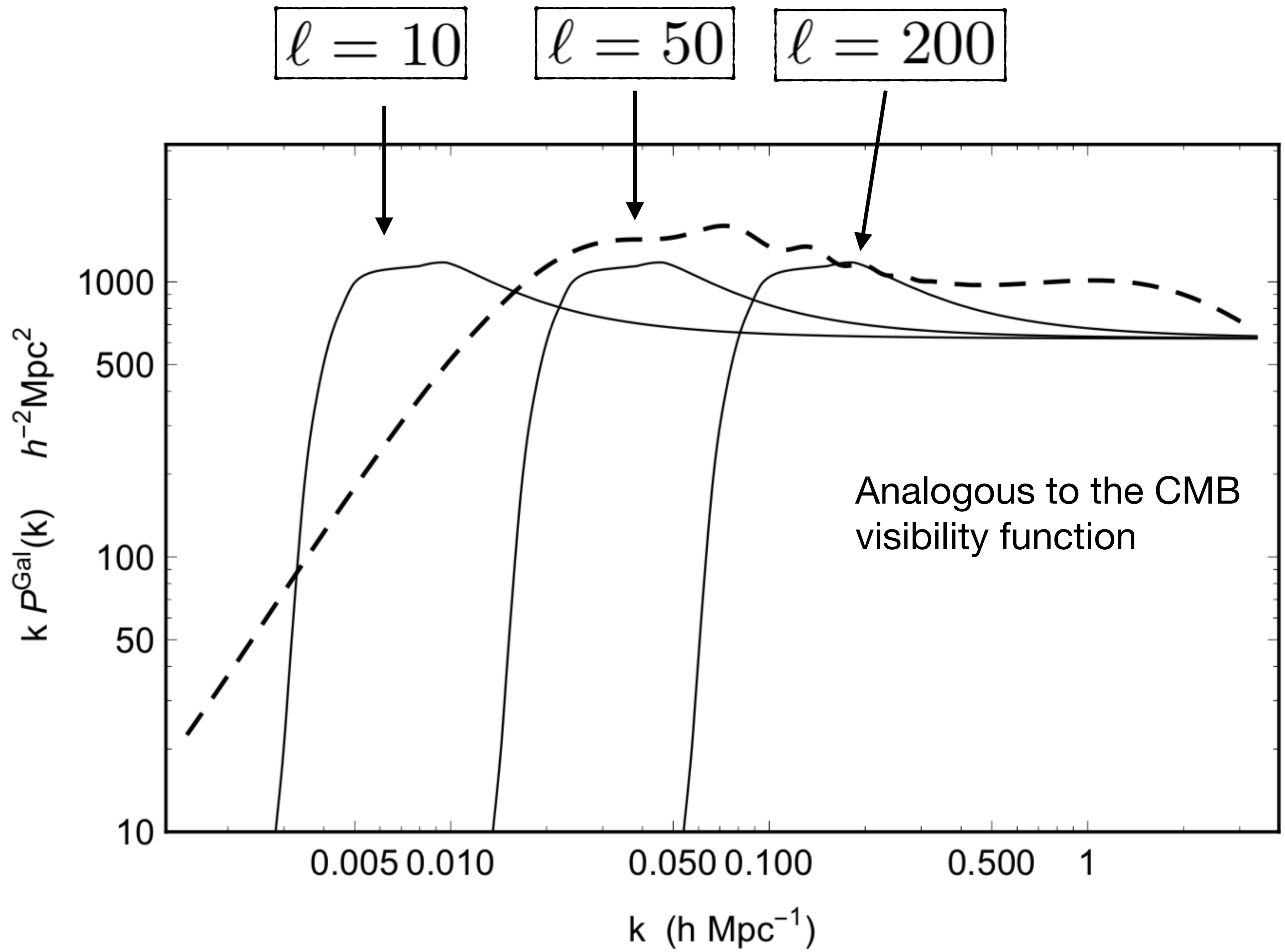
f_ref=25 Hz

Scale - multipole correspondence

$$C_\ell^{\rm Limber} \simeq \left(\ell+\tfrac{1}{2}\right)^{-1} \times \int {\rm d}\log k \, k P_{\rm Gal}(k) \left|\partial_\eta \left(\frac{\bar{\Omega}_{\rm GW}}{4\pi}\right)\right|^2$$

Limber constraint: $$k\,\Delta\eta = \ell + \frac{1}{2}$$

Window function evaluated at a conformal time to satisfy Limber constraint for:



Galaxy power spectrum today (multiplied by k) in dashed line and the window function $\sim |\partial_\eta \bar{\Omega}_{\rm GW}|^2$