The Initial Spin Probability Distribution of Primordial Black Holes

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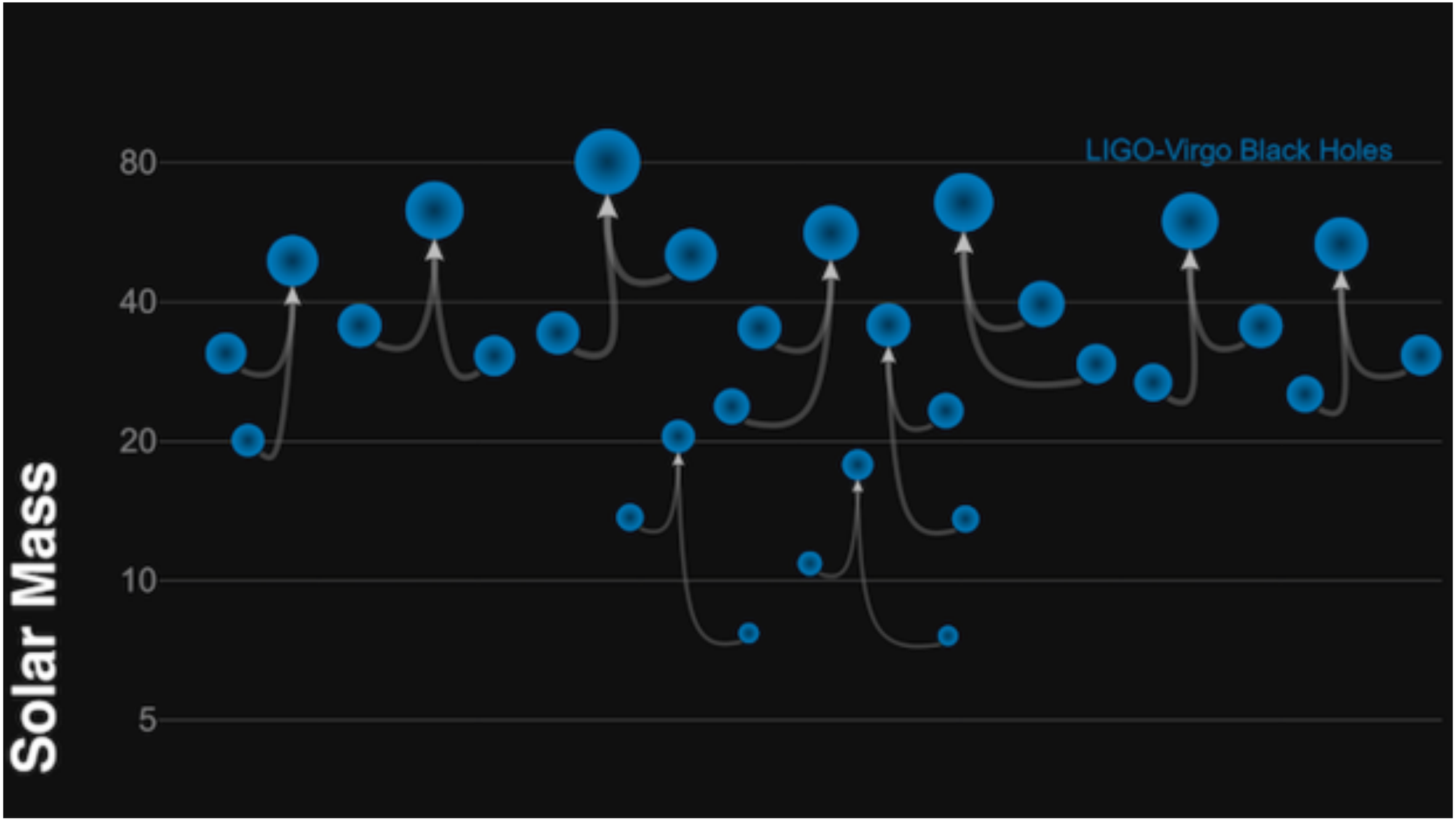
Valerio De Luca

De Luca, Desjacques, Franciolini, Malhotra, Riotto [1903.01179] (JCAP)

Motivation



LIGO

Astrophysical vs Primordial BH

Astrophysical BHs

They form from the gravitational collapse of a star, with mass bigger than the Chandrasekhar mass,

$$M > \mathcal{O}(1) M_\odot$$

PBHs

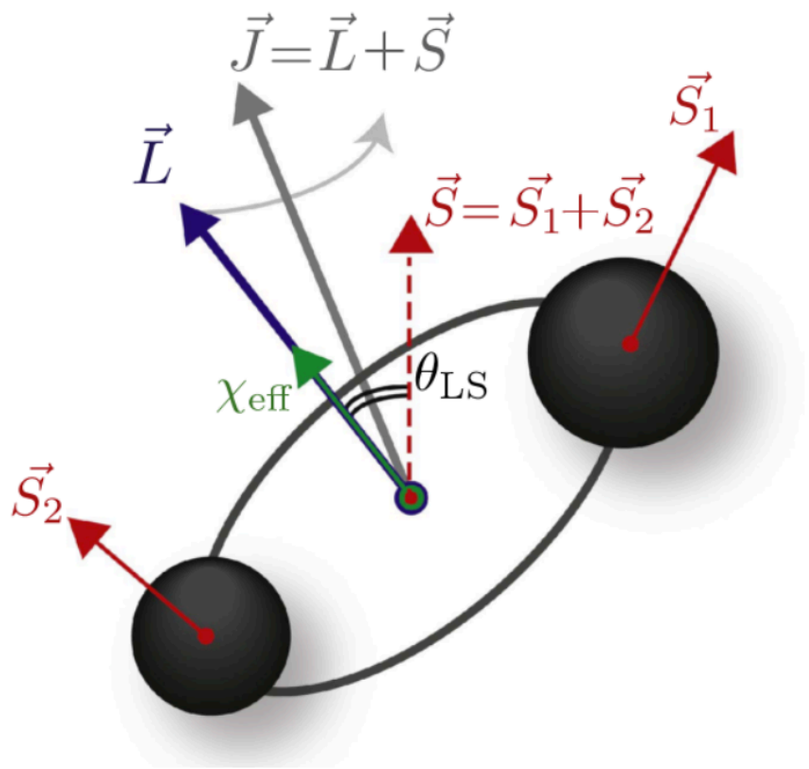
They form in the early universe from the collapse of large inhomogeneities and are not evaporated until today,

$$M > 10^{-18} M_\odot$$

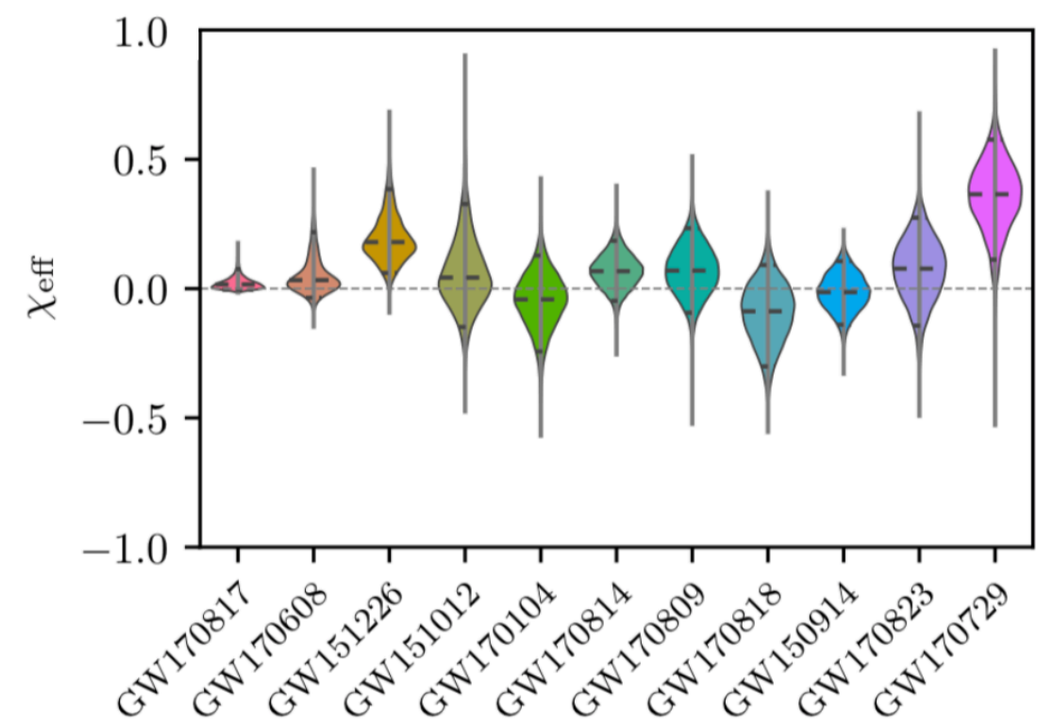
One can distinguish between them since PBHs are expected to be born with low spins

BHs merging

In the merging of two black holes, the gravitational waveforms depend on the final spin state and the orbital angular momentum



Rodriguez et al. (2016)



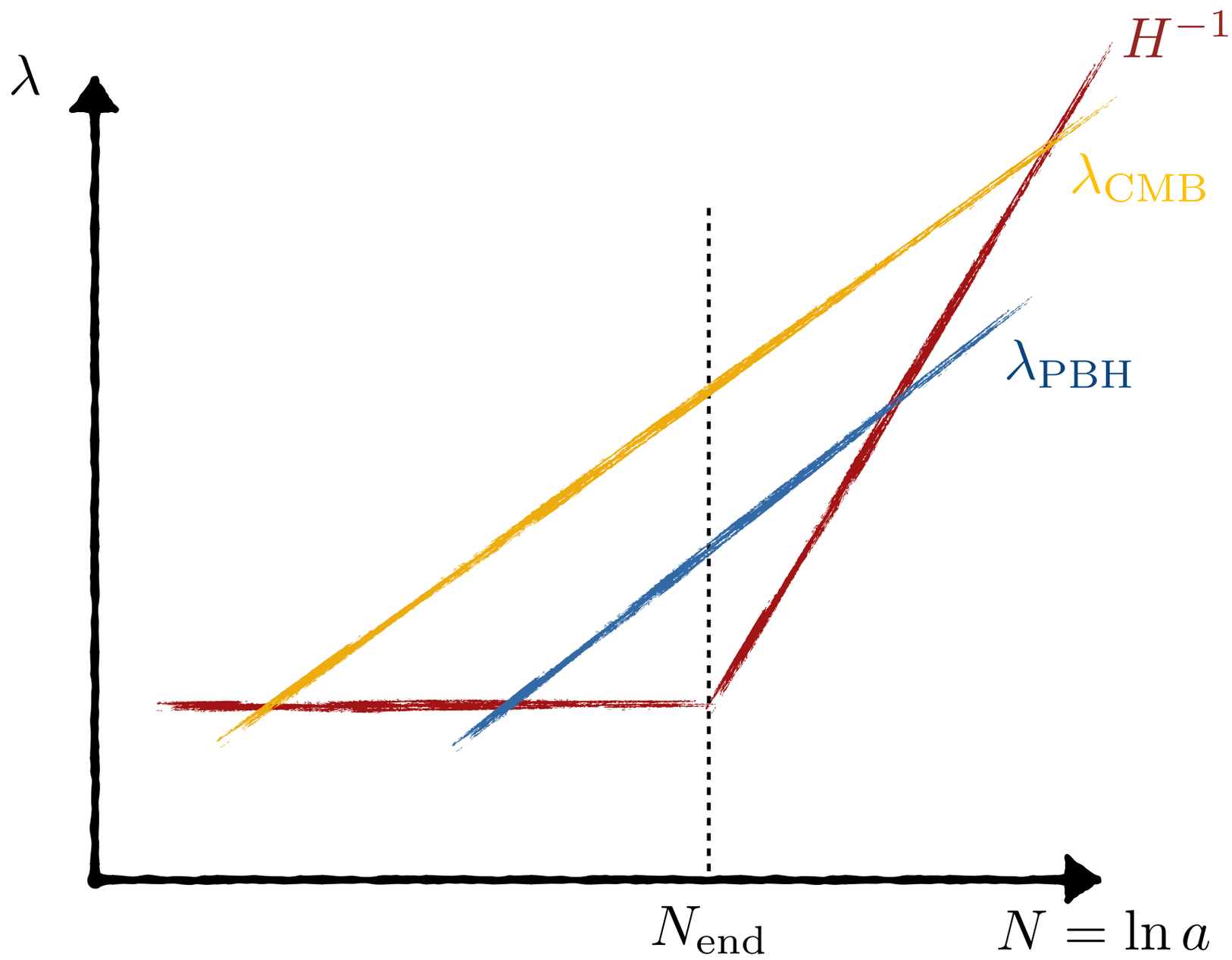
LIGO,VIRGO (2018)

$$\chi_{\rm eff} = \frac{M_1\vec{S}_1 + M_2\vec{S}_2}{M_1 + M_2} \cdot \vec{L} \simeq 0$$

Primordial Black Holes

PBH: inflation

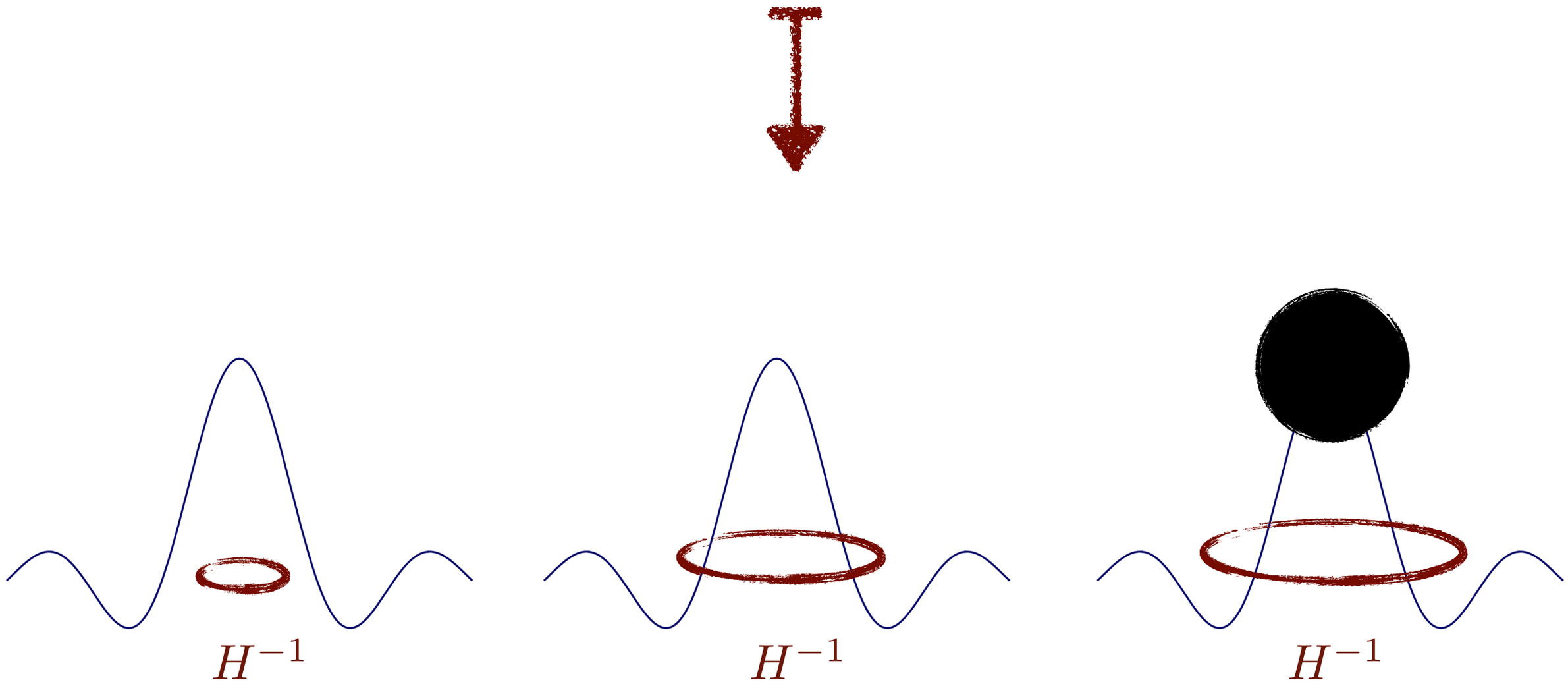
Inflation can lead to an enhancement of the comoving curvature perturbation $\mathcal{R}$ on small scales, which is then transferred to radiation during reheating and will collapse in PBH at horizon re-entry



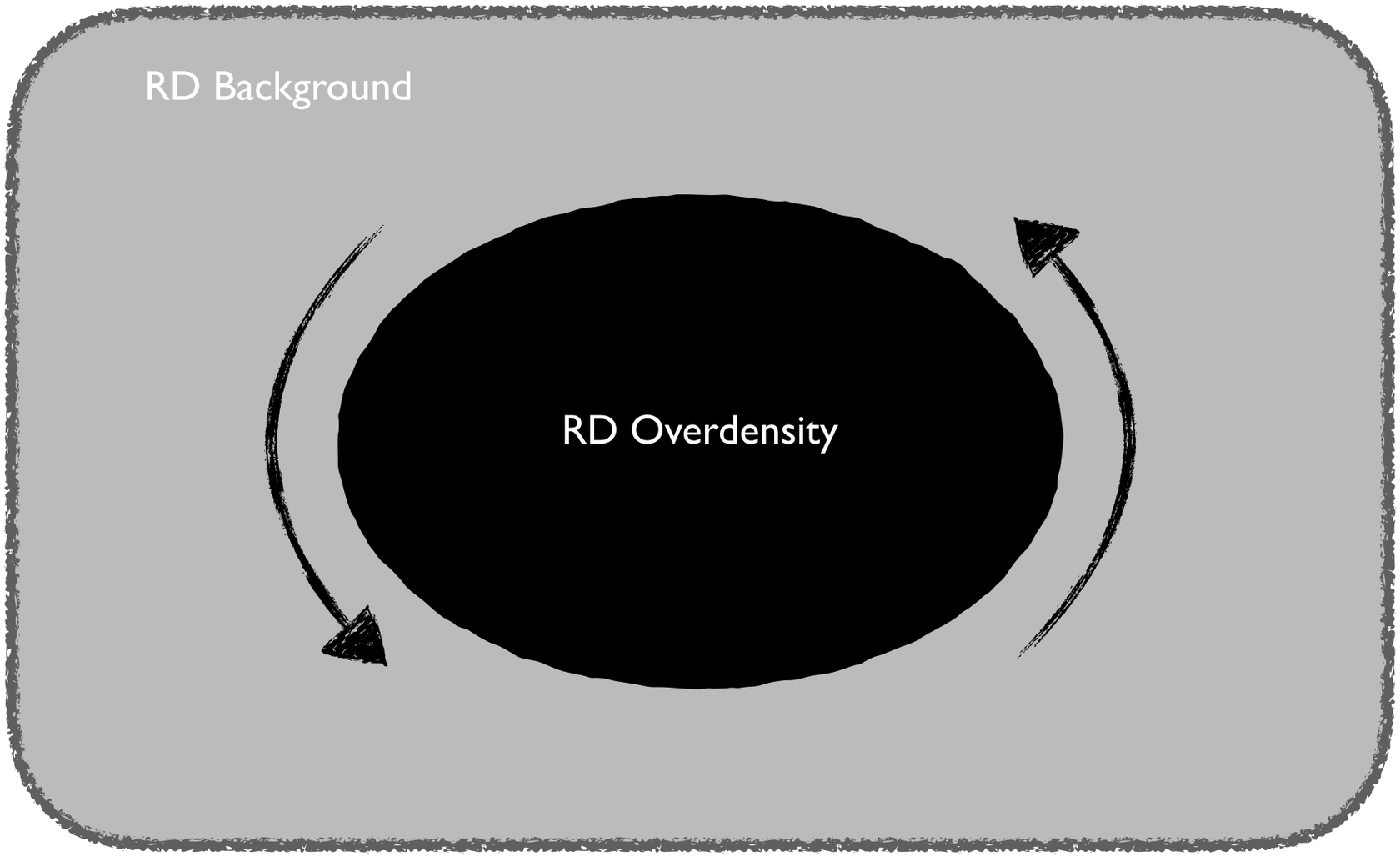
PBH: density contrast

PBHs are originated from peaks of the density contrast:

$$\delta \equiv \frac{\delta\rho}{\rho} > \delta_c$$



PBH Spin: formation process in RD



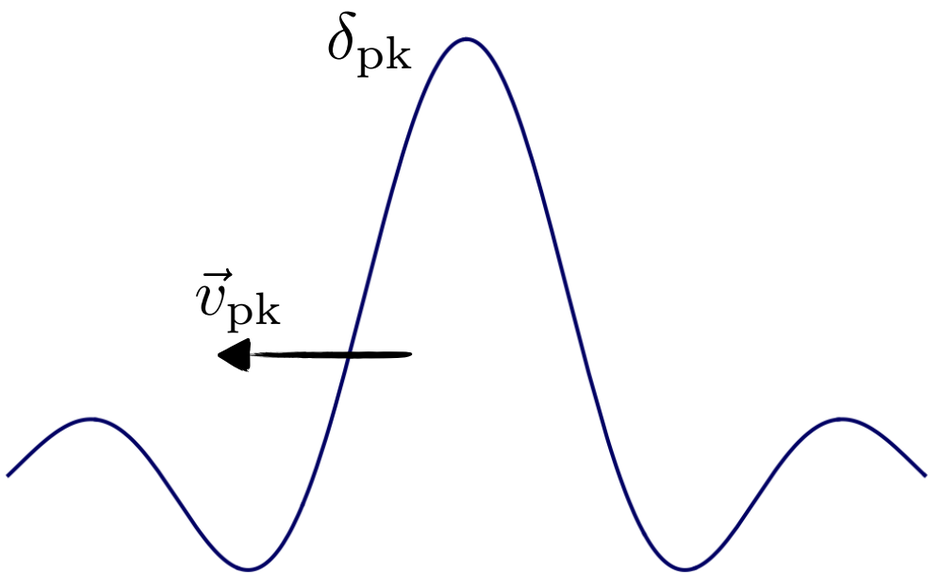
- PBHs originate from local maxima of the radiation overdensity field
- Spin originates from the action of torques generated by gravitational tidal forces upon horizon re-entry
- 1st order effect in perturbation theory due to non-spherical collapse

Computation

Angular momentum

$$S_i = \frac{4}{3} a^4(\eta) \epsilon_{ijk} \overline{\rho}_{\rm rad}(\eta) \int_{V_{\rm e}} d^3x (x - x_{\rm pk})^j (v - v_{\rm pk})^k$$

Volume of the ellipsoid



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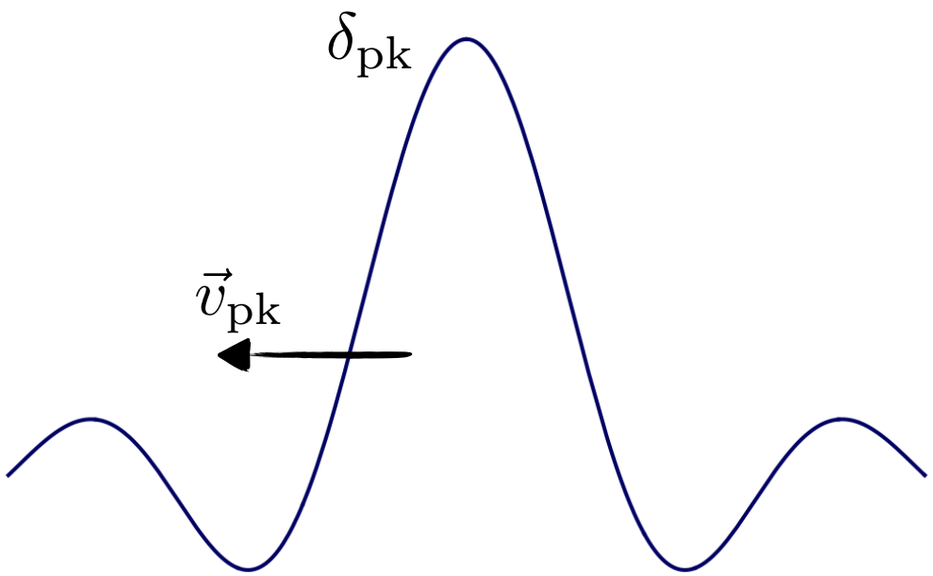

Peak theory

$$\delta(\vec{x}) \simeq \delta_{\rm pk} + \frac{1}{2} \zeta_{ij} (x - x_{\rm pk})^i (x - x_{\rm pk})^j, \quad \zeta_{ij} = \left. \frac{\partial^2 \delta}{\partial x^i \partial x^j} \right|_{\rm pk}$$

$$(v - v_{\rm pk})^k \simeq v^k_l (x - x_{\rm pk})^l, \quad v^k_l = \frac{\partial v^k}{\partial x^j}$$

Computation

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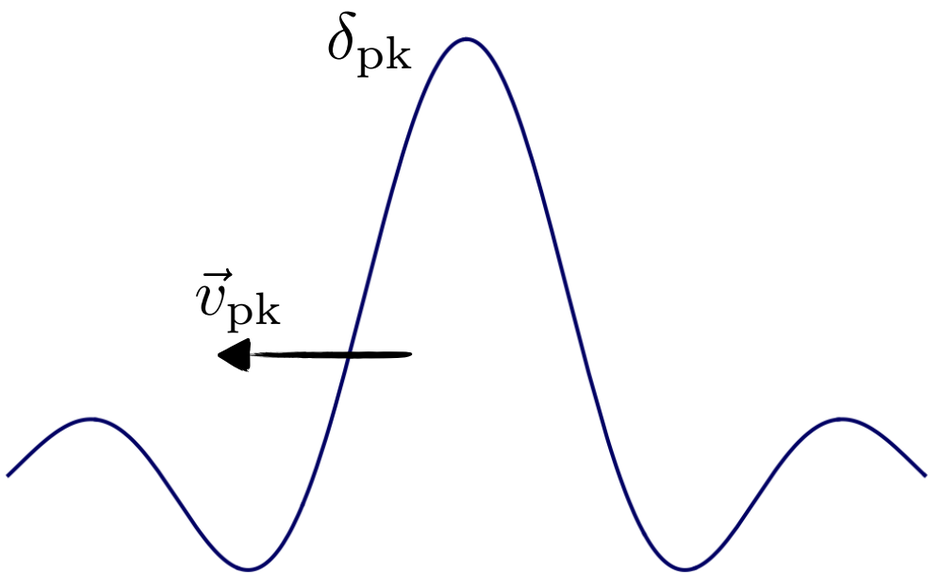

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$$(v - v_{\rm pk})^k \simeq v_l^k (x - x_{\rm pk})^l, \quad v_l^k = \frac{\partial v^k}{\partial x^j}$$

Perform a rotation of the coordinate axes to be aligned with the principal axes of the ellipsoid and find the boundary solutions for the constant-overdensity ellipsoid

$$a_i^2 \simeq 2 \frac{\sigma_\delta}{\sigma_\zeta} \frac{1}{\lambda_i} \nu$$

$$\gamma = \frac{\sigma_\times^2}{\sigma_\delta \sigma_\zeta}$$

Power spectrum width

$$\lambda_i \simeq \frac{\gamma \nu}{3}(1 + \epsilon_i), \quad \nu = \frac{\delta_{\rm pk}}{\sigma_\delta}, \quad v_l^k$$

Peak theory parameters

Computation

Angular momentum + Peak theory

Heavens and Peacock (1988)

Non-spherical

$$\vec{S}(\eta) = \frac{16\pi}{45} a^4(\eta)\overline{\rho}_{\rm rad}(\eta) a_1 a_2 a_3 \left([a_2^2 - a_3^2]v_{23}, [a_3^2 - a_1^2]v_{13}, [a_1^2 - a_2^2]v_{12}\right)$$

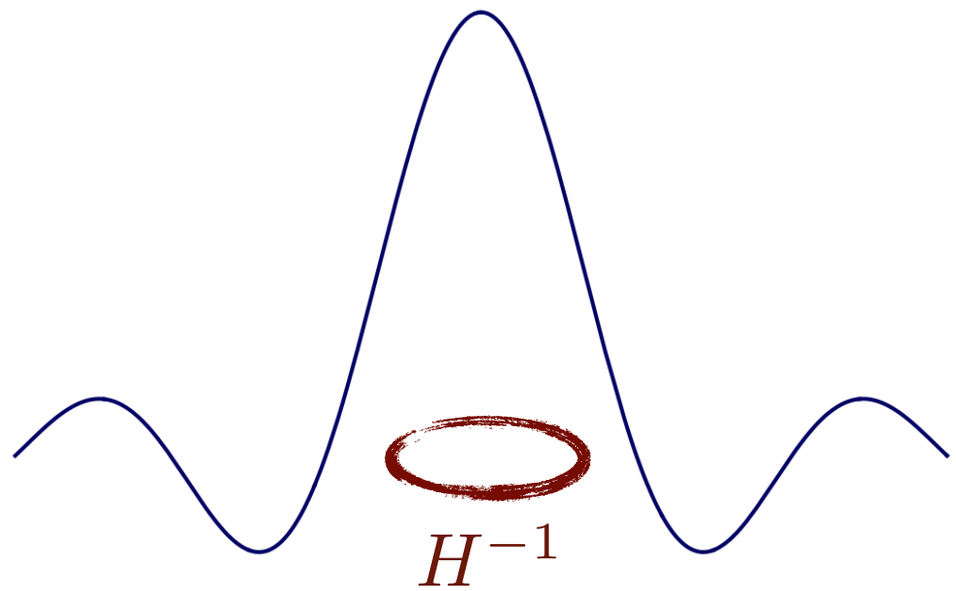
Velocity shear misaligned with the inertia tensor

$$\vec{S}(\eta) = S_{\rm ref}(\eta)\vec{s}_e$$

Time dependent part, independent on the shape of the peak

Time independent part, dependent on the shape of the peak

Time dependence: super-horizon



Using the separate universe approach, at any order in perturbation theory

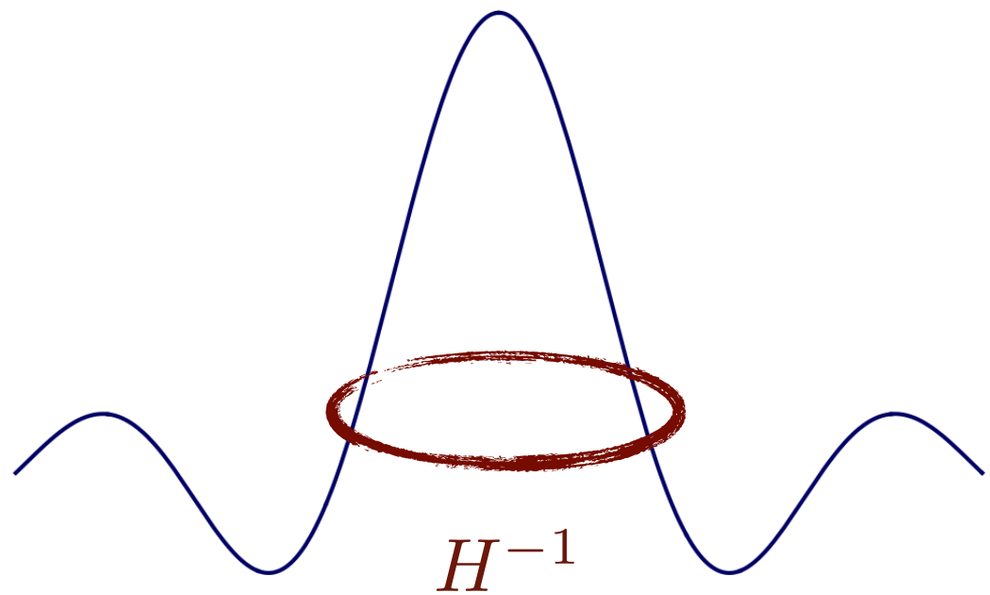
$$\delta(\vec{x},\eta) = -\frac{4}{3\mathcal{H}^2} e^{-5\mathcal{R}(\vec{x})/2} \nabla^2 e^{\mathcal{R}(\vec{x})/2}$$

$$v^i(\vec{x},\eta) = \frac{1}{12\mathcal{H}} \partial^i \delta(\vec{x},\eta)$$

$\downarrow$

Spin zero: velocity shear aligned with the inertia tensor

Time dependence: horizon crossing



The spin can grow briefly after horizon crossing due to the linear tidal torque until the perturbation decouples from the background

$$v_{ij}(\vec{k}, \eta_{\rm H}) = -3\mathcal{R}\frac{k_i k_j}{k} c_s \left(\frac{k}{k_{\rm H}}\right)^2 \frac{\sin(k c_s/k_{\rm H})}{6 + (k/k_{\rm H})^2}$$

$\downarrow$

Spin non-zero: velocity shear misaligned with the inertia tensor

Time dependence: estimate

We evaluate the spin at the turnaround (initial time)

close to the horizon crossing time

Kerr parameter $$a_s(\eta_{\rm H}) = \frac{S_{\rm ref}(\eta_{\rm H})}{GM^2} s_e = \left[\frac{\Omega_{\rm dm}\sigma_\delta}{\pi}\right] s_e$$

The Kerr parameter distribution depends on the peak distribution

$$s_e = s_e(\nu, \lambda_i, v_{jk})$$

We resort to peak theory to find it

Peak distribution

Peak theory: sixteen correlated variables

BBKS (1986)

$$V_i : \ \delta, \quad \zeta_i = \frac{\partial \delta}{\partial x_i}, \quad \zeta_{ij} = \frac{\partial^2 \delta}{\partial x_i \partial x_j}, \quad v_i^j = \frac{\partial v^j}{\partial x^i}$$

Joint Gaussian distribution

$$f(V_i) d^{16} V_i = \frac{1}{(2\pi)^8 |\mathbf{M}|^{1/2}} e^{-\frac{1}{2}(V_i - \langle V_i \rangle) \mathbf{M}_{ij}^{-1} (V_j - \langle V_j \rangle)} d^{16} V_i,$$

$$\mathbf{M}_{ij} = \langle (V_i - \langle V_i \rangle)(V_j - \langle V_j \rangle) \rangle$$

Peak distribution

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$$\mathbf{M}_{ij} = \langle (V_i - \langle V_i \rangle)(V_j - \langle V_j \rangle) \rangle$$

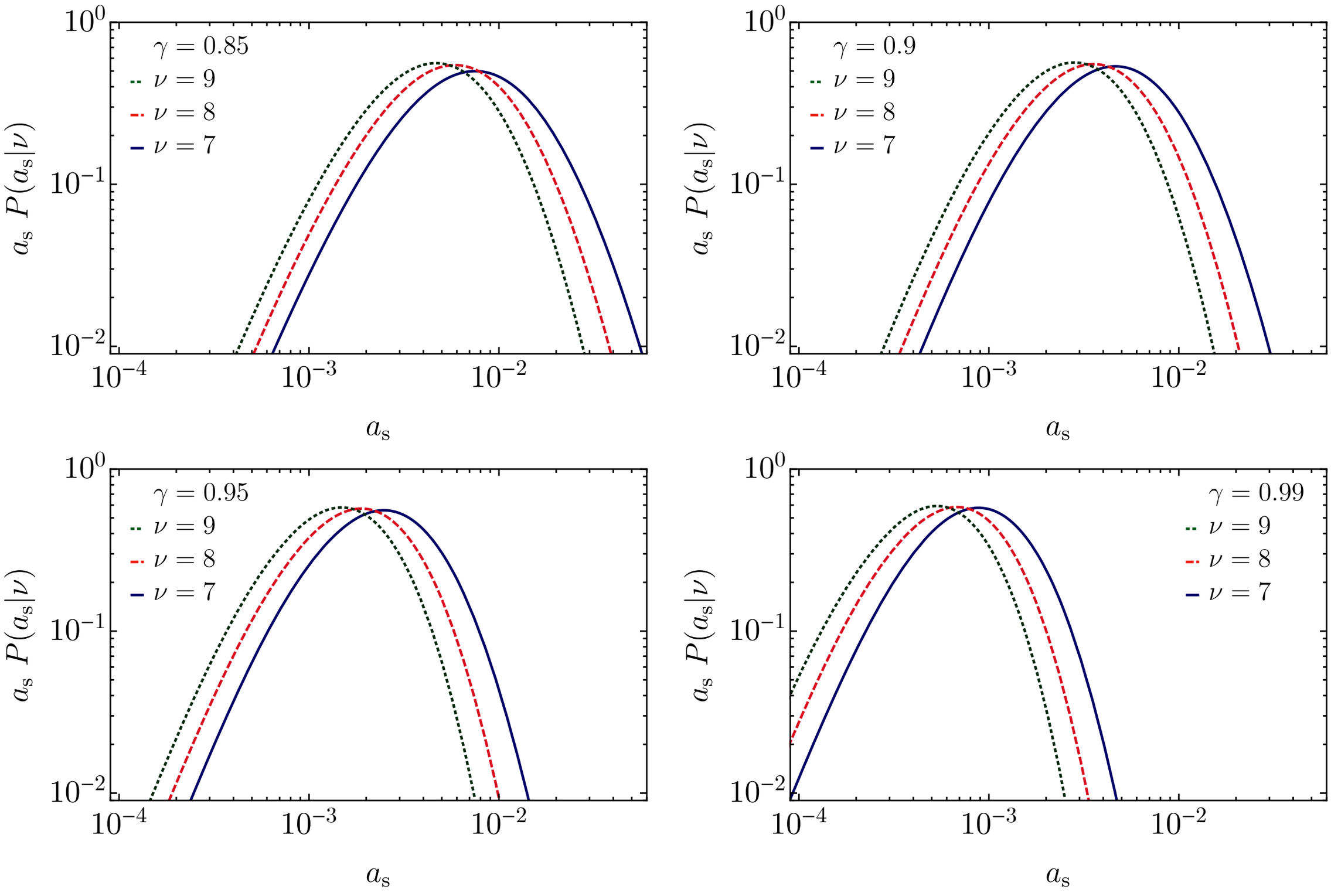
$$\downarrow$$

$$f(\nu, \lambda_i, v_{jk}) d\nu d^3\lambda_i d^3 v_{jk}$$

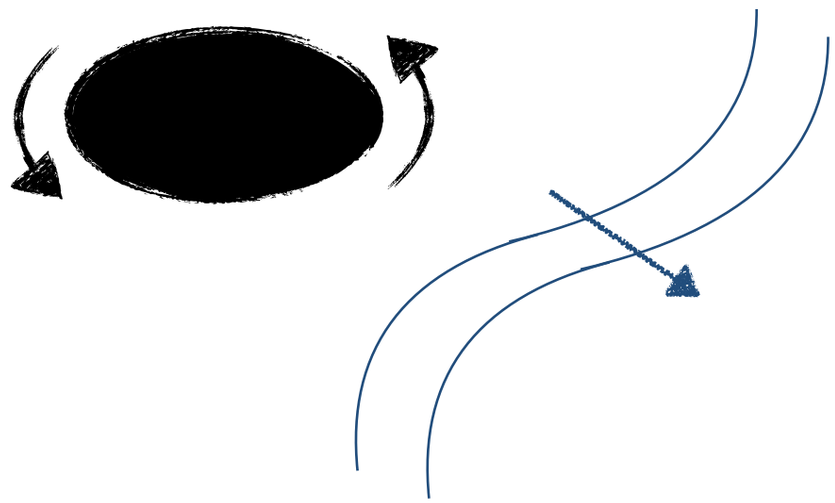
Properties:

- Distribution suppressed for high peaks (high peaks tend to be spherical)
- In the limit of monochromatic power spectrum ($\gamma \to 1$) the velocity shear is aligned with the inertia tensor

Conditional probability distribution

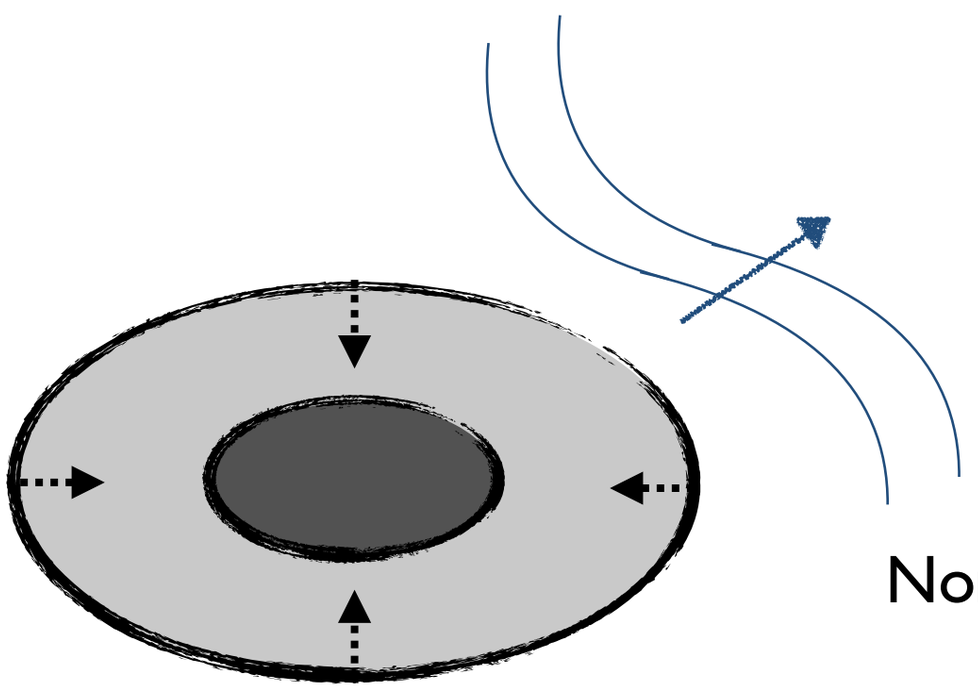


Application: GWs from peaks



Non-spherical rotation of a rigid body

GWs emission!



Non-spherical contraction leads to a quadrupole moment

De Luca et al. [1905.13459]

Conclusions

We computed the spin probability distribution of PBHs at formation time in RD

↓

Non spherical collapse and misalignment between the inertial tensor and the velocity shear lead to a non-zero spin

$$a_s = \frac{\Omega_{\rm dm}\sigma_\delta}{\pi}\sqrt{1-\gamma^2} \sim 10^{-2}\sqrt{1-\gamma^2}$$

Future directions

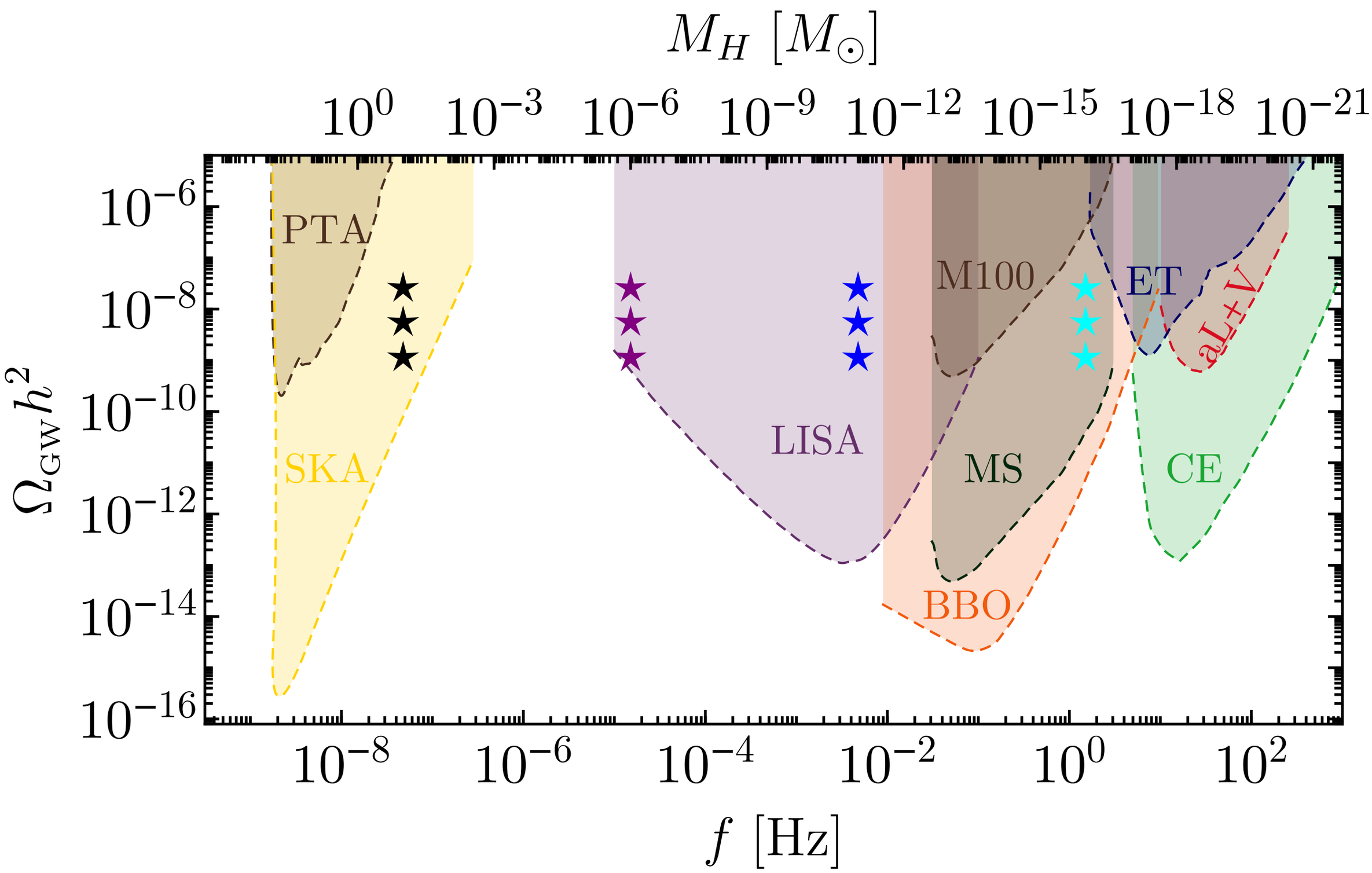
- Numerical simulation for non-spherical collapse
- PBH spin evolution

∗ In MD the result can be different (see for example Harada et al. [1707.03595])

THANK YOU !

Backup slides

Application: GWs from peaks



Spin evolution in RD: rough estimate

Accretion of the background radiation has the effect of reducing the spin

Chiba and Yokoyama (2018)

$$\dot{S} \sim -M F_{\rm rad} \sim -H^2 M^3 a_s$$

Force generating torque $F_{\rm rad} \sim \overline{\rho}_{\rm rad} M^2 a_s$

$$a_s \sim a_{s,\rm H} \exp\left[\mathcal{O}(1)\left(\frac{t_{\rm H}}{t} - 1\right)\right]$$

PBH spin evolution in RD does not appreciably affect our result